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The interactive vibration behavior in a suspension bridge system under moving vehicle loads and vertical seismic excitations

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ABSTRACT

In this paper, the vibration behavior of a suspension bridge due to moving vehicle loads with vertical support motions caused by earthquake is studied. The suspension bridge system is presented here by two coupled nonlinear cable–beam equations aiming to describe both the dynamic characteristics for the supporting cable and the roadbed, respectively. The dynamic effect of traffic vehicles are modeled as a row of equidistant moving forces, while the earthquake movement is simulated as the vertical oscillation of boundary supports. The governing integro-differential equations are transferred into a set of ordinary differential equations, which can be solved analytically in the present study. Furthermore, the world's largest designed suspended bridge – Messina Bridge – is examined (central span of length 3.3 km) and the modified Kobe earthquake records is applied to the calculations in order to validate the present study and the proposed methodology. As a result, the deformation of the cable produces more oscillations than that of the beam since the material property of the cable is more flexible. It is shown that the interaction of both the moving loads and the seismic forces can substantially amplify the response of long-span suspension bridge system especially in the vicinity of the end supports.

1. Introduction

Structural engineers often confront a dynamic problem of multiple support motions when coping with the analysis of long-span structures under earthquake excitations [1–6]. For example, the earthquake-induced response of a suspension bridge is a typical multipoint support vibration problem due to the propagation effect of seismic waves at construction site. Along with the rapid development of modern transportation networks, suspension bridges are often adopted to span wide rivers or deep valleys in the infrastructure of a country. Recently, some researchers have studied the dynamic behavior of suspension bridges subjected to moving loads [4,7–12]. Based on the conclusion in these studies, the cable tensions of short-span suspension bridges induced by moving loads would be amplified significantly. Few studies have been performed on the train-induced vibration for suspension bridges shaken by earthquake support excitations. Using an analytical approach, Fŕyba [13], Yang et al. [14], and Xia et al. [15] presented a resonant condition for the train-induced response of simply supported bridges. Such a condition provides a useful criterion for predicting the resonant speeds of a high speed train traveling on railway bridges. As for the stability problem of a train moving over a bridge shaken by earthquakes, Yang et al. [16] pointed out that the presence of vertical ground excitations would affect drastically the stability of a moving train. Xia et al. [17] demonstrated that for a train traveling over a continuous seven-span bridge shaken by earthquakes, it might

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lead the train manipulation to an unsafe conclusion due to lack of considering seismic traveling wave effect. Yau and Yang [18] performed the vibration analysis of a suspension bridge installed with a water pipeline and subjected to moving trains; in addition, Yau [19] investigated the dynamic response of suspended beams subjected to moving vehicles and multiple support excitations.

In the present study, the vibration behavior of a suspension bridge due to moving vehicle loads with vertical support motions caused by earthquake is studied. The suspension bridge system is presented here by two coupled nonlinear cable–beam equations aiming to describe both the dynamic characteristics for the supporting cable and the roadbed respectively. Hopefully the results of the present paper might be served as a design basis for the selection of span length of a suspension bridge located at construction site in seismic regions.

2. Mathematical modeling

The physical model of a suspension bridge is schematically drawn in Fig. 1, where a sequence of identical vehicle P with equal distance d is moving along the roadbed at a constant speed v. The suspension bridge is composed of a uniform beam and a suspended cable that is anchored at the tip points of two undeformable pylons, which implies that cable and beam can be treated as being simply supported at both rigid pylons. Let $u(x, t)$ and $w(x, t)$ denotes the downward deflections of the cable and the roadbed, respectively. Thus, the following partial differential equations describing the dynamic behavior of the suspension bridge model was proposed by Lazer and MaKenna [20], and the traveling waves in the suspension bridge system was studied by Ding [21].

For $0 < x < L$

$$m_c u_{tt} - Q u_{xx} - K(w-u)^+ = m_c g + f_1(x,t), \tag{1}$$

$$m_b w_{tt} + C_d w_t + EI w_{xxxx} + K(w-u)^+ = m_b g + f_2(x,t), \tag{2}$$

where m_c and m_b are the mass densities of the cable and the roadbed, respectively; Q is the coefficient of cable tensile strength; C_d is the damping constant of the roadbed; EI is the roadbed flexural rigidity; K is Hooke's constant of the stays; $(w-u)^+ \equiv \max\{w-u, 0\}$ is the difference between beam and cable deflections; and $f_1(x, t)$ and $f_2(x, t)$ represent the external dynamic forces which might be caused by passing wind or moving vehicles.

It should be noted that the term for cable tensile strength, Q, stated in Eq. (1) represents actually the axial tension in the cable for a specific location x, however, when a parabolic-shaped cable is under consideration as describes in Fig. 2, there should be an extra increment in the horizontal component of the axial tensile in the suspended cable due to the external loading, i.e., the governing equation for suspended cable should be further modified into

$$m_c u_{tt} - T u_{xx} - \Delta T y'' - K(w-u)^+ = m_c g + f_1(x,t), \tag{3}$$

where T represents the horizontal component of the cable axial tensile and ΔT is the increase of horizontal component in the cable under the action of moving loads and vertical support movements.

By cutting the free-body diagram and taking the moment on one of the hinged ends at any position $x = \eta$, the aforementioned horizontal component T can be easily derived to be

$$T = -\frac{m_c g}{y''} = \frac{m_c g L^2}{8 y_0}, \tag{4}$$

where g is the gravity constant, y is the so-called sag function of the cable, which is set to be the following parabolic function,

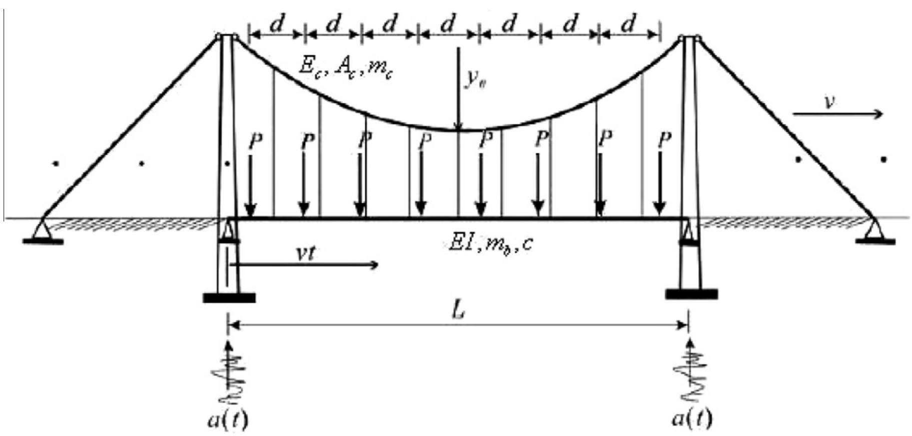

Fig. 1. A suspended bridge – cable–beam system.

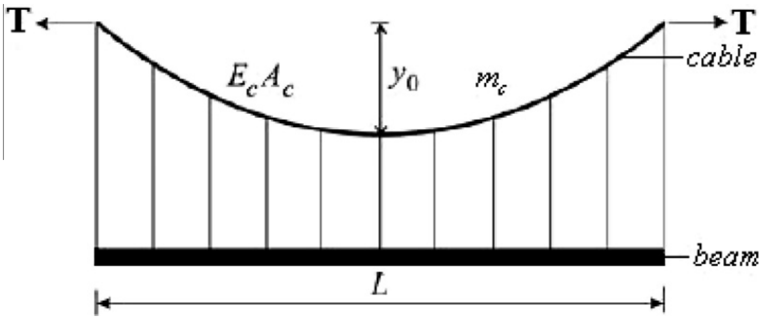

Fig. 2. Deflected shape of cable due to gravity.

$$y(x) = 4y_0\left[x/L - (x/L)^2\right], \tag{5}$$

herein y_0 indicates the cable sag at mid span, L is the span length of the system and E_CA_C represents the axial rigidity of the cable as specified in Fig. 2.

To express the incremental force ΔT in the suspended cable, the following equations and symbols for a parabolic cable shape are adopted in **Fŕyba** and Yau's study [22]. Let us denotes the original length $ds_0 = \sqrt{[(dx)^2 + (dy)^2]}$ as the static length while the cable is in equilibrium state, and consider the deformed length $ds = \sqrt{[(dx)^2 + (dy + du)^2]}$ for an infinitesimal element of the cable under oscillation as shown in Fig. 3. According to the Hook's law for the deformed cable element due to the horizontal force increment, ΔT, it can be found that, [23],

$$\frac{\Delta T}{E_CA_C}\frac{ds_0}{dx} \approx \frac{ds - ds_0}{ds_0} \approx \frac{dy}{ds_0}\frac{dw}{ds_0} = \left(\frac{dx}{ds_0}\right)^2 \frac{dy}{dx}\frac{dw}{dx}. \tag{6}$$

Considering the boundary conditions for the cable with two-hinged ends under vertical support motions and multiplying Eq. (6) by $(ds_0/dx)^2$, one can integrate this equation from 0 to L to obtain

$$\frac{\Delta T}{E_CA_C}\int_0^L \left(\frac{ds_0}{dx}\right)^3 dx = \int_0^L y'(x,t)u'(x,t)dx = y'(x,t)u(x,t)|_0^L - y''(x,t)\int_0^L u(x,t)dx. \tag{7}$$

If we define the effective length of the cable, L_C, to be as follows:

$$L_C \equiv \int_0^L \left(\frac{ds_0}{dx}\right)^3 dx = \int_0^L \left(\sqrt{1+y'^2}\right)^3 dx, \tag{8}$$

then the horizontal force increment ΔT in Eq. (7) can be evaluated as,

$$\begin{aligned}\Delta T &= \frac{E_CA_C}{L_C}\int_0^L y'w'\,dx = \frac{E_CA_C}{L_C}\left[y'(x,t)u(x,t)|_0^L - y''(x,t)\int_0^L u(x,t)dx\right] \\ &= \frac{E_CA_C}{L_C}\left[-\frac{4y_0}{L}(u(0,t)+u(L,t)) + \frac{8y_0}{L^2}\int_0^L u(x,t)dx\right],\end{aligned} \tag{9}$$

Using the fact that $y'' = -8y_0/L^2$ for a parabolic shape cable described in Eq. (5), and the force increment ΔT in Eq. (9), Eq. (3) can then be transformed into the following partial integro-differential equation:

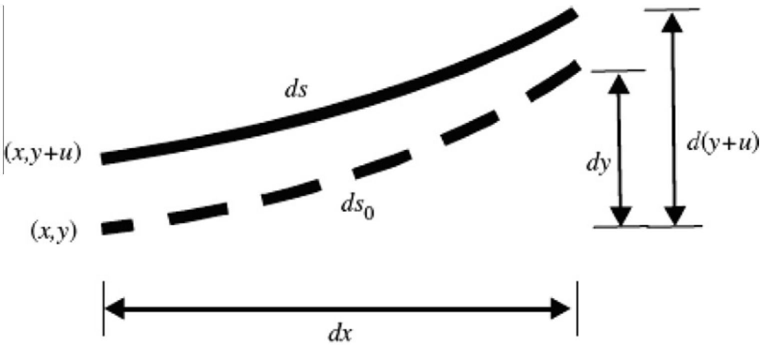

Fig. 3. An infinitesimal element of the cable.

$$m_c \frac{\partial^2 u(x,t)}{\partial t^2} - T\frac{\partial^2 u(x,t)}{\partial x^2} - K(w-u)^+ + A\cdot\int_0^L u(x,t)dx = m_c g + f_1(x,t) + \frac{AL}{2}[u(L,t)+u(0,t)], \tag{10}$$

where A is a const and defined as,

$$A \equiv \left(\frac{8y_0}{L^2}\right)^2 \frac{E_C A_C}{L_C}, \tag{11}$$

As shown in Fig. 1, assume a row of moving vehicle with identical weight P and equal interval d is crossing the proposed suspension cable–beam system at a constant speed ν. The external loading $f_2(x, t)$ describing the action of vehicle movements on the system can be expressed as, see Ref. [24],

$$f_2(x,t) = P\cdot\sum_{k=1}^{N}\{\delta[x-\nu(t-t_k)]\cdot[H(t-t_k)-H(t-t_k-L/\nu)]\} \tag{12}$$

in which $\delta(x)$ is the Dirac delta function, $H(t)$ is the Heaviside unit step function with $H(t) = 0$ for $t < 0$ and $H(t) = 1$ for $t \geqslant 0$, N represents the Nth moving force acting on the roadbed, and $t_k = (k-1)d/v$ indicates the arriving time of the kth load on the beam.

The boundary conditions for the suspended cable–beam system with two-hinged ends under vertical support movements due to earthquake can be expressed as

$$u(0,t) = a(t), \quad u(L,t) = a(t) \tag{13}$$

for the cable part, and

$$w(0,t) = a(t), \quad w(L,t) = a(t), \tag{14}$$
$$EIw''(0,t) = EIw''(L,t) = 0, \tag{15}$$

for the beam part, respectively. It should be noted here that $a(t)$ and $b(t)$ represents the vertical displacement at the two bridge supports due to the seismic action and we suppose that cable and beam are having the same vertical shift during vertical shaking as stated in Fig. 1. Meanwhile, we assume that the initial conditions for these two quantities are both zero, i.e. when the first moving vehicle enters the suspension bridge, we impose

$$u(x,0) = \dot{u}(x,0) = 0, \tag{16}$$

and

$$w(x,0) = \dot{w}(x,0) = 0 \tag{17}$$

Under the consideration of governing equations for the cable–beam system, Eq. (10) and (2), accompanied by boundary conditions, Eqs. (13)–(15), and initial conditions, Eqs. (16) and (17), it can be observed that we are now facing a coupled cable–beam interactive vibration problem with time-dependent boundary conditions. In order to obtain the total response of the suspended cable–beam system, a quasi-static decomposition method [2] will be employed to solve the set of integro-differential equations in the following sections.

3. Quasi-static decomposition method

By using the same technique adopted in Ref. [24] for the time-dependent boundary value problem in beam vibration, the total deflections of the cable as well as the beam, i.e., $u(x, t)$ and $w(x, t)$ can both be decomposed into two parts: the quasi-static components, $U(x, t)$, $W(x, t)$, and the dynamic components, $u_d(x, t)$, $w_d(x, t)$, i.e.,

$$u(x,t) = U(x,t) + u_d(x,t), \tag{18}$$
$$w(x,t) = W(x,t) + w_d(x,t). \tag{19}$$

Here, the quasi-static part component, $U(x, t)$ and $W(x, t)$, can be regarded as the cable or beam displacement induced by the static effect of support movements, nevertheless, the remaining dynamic part, $u_d(x, t)$ or $w_d(x, t)$ can be considered as the dynamic behavior of the cable–beam system under the vibration status.

Substituting Eqs. (18) and (19) into Eq. (10) and (2), the following two sets of equations can be obtained,

$$m_b\frac{\partial^2 W(x,t)}{\partial t^2} + m_b\frac{\partial^2 w_d(x,t)}{\partial t^2} + C_d\frac{\partial W(x,t)}{\partial t} + C_d\frac{w_d(x,t)}{\partial t} + EI\frac{\partial^4 W(x,t)}{\partial x^4} + EI\frac{\partial^4 w_d(x,t)}{\partial x^4}$$
$$+ K(W(x,t) + w_d(x,t) - U(x,t) - u_d(x,t))^+ = m_b g + f_2(x,t), \tag{20}$$

and

$$m_c \frac{\partial^2 U(x,t)}{\partial t^2} + m_c \frac{\partial^2 u_d(x,t)}{\partial t^2} - T\frac{\partial^2 U(x,t)}{\partial x^2} - T\frac{\partial^2 u_d(x,t)}{\partial x^2} - K(W(x,t) + w_d(x,t) - U(x,t) - u_d(x,t))^+$$
$$+ A \cdot \int_0^L U(x,t)dx + A \cdot \int_0^L u_d(x,t)dx = m_c g + f_1(x,t) + \frac{AL}{2}[u(L,t) + u(0,t)]. \tag{21}$$

Eliminating the common term in the above equations and re-arranging the terms by moving the quasi-static components to the right hand side can yield the following four equations if no wind force is undergoing,

$$EI\frac{\partial^4 W(x,t)}{\partial x^4} = 0, \tag{22}$$
$$m_b \ddot{W} + C_d \dot{W} + m_b \frac{\partial^2 w_d(x,t)}{\partial t^2} + C_d \frac{w_d(x,t)}{\partial t} + EI\frac{\partial^4 w_d(x,t)}{\partial x^4} = m_b g + f_2(x,t), \tag{23}$$

and

$$-T\frac{\partial^2 U(x,t)}{\partial x^2} + A \cdot \int_0^L U(x,t)dx = 0, \tag{24}$$
$$m_c \ddot{U} + m_c \frac{\partial^2 u_d(x,t)}{\partial t^2} - T\frac{\partial^2 u_d(x,t)}{\partial x^2} + A \cdot \int_0^L u_d(x,t)dx = m_c g + \frac{AL}{2}[u(L,t) + u(0,t)], \tag{25}$$

with the boundary conditions be modified as

$$W(0,t) = W(L,t) = a(t), \tag{26}$$
$$W''(0,t) = W''(L,t) = 0, \tag{27}$$

and

$$w_d(0,t) = w_d(L,t) = 0, \tag{28}$$
$$w''_d(0,t) = w''_d(L,t) = 0. \tag{29}$$

By solving Eq. (22) in accordance with conditions (26) and (27), we can easily reach the exact quasi-static solution of the bridge road (i.e., beam part) as

$$W(x,t) = w(x)T(t) = 1 \cdot a(t) = a(t). \tag{30}$$

Plug back the quasi-static solution into Eq. (23), one can get the following uncoupled differential equation for the dynamic part,

$$m_b \frac{\partial^2 w_d(x,t)}{\partial t^2} + C_d \frac{w_d(x,t)}{\partial t} + EI\frac{\partial^4 w_d(x,t)}{\partial x^4} = m_b g - m_b \ddot{a}(t) - C_d \dot{a}(t) + f_2(x,t), \tag{31}$$

after imposing the Galerkin's procedure, it can be further expressed as

$$m_b \sum_{n=1}^{\infty} \ddot{q}_n(t)\sin\frac{n\pi x}{L} + C_d \sum_{n=1}^{\infty} \dot{q}_n(t)\sin\frac{n\pi x}{L} + \left[EI\left(\frac{n\pi}{L}\right)^4\right] \sum_{n=1}^{\infty} q_n(t)\sin\frac{n\pi x}{L} = m_b g - m_b \ddot{a}(t) - C_d \dot{a}(t) + f_2(x,t), \tag{32}$$

and can be stated as below after performing the orthogonal properties,

$$m_b \ddot{q}_n(t) + C_d \dot{q}_n + \left[EI\left(\frac{n\pi}{L}\right)^4\right] q_n(t) = [m_b g - m_b \ddot{a}(t) - C_d \dot{a}(t)]\frac{2}{n\pi}(1 - \cos n\pi) + \left[\sum_{k=1}^{N} F_k(\omega_n, v, t)\right]. \tag{33}$$

where the generalized force $\sum_{k=1}^{N} F_k(\omega_n, v, t)$ of the kth moving force is expressed as

$$F_k(\omega_n, v, t) = \frac{2P}{L}[\sin\omega_n(t - t_k)H(t - t_k) + (-1)^{n+1}\sin\omega_n(t - t_k - L/v)H(t - t_k - L/v)],$$

and $\omega_n = n\pi v/L$ represents the driving frequency of the moving loads.

Eq. (33) can be numerically solved by adopting the Newmark's method in the quantity $q_n(t)$, in so doing, the quasi-static and dynamic solutions for the road bed are accomplished successfully.

As for the other two differential equations describing the responses of the supporting cable, i.e., Eq. (24) and (25), the technique of separation of variables can be used and the analytical solution of the quasi-static solution, $U(x,t)$ can be found according to the following boundary conditions,

$$U(0,t) = U(0)T(t) = a(t), \tag{34}$$
$$U(L,t) = U(L)T(t) = a(t), \tag{35}$$

that is,

$$T(t) = a(t), \tag{36}$$

$$U(x) = \frac{AL}{2T + AL^3/6}x^2 - \frac{AL^2}{2T + AL^3/6}x + 1, \tag{37}$$

and this reaches the exact solution for $U(x, t)$,

$$U(x,t) = a(t)\left\{\frac{AL}{2T + AL^3/6}x^2 - \frac{AL^2}{2T + AL^3/6}x + 1\right\}. \tag{38}$$

Having the above solution as a prior, we can easily solve the last differential equation as stated in Eq. (25) with the boundary conditions

$$u_d(0,t) = u_d(L,t) = 0. \tag{39}$$

By adopting the superposition method, the following series solution which satisfies Eq. (39) is assumed

$$u_d(x,t) = \sum_{m=1}^{\infty} q_m(t)\sin\frac{m\pi x}{L}, \tag{40}$$

and after substituting back to Eq. (25) in accordance with Eq. (38) one can have

$$m_c \cdot \sum_{m=1}^{\infty} \ddot{q}_m(t) \cdot \sin\frac{m\pi x}{L} - T \cdot \sum_{m=1}^{\infty} q_m(t) \cdot \left(\frac{k\pi}{L}\right)^2 \cdot \sin\frac{m\pi x}{L} + A \cdot \sum_{m=1}^{\infty} q_m(t)\int_0^L \sin\frac{m\pi x}{L}dx = m_c g - m_c\ddot{U} + ALa(t), \tag{41}$$

Table 1
Properties of the suspended beam (Messina Bridge).

L (m)	EI (kN m^2)	E_CA_C (kN)	c (Ns m^{-1})	m_c (kg m^{-1})	m_b (kg m^{-1})	y_0 (m)
3300	8.6×10^7	1.6×10^8	0.78	576	21,600	335

Table 2
Properties of moving loads.

P (kN)	d (m)	N
360	18	30

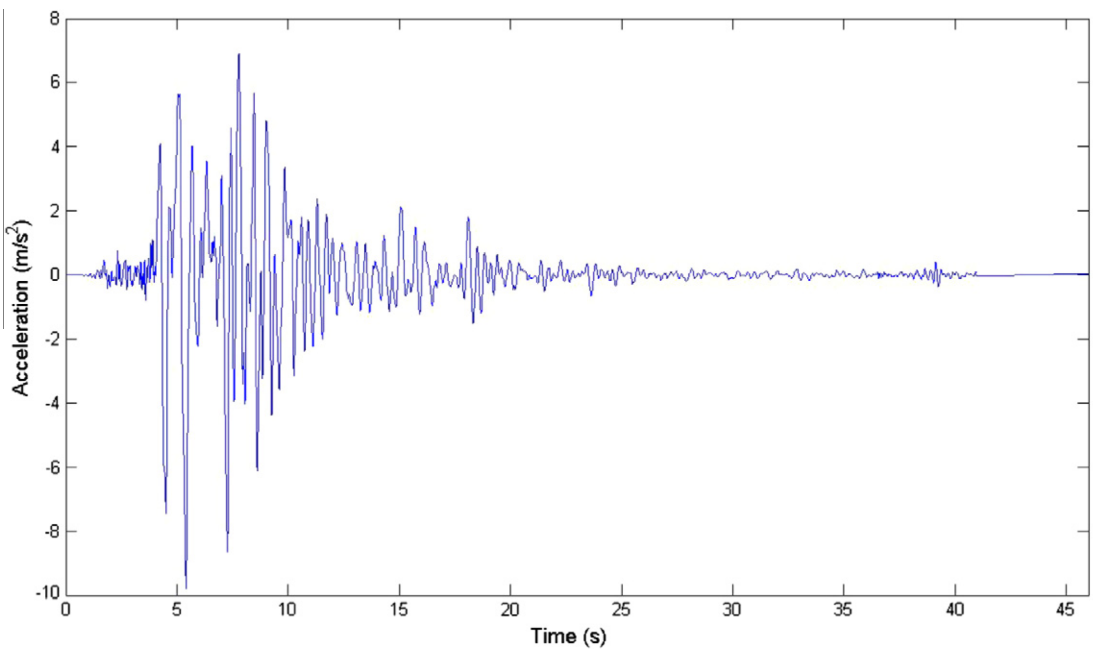

Fig. 4. Time history of the vertical acceleration during the Kobe earthquake.

further, in performing Galerkin's method, it can be derived that, for $k = 1, 2, 3, \ldots$, we have

$$m_c \cdot \ddot{q}_k(t)\frac{L}{2} - T \cdot q_k(t) \cdot \left(\frac{k\pi}{L}\right)^2 \frac{L}{2} + \frac{AL^2}{k\pi^2} \cdot (1 - \cos k\pi) \sum_{m=1}^{\infty} q_m(t)(1 - \cos m\pi)$$
$$= \frac{L}{k\pi}(1 - \cos k\pi)\{ALa(t) + m_c g\} - m_c G(t) \quad (42)$$

in which

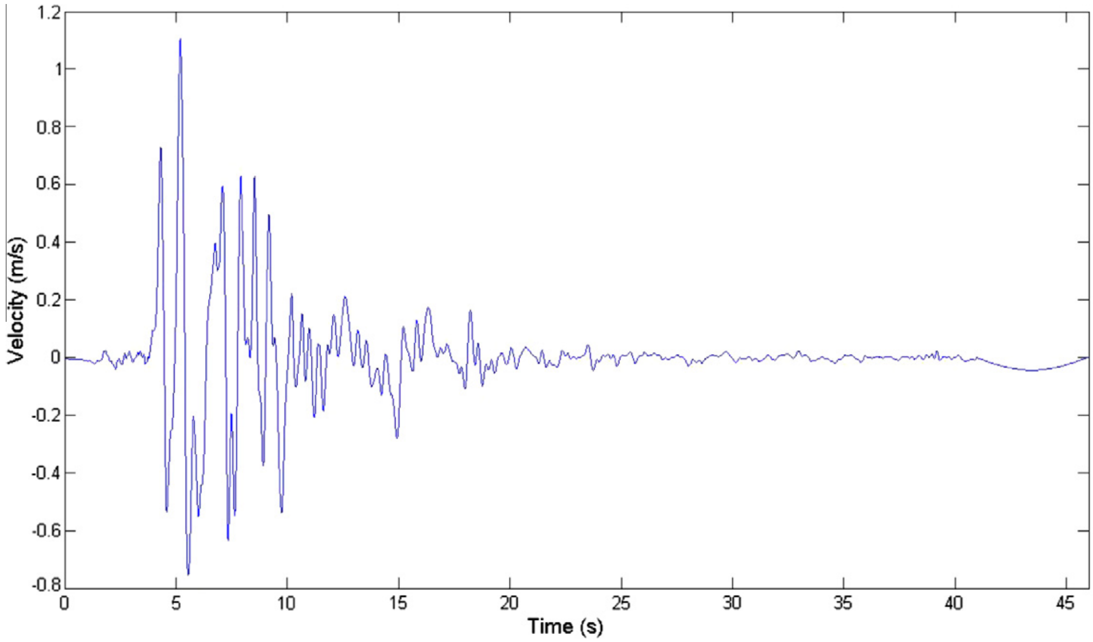

Fig. 5. Time history of the vertical velocity during the Kobe earthquake.

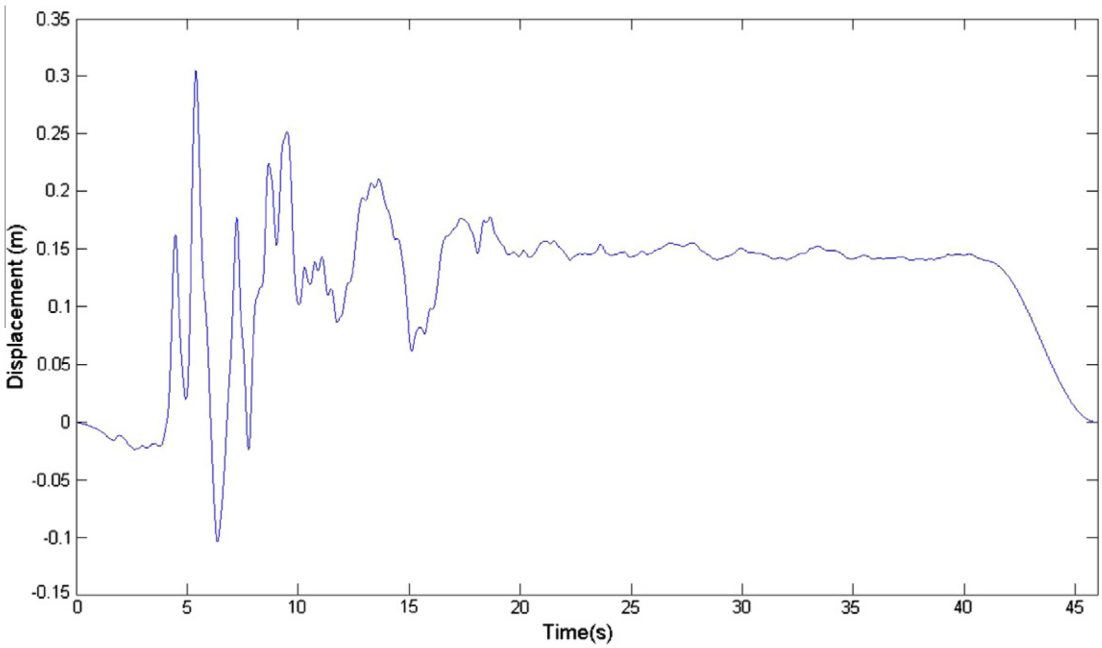

Fig. 6. Time history of the vertical displacement during the Kobe earthquake.

$$G(t) \equiv \ddot{a}(t)\frac{L}{k\pi}\left\{\cos k\pi \cdot \left[2\lambda\left(\frac{L}{k\pi}\right)^2 - 1 - \lambda L - \lambda L^2\right] - 2\lambda\left(\frac{L}{k\pi}\right)^2 + 1\right\}. \tag{43}$$

where $\lambda = \frac{AL}{2T+AL^3/6}$.

The equation can also be solved numerically by using the Newmark's method again, however, it should be noted that the physical meaning of the quantity $q_k(t)$ here is different with that appeared in Eq. (33) for a beam equation, instead it represents the time variation of the supporting cable while the system is undergoing earthquake excitation at the boundaries.

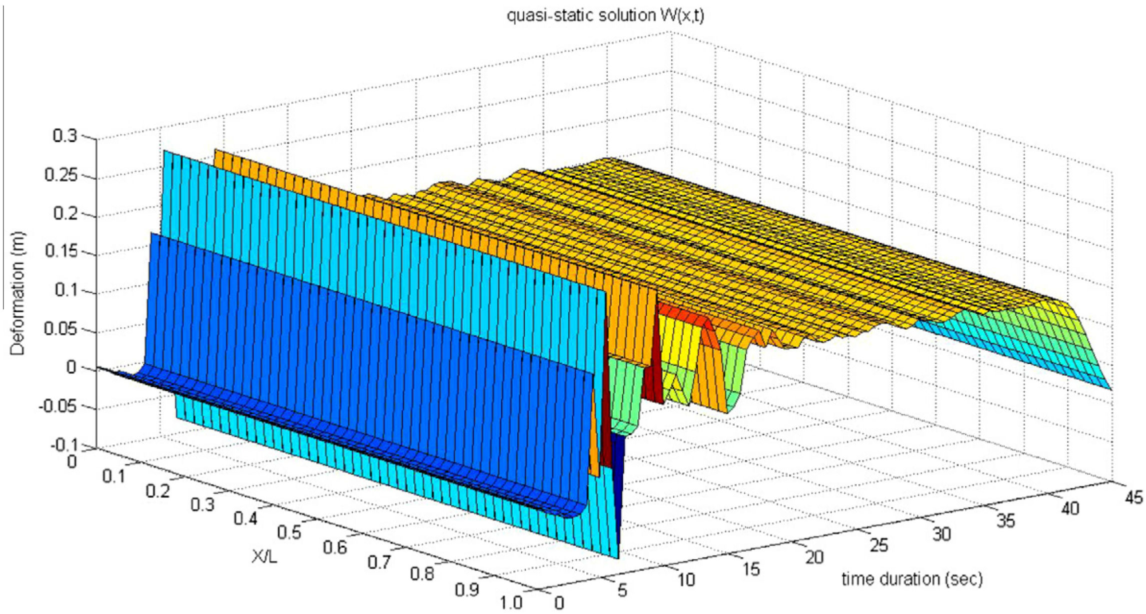

Fig. 7. Quasi-static deformation of the beam.

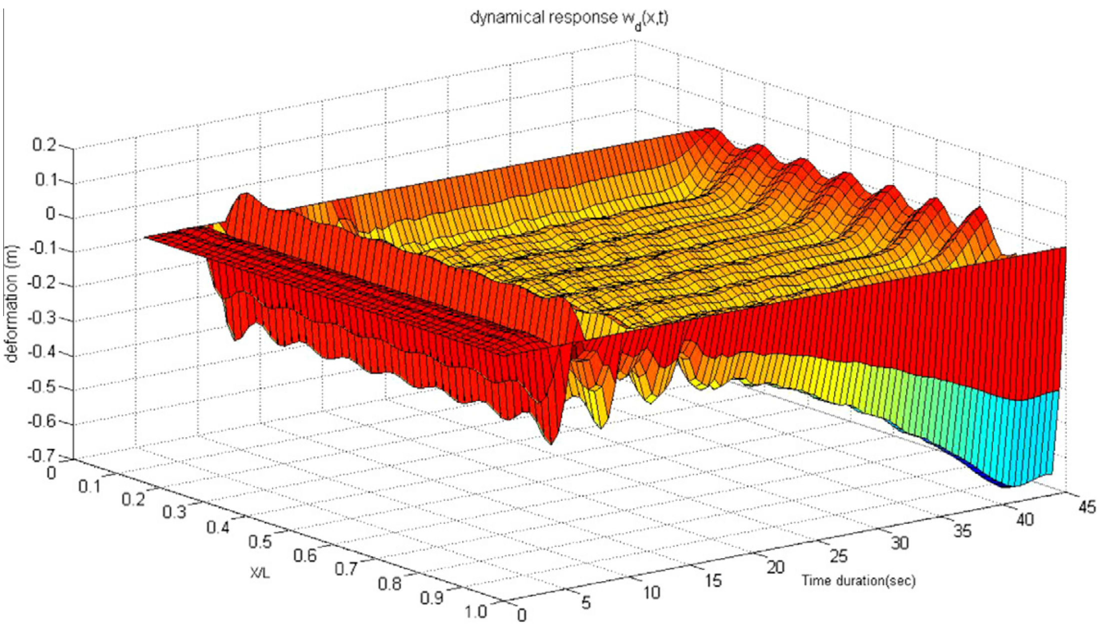

Fig. 8. Dynamic deformation of the beam.

4. Numerical example

In order to validate the proposed methodology, the numerical Newmark's differential scheme with $\beta = 0.25$ and $\gamma = 0.5$ is applied to the solution of Eqs. (33) and (42). The properties of the designed Messina Bridge and of moving loads are listed in Tables 1 and 2, see also Fig. 1 with $L = 3300$ m, $y_0 = 335$ m and $L_C = 3577$ m. Figs. 4–6 are the time histories of the vertical acceleration, velocity, and displacement, respectively, recorded during the Kobe earthquake. In the present study, the first 16 shape functions are applied to study the dynamic responses of the suspended cable–beam system. We demonstrate the case for a coupled cable–beam system under the moving vehicle loads and earthquake excitation. Since the dynamic

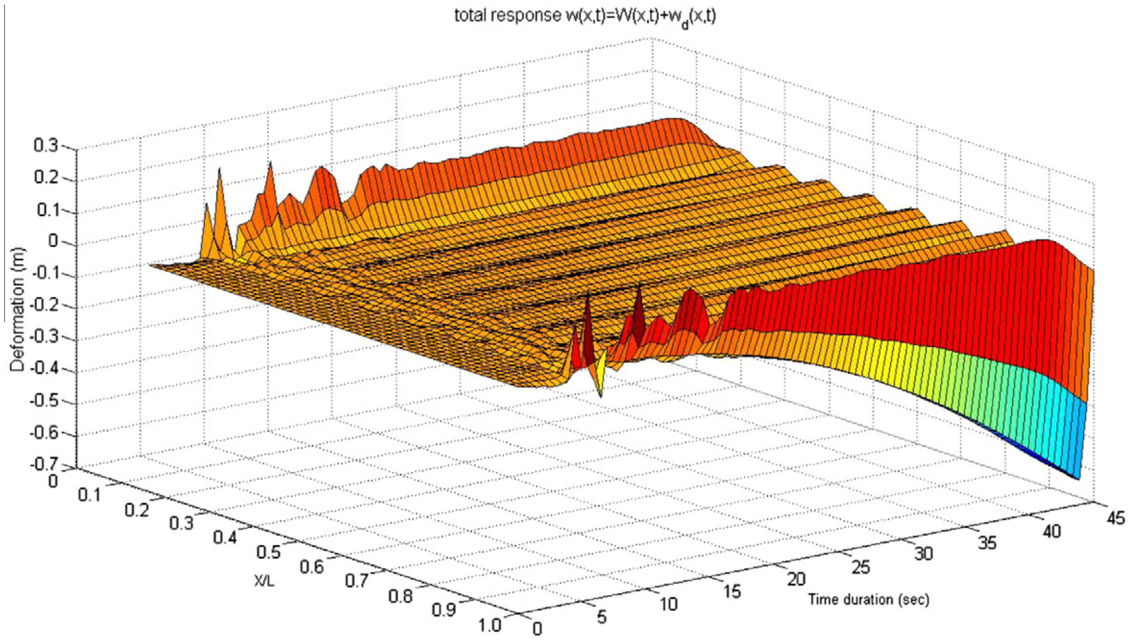

Fig. 9. Total deformation of the beam.

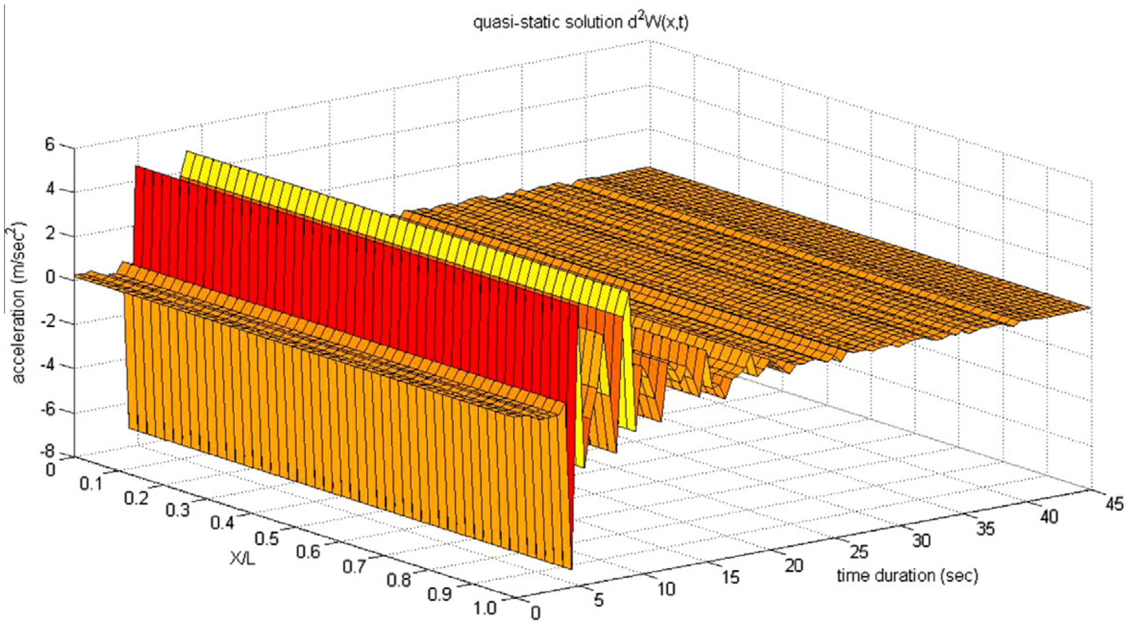

Fig. 10. Quasi-static acceleration of the beam.

responses for the beam as well as the cable have been successfully separated as shown in Eq. (33) and (42), hereinafter we will present these quantities individually. The deformation distribution of quasi-static solution versus time for beam part is depicted in Fig. 7, and the corresponding ones of the dynamic response and total response are presented in Figs. 8 and 9 individually. As it can be seen from Fig. 9, the maximum deformation of the beam occurs at both end supports of the beam that is fairly reasonable since the earthquake loading is applied at the supports. Followed by the acceleration distribution versus time of quasi-static solution, dynamic response and the total response are demonstrated in Figs. 10–12 separately. Once again, as it can be detected from Fig. 12, the maximum acceleration of the beam occurs at both end supports of the beam and at time near 5.0 seconds that is quite rational due to earthquake excitation at the supports. Meanwhile, the dynamic

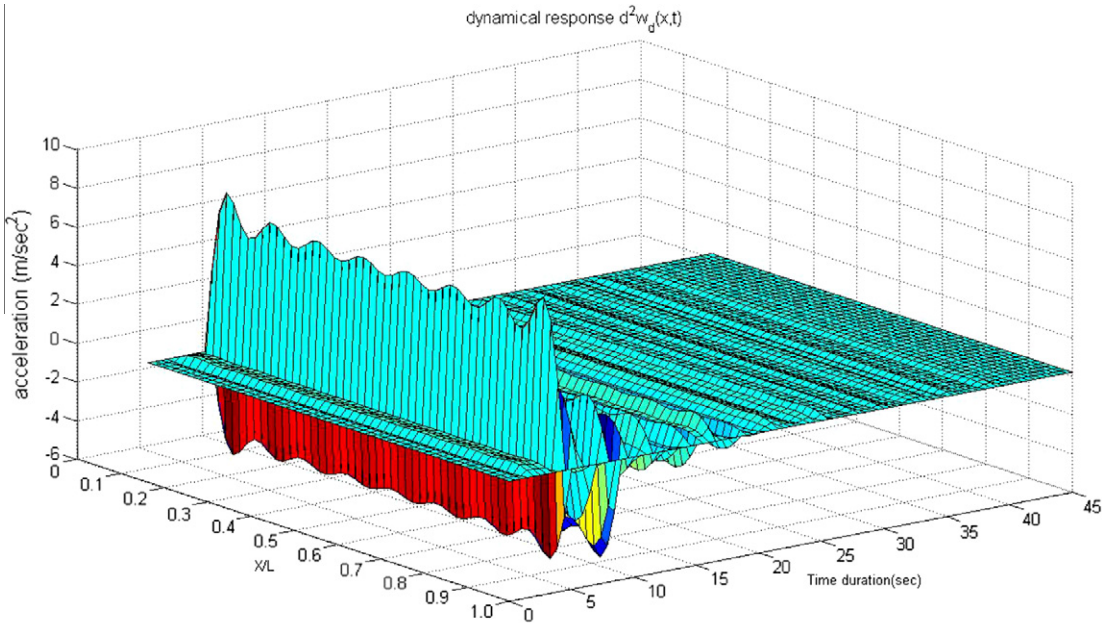

Fig. 11. Dynamic acceleration of the beam.

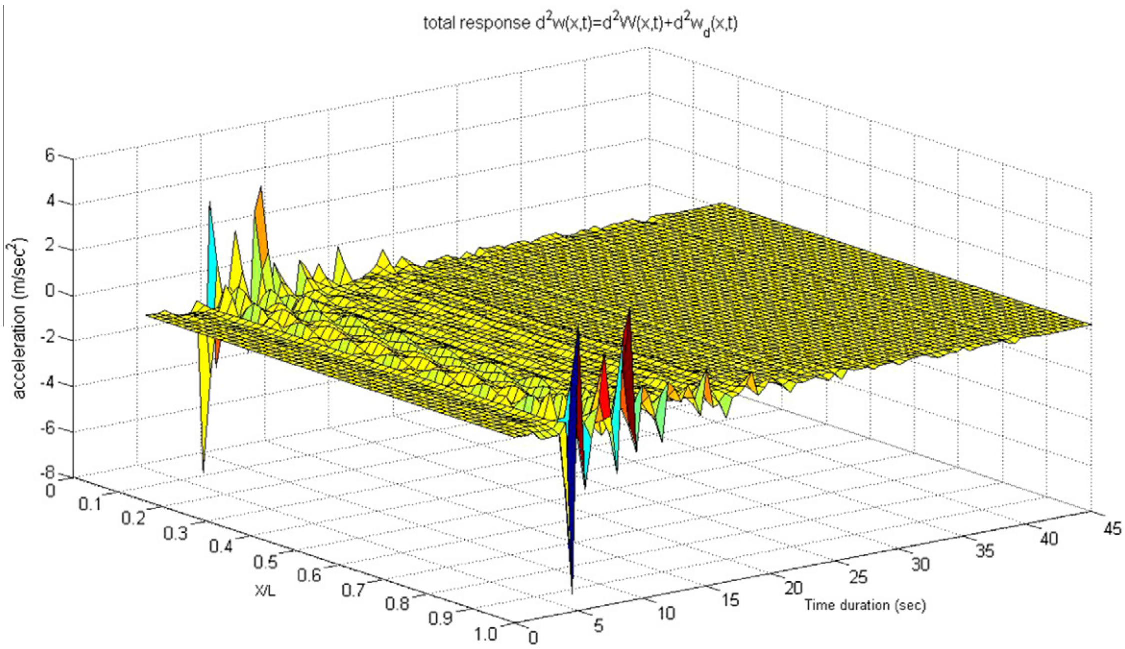

Fig. 12. Total acceleration of the beam.

responses including the deformation and acceleration of the cable part are presented in Figs. 13–18. In Fig. 13, the deformation distribution of quasi-static solution versus time for cable is depicted, and the corresponding ones of the dynamic response and total response are presented in Figs. 14 and 15 individually. Followed by the acceleration distribution versus time of quasi-static solution, dynamic response and the total response are demonstrated in Figs. 16–18 separately. As it can be detected from Figs. 9 and 15, the total deformation of the cable is quite different from that of the beam when the whole system is subjected to the interaction of moving vehicle loads and earthquake loadings. Obviously, the deformation of the cable produces more oscillations than that of the beam since the material property of the cable is more flexible. Based on Figs. 9, 12, 15 and 18, it can be concluded that the interaction of both the moving loads and the seismic forces can substantially amplify the response of long-span suspension bridge system especially in the vicinity of the end supports.

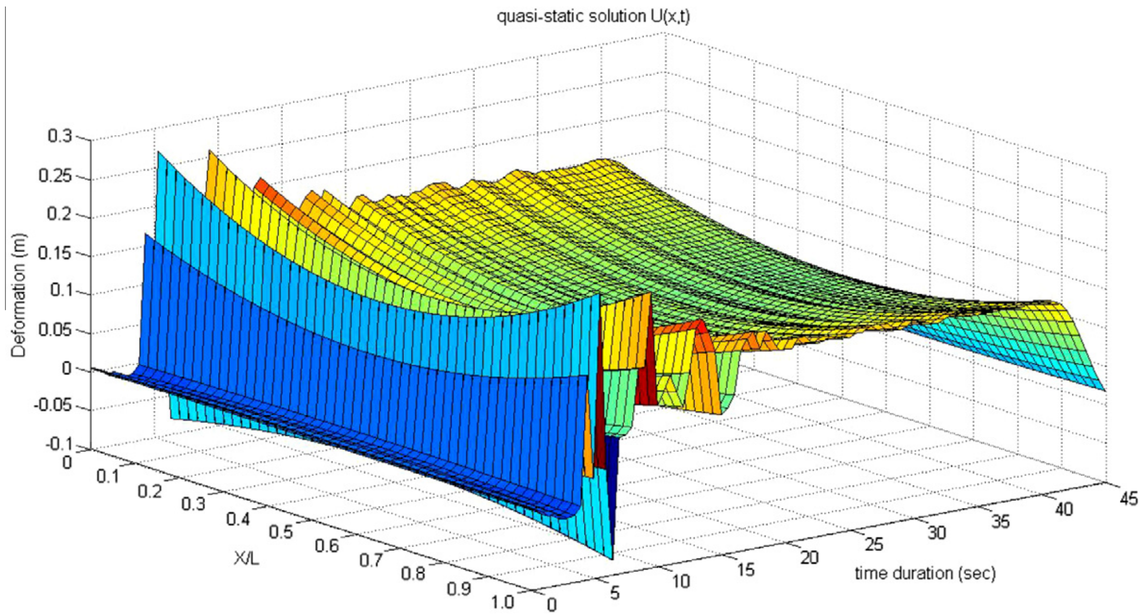

Fig. 13. Quasi-static deformation of the cable.

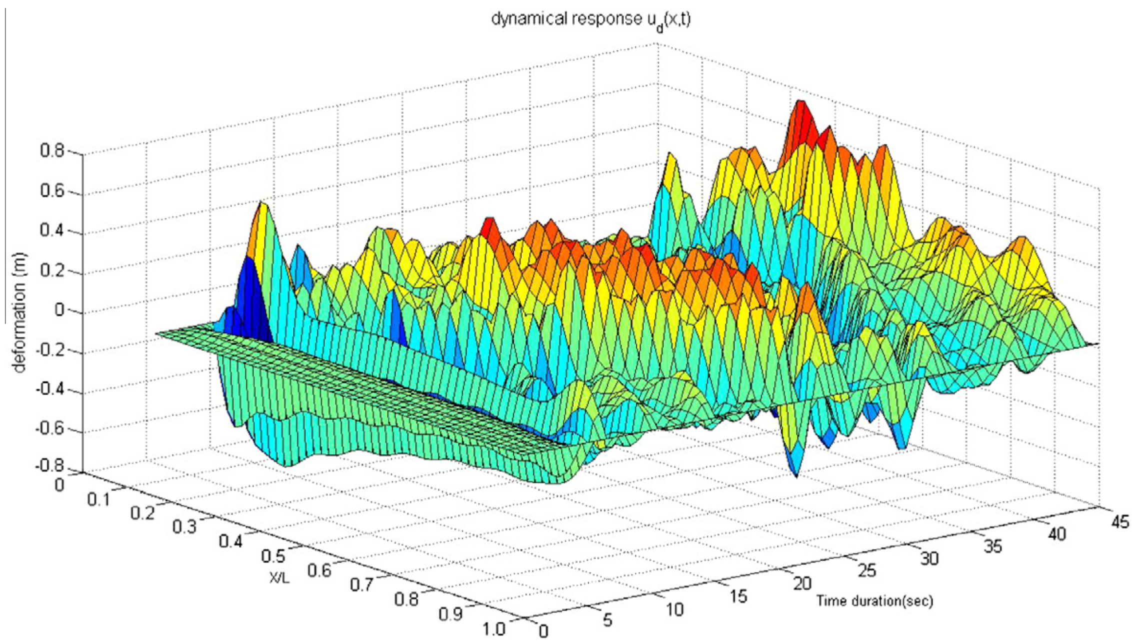

Fig. 14. Dynamic deformation of the cable.

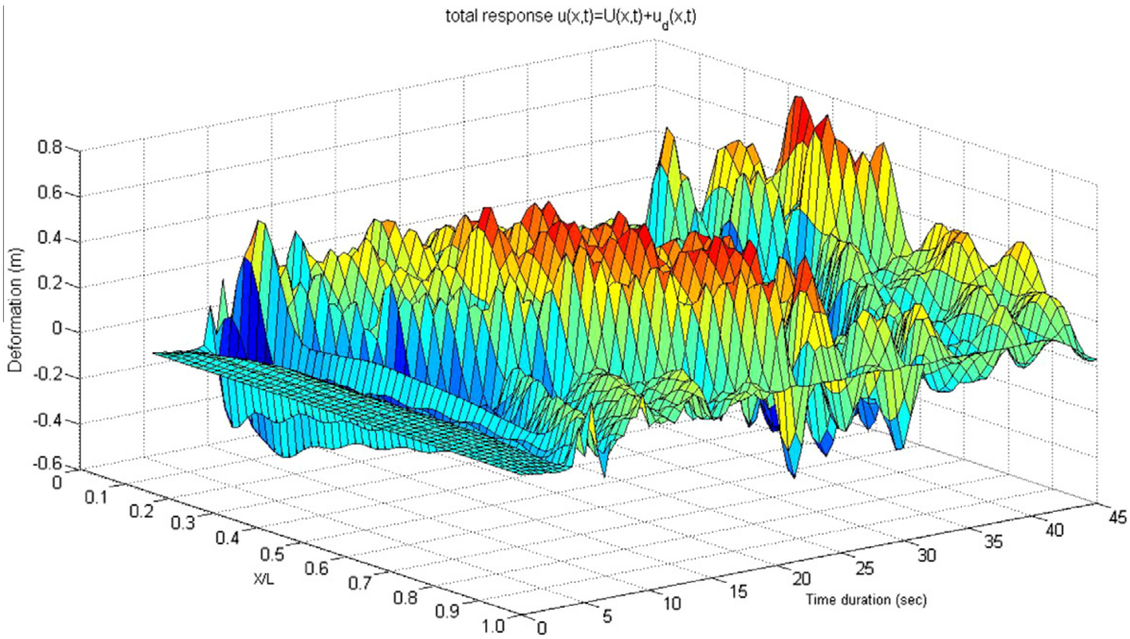

Fig. 15. Total deformation of the cable.

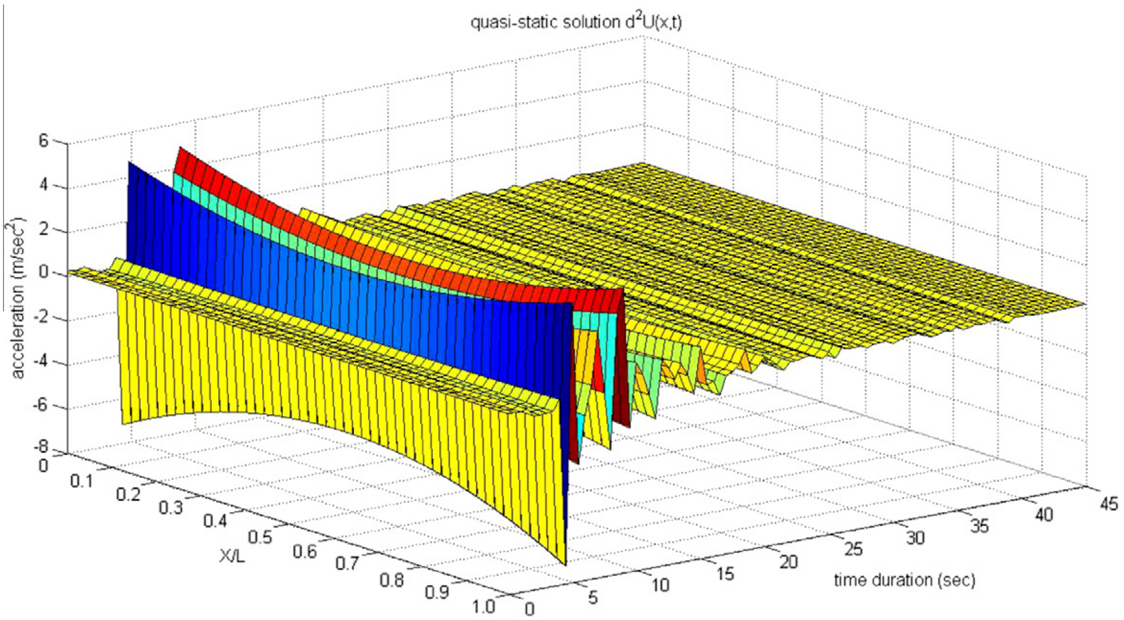

Fig. 16. Quasi-static acceleration of the cable.

5. Conclusions

In the present study, the vibration behavior of a suspension bridge due to moving vehicle loads with vertical support motions caused by earthquake is studied. The suspension bridge system is presented here by two coupled nonlinear cable–beam equations aiming to describe both the dynamic characteristics for the supporting cable and the roadbed, respectively. The dynamic effect of traffic vehicles are modeled as a row of equidistant moving forces, while the earthquake movement is simulated as the vertical oscillation of boundary supports. In order to conduct the interactive traveling waves between the cable and the beam system which is subjected to time-dependent boundary conditions, the total responses of the suspension

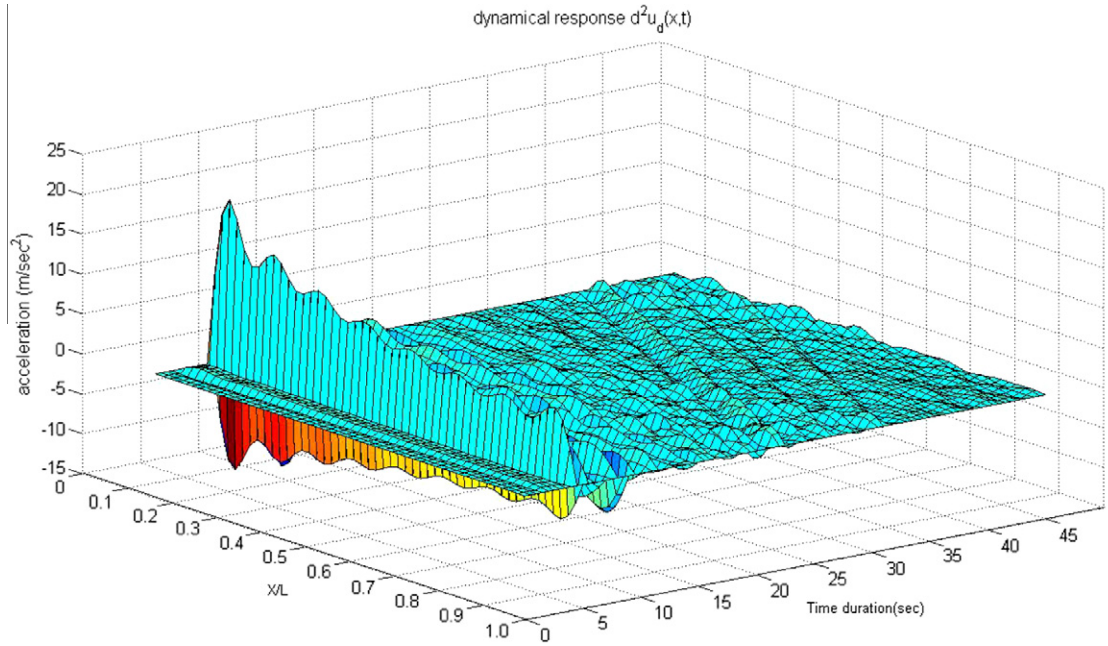

Fig. 17. Dynamic acceleration of the cable.

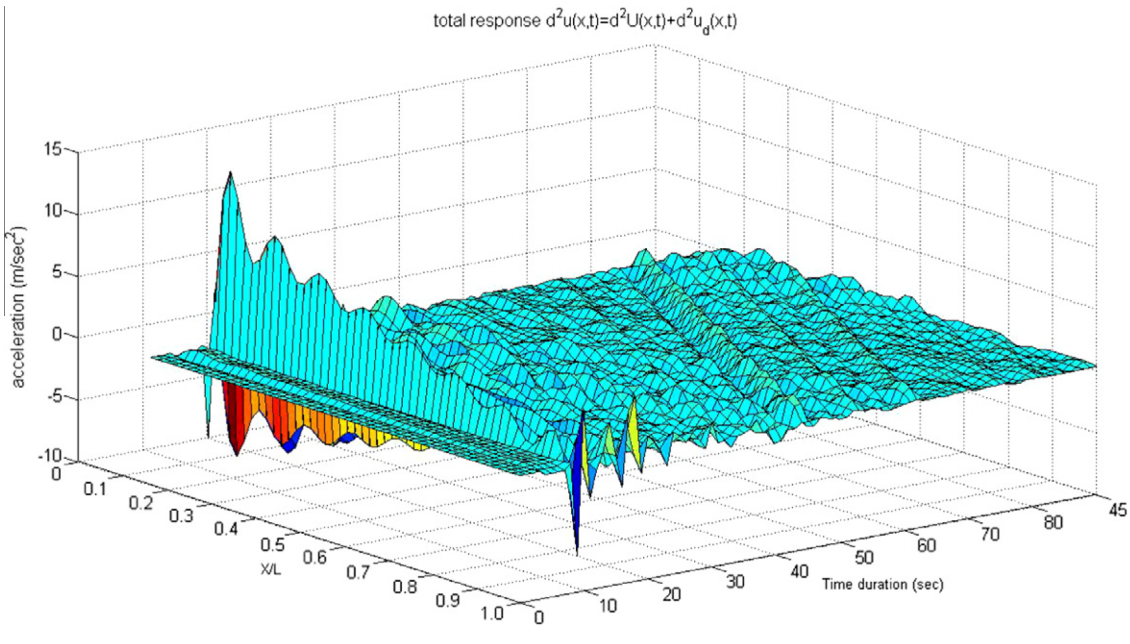

Fig. 18. Total acceleration of the cable.

bridge system are decomposed into two parts: the quasi-static components and the dynamic ones by adopting the decomposition method. In so doing, the governing integro-differential equations are transferred into a set of ordinary differential equations, which can be solved analytically in the present study. After the aforementioned quasi-static components of the cable–beam system under static action of multiple support motions is obtained, the remaining dynamic part can therefore be evaluated numerically by imposing the Galerkin's method. Moreover, the world's largest designed suspended bridge – Messina Bridge – is examined (central span of length 3.3 km) and the modified Kobe earthquake records is applied to the calculations in order to validate the present study and the proposed methodology. As a result, the deformation of the cable produces more oscillations than that of the beam since the material property of the cable is more flexible. It is shown that the interaction of both the moving loads and the seismic forces can substantially amplify the response of long-span suspension bridge system especially in the vicinity of the end supports.

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