Seismic Response of Alluvial Valleys to SH Waves

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Abstract. This paper presents a theoretical study on the seismic response of alluvial valleys. The considered model consists of a two-dimensional elastic inclusion of arbitrary shape embedded in a stiffer half-plane excited by vertically or obliquely incident SH waves. Computations are conducted using a procedure based on the boundary element method. As known, this numerical technique is well suited to deal with wave propagation in infinite media as it avoids the introduction of fictitious boundaries and reduces by one the dimensions of the problem. This provides significant advantages from a computational point of view. A one-dimensional closed form solution is also used for comparison, and the most significant differences between the results obtained using the two methods are highlighted.

Keyword: microzonation studies, site effects, one and two-dimensional analyses.

INTRODUCTION

Macroseismic observations during various historical and recent earthquakes have shown that the local geological conditions can generate large amplifications and important spatial variations in the ground motion. Consequently, prediction of the local site effects is of great importance for the microzonation studies and the analysis of the seismic response of engineering works. To this purpose, it is necessary to understand the physical phenomena associated with the seismic wave propagation, and at the same time to develop methods capable of predicting reasonably the ground motion at a given site. In many situations, the simple shear beam model is not completely appropriate, thus use of two and three dimensional solutions is generally mandatory. In this context, both analytical and numerical methods are available.

The analytical solutions deal with simple geometric situations, such as semi-cylindrical or semi-elliptical alluvial valleys subjected to incident SH-waves [1-2]. Their application has allowed the role of the parameters involved to be highlighted. In addition, they represent unfailing references against which numerical solutions can be tested.

The numerical methods that are widely used for the analysis of seismic wave propagation can be classified into domain, boundary and asymptotic methods. The finite difference method [3] and the finite element method [4] fall within the first class as these numerical techniques require that the entire domain be discretized. On the

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other hand, when use is made of the methods falling within the second class such as the boundary element method, discretization of the boundaries needs only to be performed. In this context, two main approaches can be distinguished: one is based on the use of complete systems of solutions [5], and the other on the boundary integral equations [6]. Finally, the asymptotic methods are useful when the solution has an interest in the high frequency range and the diffraction effects may be ignored [7]. These methods solve the differential equations governing wave propagation under two or three-dimensional conditions [8-12]. However, engineering applications usually rely on the one-dimensional analysis to predict the surface motion at a site. As a consequence, the effects of the geometric shape and limited lateral extent of the soil deposit under consideration are completely ignored in the analyses.

In this paper, the boundary element method is used to analyse SH wave scattering by alluvial valley of arbitrary shape under conditions of plane strains. This numerical technique is well suited to deal with wave propagation problems, because it avoids the introduction of fictitious boundaries and reduces by one the dimensions of the problem. This provides significant advantages from a computational point of view. However, the involved materials are assumed to behave as linear elastic media. To assess the accuracy of the proposed method, the results in terms of surface displacement amplitude are compared with those calculated using the analytical solution derived by Trifunac [1] for semi-cylindrical alluvial valleys. In addition, a simple one-dimensional solution is also considered with the purpose of highlighting the main differences between the results.

PROBLEM FORMULATION

The problem considered in this study concerns a two-dimensional alluvial valley of arbitrary shape embedded in a half-plane excited by incident harmonic SH waves with frequency ω, and angle of incidence γ (Fig. 1). The material of the valley and that of the half-plane is assumed to be homogeneous, isotropic and linearly elastic. It is also assumed that the valley is perfectly bonded to the half-plane at the interface Γ_j indicated in Fig. 1. Under conditions of plane strain, the differential equation governing the propagation of SH waves is:

$$\frac{\delta^2 \overline{u}_j}{\delta x^2} + \frac{\delta^2 \overline{u}_j}{\delta y^2} = \frac{1}{\beta_j^2} \frac{\delta^2 \overline{u}_j}{\delta t^2} \tag{1}$$

where x and y are the spatial coordinates, t is the time, $\overline{u}_j$ indicates the out-plane displacement field of the half-plane (when j=1) or the valley (when j=2), $\beta_j = (G_j/\rho_j)^{1/2}$ is the shear wave velocity, G_j is the shear modulus and ρ_j is the mass density of the half-plane (j=1) or the valley (j=2). For harmonic motion (i.e. $\overline{u}_j = u_j e^{i\omega t}$, with $i = \sqrt{-1}$), Eq. 1 reduces to the Helmholtz equation, that is:

$$\frac{\delta^2 u_j}{\delta x^2}+\frac{\delta^2 u_j}{\delta y^2}+k_j^{\,2} u_j=0 \tag{2}$$

in which $k_j=\omega/\beta_j$ and ω is the excitation frequency. Owing to the linearity of the problem, the displacement field of the half-plane can be cast in the form:

$$u_1 = u_o + u_d \tag{3}$$

where u_o is the amplitude of the displacement field due to the incident and reflected waves, and u_d is the displacement amplitude due to the diffracted waves caused by the presence of the valley. The former represents the free-field motion amplitude and is provided by the following equation, under the assumption that the displacement amplitude due to the incident waves is equal to 1:

$$u_o = 2\cos(k_1\, y\cos\gamma)e^{ik_1\, x\sin\gamma} \tag{4}$$

The displacement of the valley, u_2, is due to the diffracted waves at the boundary Γj. In the case under consideration, the unknown fields are u_d and u_2. Both these fields satisfy Eq. 2.

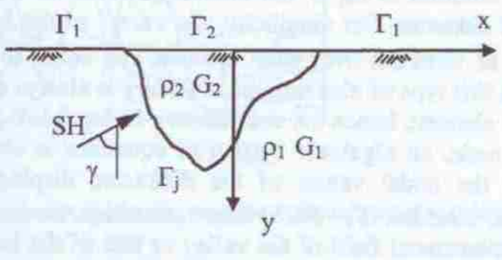

FIGURE 1. The problem considered

In the present study, the solution to Eq. 2 is achieved using the boundary element method. Following Brebbia et al. [6], this method is based on the integral equation:

$$c(P)u(P)=\int_{\Gamma}\left[u^*(P,Q)\frac{\delta u(Q)}{\delta n}-u(Q)\frac{\delta u^*(P,Q)}{\delta n}\right]\mathrm{d}\Gamma \tag{5}$$

where P is the point under consideration and Q is a point located on the boundary Γ (where $\Gamma=\Gamma_1 \cup \Gamma_2 \cup \Gamma_j$, as shown in Fig. 1), c is a coefficient depending on the position of P, n indicates the normal direction to Γ, and u^* is the fundamental solution of the Helmholtz equation, which for two dimensions results in

$$u^* = \frac{1}{4} i\, \mathrm{H}_o^1(k_j r) \tag{6}$$

in which H_o^1 is the Hankel function of first kind and zero order, and r is the distance between P and Q. It is worth noting that Eq. 5 only involves boundary integrals. In addition, it directly accounts for the radiation condition for infinite media owing to the presence of the fundamental solution. This avoids the introduction of fictitious boundaries, unlike other numerical techniques such as the finite element method or the finite difference method. As suggested by Kobayashi [13] and Conte et al. [14], it is more convenient, from a computational point of view, to use the following equation instead of Eq. 6:

$$u^{*} = \frac{1}{4} i \left[H_{o}^{1}(k_{j} r) + H_{o}^{1}(k_{j} r') \right] \tag{7}$$

where r' is the distance between P and the image point of Q with respect to the horizontal ground surface (Γ_1 and Γ_2 in Fig. 1). Unlike Eq. (6), this latter directly satisfies the boundary condition on Γ_1 and Γ_2 (i.e., $\partial u_d / \partial n = 0$ at Γ_1, and $\partial u_2 / \partial n = 0$ at Γ_2). As a consequence, only the interface Γ_j needs to be considered in Eq. 5.

The basic integral equation (Eq.5) is first applied separately to the valley (in terms of u_2) and the half-plane (in terms of u_d), and then the compatibility and equilibrium conditions are enforced at the interface Γ_j. These conditions are expressed by the equations:

$$u_1 = u_2 \tag{8a}$$

$$G_1 \left(\frac{\delta u_1}{\delta n} \right) = -G_2 \left(\frac{\delta u_2}{\delta n} \right) \tag{8b}$$

To achieve a solution to Eq. 5, the interface Γ_j is divided into a finite number of one-dimensional elements. For simplicity, the values of the unknowns u and $\partial u / \partial n$ are assumed to be constant over each element and equal to the value at the mid-element node. In this type of element, the boundary is always smooth as the node is at the centre of the element, hence the coefficient c in Eq. 5 is 0.5 [6]. After discretizing Eq. 5 for each node, an algebraic system of equations is obtained, the solution of which provides the nodal values of the diffracted displacement and its normal derivative at the interface Γ_j. Once these quantities are known, it is possible to calculate the displacement field of the valley or that of the half-plane using Eq. 2 in which c is 1, together with Eqs. 3 and 8. It is worth noting that the resulting values are complex. They provide the normalized amplitude of the out-plane displacements due to wave propagation with respect to that due to the incident waves. As already said, this latter amplitude is assumed equal to 1. The variation of these quantities with time is obtained multiplying them by the factor $e^{i\omega t}$.

To assess the accuracy of the method, comparisons were performed with the analytical solution derived by Trifunac [1] for a semi-cylindrical alluvial valley excited by incident SH waves. Some results are presented in Fig. 2, in terms of the displacement amplitude at the ground surface for different values of the incidence angle and two values of the dimensionless frequency (η=0.5 and 1). This latter is defined as

$$\eta = \frac{\omega a}{\pi \beta_1} \tag{9}$$

where a is the radius of the valley. The other data assumed in the calculations are ρ_1/ρ_2=1.5 and G_1/G_2=6. As can be seen from Fig. 2, the results are in very close agreement.

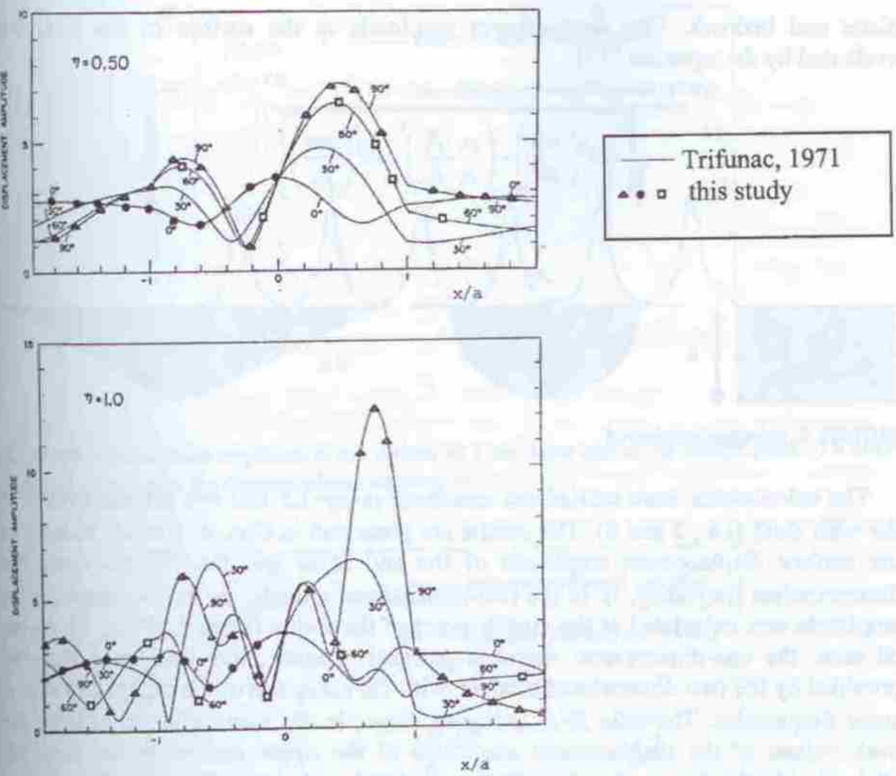

FIGURE 2. Comparison between the present method and Trifunac's solution (adapted from [1])

APPLICATIONS

In the engineering applications, the local site response is often evaluated using a one-dimensional model consisting of a soil deposit resting on a deformable or rigid bedrock excited by vertically incident shear waves. This implies that the effects of wave scattering due to the actual shape and the limited lateral extent of the soil deposit are ignored. Consequently, it is of interest to compare the results obtained by such a simple model with those calculated using the two-dimensional solution presented in the previous section. To this purpose, a semi-cylindrical valley embedded in a homogeneous half-plane is considered, and the results are compared with those obtained for a soil layer with thickness equal to the radius of the valley, resting on a deformable bedrock. In addition, a valley with triangular cross-section is also considered (Fig. 3). This latter is characterized by a height equal to the radius of the semi-cylindrical valley, and a slope of ½ with respect to the horizontal direction. For all these soil systems, the excitation consists of a train of SH harmonic waves with unit amplitude and vertical incidence. Using the same notation for the one and two-dimensional models, ρ_2 and β_2 are the mass density and the shear wave velocity of the valley and soil layer, whereas ρ_1 and β_1 are the above material properties for the half-plane and bedrock. The displacement amplitude at the surface of the layer was evaluated by the equation [15]:

$$u=\frac{2}{\sqrt{\left[\cos^2\left(\frac{\omega a}{\beta_2}\right)+\left(\frac{\mu_2}{\mu_1}\frac{\beta_1}{\beta_2}\right)^2\sin^2\left(\frac{\omega a}{\beta_2}\right)\right]}} \tag{10}$$

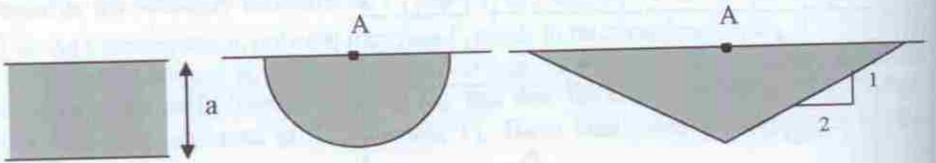

FIGURE 3. Schemes considered

The calculations were carried out assuming ρ_1/ρ_2=1.5 and two different values of the ratio β_1/β_2 (i.e., 2 and 3). The results are presented in Figs. 4, 5 and 6, in terms of the surface displacement amplitude of the soil layer and the valleys, versus the dimensionless frequency, η. In the two-dimensional models, the surface displacement amplitude was calculated at the middle point of the valley (point A in Fig. 3). As can be seen, the one-dimensional response generally results more attenuated than that provided by the two-dimensional models, with the exception of the triangular valley at some frequencies. The ratio β_1/β_2 is higher, larger is this attenuation. In addition, the peak values of the displacement amplitude at the upper surface of the layer are attained at lower frequencies than those calculated under two-dimensional conditions. In other words, using the one-dimensional model leads to a reduction of the surface displacement amplitude accomplished by a shifting in the fundamental frequencies with respect to the two-dimensional case. This is due to the wave scattering effects that are ignored when a one-dimensional solution is employed.

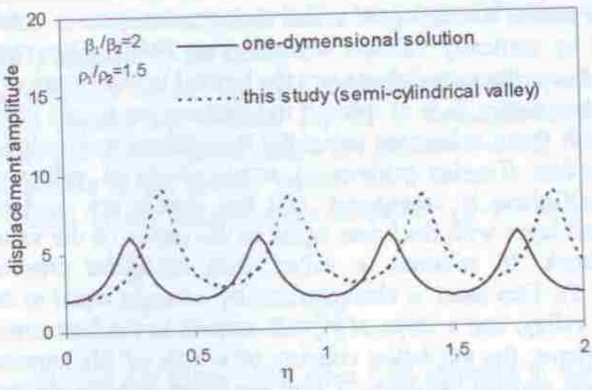

FIGURE 4. Displacement amplitude at the surface of a soil layer and at the middle point of a semi-cylindrical valley versus the dimensionless frequency η.

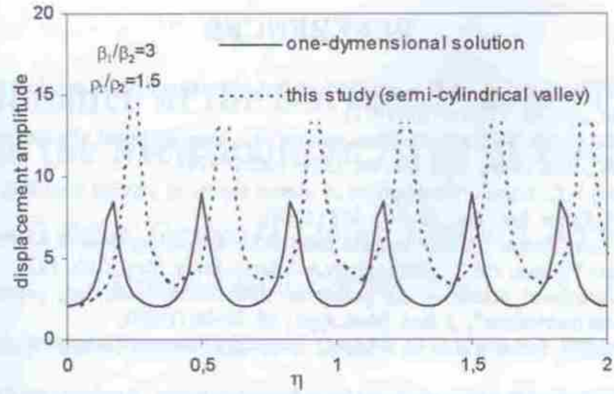

FIGURE 5. Displacement amplitude at the surface of a soil layer and at the middle point of a semi-cylindrical valley versus the dimensionless frequency η.

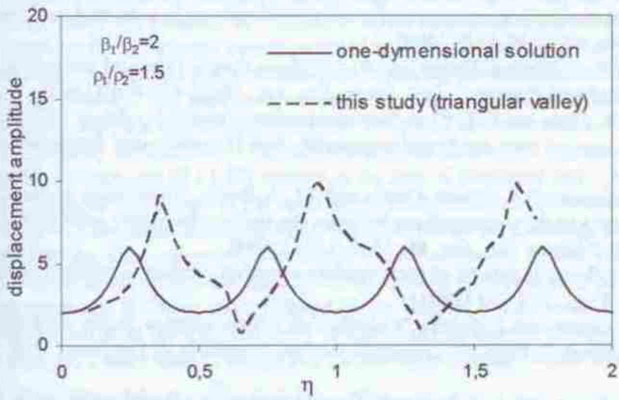

FIGURE 6. Displacement amplitude at the surface of a soil layer and at the middle point of a triangular valley versus the dimensionless frequency η.

CONCLUDING REMARKS

A numerical solution based on the boundary element method has been presented for the dynamic response of alluvial valleys to SH waves. Taking advantage of a special fundamental solution of the Helmholtz equation, the interface between the valley and the underlying half-plane needs only to be considered for solving the integral equation on which the method is based. Comparisons have been made with a well-known one-dimensional solution to highlight the most significant differences between the results. It has been shown that the one-dimensional model in principle leads to a reduction of the surface displacement amplitude and a shifting in the fundamental frequencies towards lower values than those obtained using the two-dimensional method presented in this paper.

REFERENCES

1. M. D. Trifunac "Surface motion of a semi-cylindrical alluvial valley for incident plane SH-waves", *Bull. Seism. Soc. Am.*, **61**, 1755–1770 (1971).
2. H. L. Wong and M. D. Trifunac "Surface motion of a semi-elliptical alluvial valley for incident plane SH waves", *Bull. Seism. Soc. Am.*, **64**, 1389–1408 (1974).
3. Z. Alterman and F.C. Karal "Propagation of elastic waves in layered media by finite difference methods ", *Bull. Seism. Soc. Am.*, **58**, 367–398 (1968).
4. J. Lysmer and L.A. Drake "Afinite element method for seismology", in: B.A. Bolt (Ed.), Methods in Computational Physics, vol. 11, Seismology,Academic Press, New York, 1972.
5. I. Herrera, "Variational principles for problems with linear constraints, prescribed jumps and continuation type restrictions", *J. Inst. Math.Appl.*, **25**, 67–96 (1980).
6. C.A. Brebbia, J.C.F. Telles and L. C. Wrobel, Boundary Element Techniques, Spring-Verlag Ed., 1984.
7. J.A. Rial, "Caustics and focusing produced by sedimentary basins. Application of catastrophe theory to earthquake seismology, Geophys.", *J. Roy. Astr. Soc.*, **79**, 923–938 (1984).
8. J.E. Luco, H.L. Wong, F.C.P. de Barros.A. Rial, "Three-dimensional response of a cylindrical canyon in a layered half-space ", *Earthquake Eng. Struct. Dyn.* 419, 799–817 (1990).
9. F. Luzón, S. Aoi, D. Fäh, F.J. Sánchez-Sesma "Simulation of the seismic response of a 2D sedimentary basin: a comparison between the indirect boundary element method and a hybrid technique three-dimensional response of a cylindrical canyon in a layered half-space ", *Earthquake Eng. Struct. Dyn.* 419, 799–817 (1990).
10. M. A. Bravo, F. J. Sànchez-Sesma and F. J. Chàvez-Garcia "Ground motion on stratified alluvial deposits for incident SH waves", *Bull. Seism. Soc. Am.*, **78**(2), 436–450 (1988).
11. J. T. Chen, P.Y. Chen and C.T. Chen "Surface motion of multiple alluvial valleys for incident plane SH-waves by using a semi-analytical approach", *Soil Dynamics and Earthquake Engineering*, **28**, 58–72 (2008).
12. F. J. Sànchez-Sesma, F. J. Chàvez-Garcia and M. A. Bravo, "Simulation of the seismic response of a 2D sedimentary basin: a comparison between the indirect boundary element method and a hybrid technique", *Bull. Seism. Soc. Am.*, **85**, 1501–1506 (1995).
13. S. Kobayashi "Some problems of the boundary equation method in elastodynamics", Proc. 5th Int. Conf. BEM in Eng., 775-784 (1983).
14. E. Conte, G. Dente and I. Guerra "Calcolo dell'effetto sismico locale mediante il metodo degli elementi di contorno", Proc. 7th Convegno Gruppo Nazionale di Geofisica della Terra Solida 341-354 (1988).
15. F. J. Sànchez-Sesma and J. A. Esquivel "Ground motion on alluvial valley under incident plane SH waves", *Bull. Seism. Soc. Am.*, **69**, 1107–1120 (1979).