Modeling and Analysis of Spacecraft Structures Subject to Acoustic Excitation

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Abstract

Acoustic load and high frequency vibration can severely and adversely affect spacecraft structures and their payloads and is a significant issue found in spacecraft structures. For many classes of structures exhibiting a plate-like vibration behavior, such as antennas and solar panels, their low-order mode response is likely to be of greatest importance. With such motivation, structural-acoustic interaction is modeled and analyzed using boundary and finite element coupling to solve the structure-acoustic interaction problem.

The problem will be idealized in three parts; the calculation of the acoustic radiation from the vibrating structure, the finite element formulation of structural dynamic problem, and the calculation of the acousto-elastic-mechanic fluid-structure coupling using coupled BEM/FEM techniques. The development of the computational scheme for the calculation of the acoustic radiation as well as the structural dynamic response of the structure using coupled BEM/FEM will be elaborated. Some generic examples typical for spacecraft structure are elaborated.

1. Introduction

Structural-acoustic interaction, in particular the vibration of structures due to sound waves, has been addressed by the authors in previous work [1][2][3] The structural design of spacecraft is driven by the severity of the launch environment. The loads transmitted to the spacecraft structure from the launch vehicle (LV) in the first few minutes of flight are far more severe than any load that a payload experiences on orbit. Therefore, payloads are qualified by subjecting them to loads whose magnitude and frequency contents are representative of the launch environment. A more severe environment increases the cost of placing the payload into orbit because it must be designed to withstand higher launch loads.

Acoustic loads are a major component of the launch environment for spacecraft structure. Exterior sound pressure levels on a spacecraft structure during launch as depicted in Fig.1 can reach 150 dB [4,5] depending on the vehicle and the launch configuration. The magnitude of the acoustic loads transmitted to the payload is a function of the external acoustic environment as well as the design of the spacecraft structure and its sound absorbing treatments.

![Fig. 1 Acoustic loads are a major component of the launch environment for spacecraft structure](image)

For many classes of structures exhibiting a plate-like vibration behavior, such as antennas and solar panels as depicted in Fig. 2, their low-order mode response is likely to be of greatest importance. Assessment of combined acoustical and quasi-static loads may be significant. By modeling structural-acoustic interaction using boundary and finite element coupling, it is possible to couple the boundary element method and the finite element method to solve the structure-acoustic interaction problem [6-8].

The propagation of vibration through structures, the radiation of sound from vibrating structures, and fluid/structure interaction are all elements that are significant in structural acoustic problems on spacecrafts. In the case of acoustic excitation, the coupling efficiency depends on how well the sound waves interact with the structures.

The methods and related numerical computation codes in structural acoustics for the prediction of...
noise emitted by structural vibration in all the audible low-, medium- and high-frequency band, play a very important role in the design and the conception of industrial products. These methods allow the design to be improved before construction and optimization with respect to the acoustical problems.

In the Structural-Acoustic Analysis for Aerospace Structure Design problems, it is recognized that Computational Structural Mechanics (CSM) is efficient for structural-acoustic prediction in low-frequency ranges. Analysis of the detailed behavior of individual modes is possible using finite-element and classical methods.

The present work is focused on the formulation of the basic problem of acoustic excitation and vibration of elastic structure in a coupled fluid-elastic-structure interaction. The approach consists of three parts. The first is the formulation of the acoustic field governed by the Helmholtz equation subject to the Sommerfeld radiation condition for the basic acoustic problem without solid boundaries. The interface between the acoustic domain and the surface of the structure poses a particular boundary condition. Boundary element method will be utilized for solving the governing Helmholtz equation subject to the boundary conditions for the calculation of the acoustic pressure on the interface boundary.

The second part deals with the structural dynamic problem, which is formulated using finite element approach. The third part involves the calculation of the acousto-elasto-mechanic fluid-structure coupling, which is formulated using coupled BEM/FEM techniques.

The acoustic loading on the structure is calculated on the part of the boundary of the acoustic domain, which coincides with the structural surface as defined by the problem.

2. Helmholtz Integral Equation for the Acoustic Field

For an exterior acoustic problem, as depicted in Fig. 3, the problem domain \( V \) is the free space \( V_{ext} \) outside the closed surface \( S \). \( V \) is considered enclosed in between the surface \( S \) and an imaginary surface \( \Lambda \) at a sufficiently large distance from the acoustic sources and the surface \( S \) such that the boundary condition on \( \Lambda \) satisfies Sommerfeld’s acoustic radiation condition as the distance approaches infinity.

Green’s first and second theorem provide the basis for transforming the differential equations in the problem domain \( V \) and the boundary conditions on the surface \( S \) into an integral equation over the surface \( S \).

For time-harmonic acoustic problems in fluid domains, the corresponding boundary integral equation is the Helmholtz integral equation [9].

\[
cp(R) = \int_S p(R) \frac{\partial g}{\partial \eta_0} - g\left(\left|R - R_0\right|\right) \frac{\partial p}{\partial \eta_0} dS \tag{1}
\]

where \( \eta_0 \) is the surface unit normal vector, and the value of \( c \) depends on the location of \( R \) in the fluid domain, and where \( g \) the free-space Green’s function. \( R_0 \) denote a point located on the boundary \( S \), as given by

\[
g\left(\left|R - R_0\right|\right) = \frac{e^{-ik\left|R - R_0\right|}}{4\pi\left|R - R_0\right|} \tag{2}
\]

To solve Eq. (1) with \( g \) given by Eq. (2), one of the two physical properties, acoustic pressure and normal velocity, must be known at every point on the boundary surface. The specific normal impedance, which describes the relationship between pressure and normal velocity, can also serve as a boundary condition. At the infinite boundary \( \Lambda \), the Sommerfeld radiation condition in three dimensions can be written as [9]:

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Fig. 2 Typical spacecraft structure with a plate-like behavior such as antenna, solar panel and a box as main spacecraft structure

Fig. 3 Exterior problem for homogeneous Helmholtz equation
For scattering problems, Eq. (1) only gives the solution to the scattered wave. The boundary conditions are however given in terms of the total wave, which is the sum of the incident and the scattered waves. It is necessary to modify the equation to include the incident wave. For exterior scattering problems, the modification can be carried out by adding to the scattered wave integral equations the result of applying the interior Helmholtz equations to the incident wave \( p_{\text{inc}} \). Then the integral equation for the total wave is given by

\[
 \int \left[ p(R) \frac{\partial g(R-R_e)}{\partial n} \frac{\partial p}{\partial n} - g(R-R_e) \right] dS
\]

where \( p = p_{\text{inc}} + p_{\text{scatter}} \), and where

\[
 c = \begin{cases} 
 1 & R \in V_{\text{mat}} \\
 1/2 & R \in S \\
 \Omega/4\pi & R \in S \text{ (non smooth surface)} \\
 0 & R \in V_{\text{in}} 
\end{cases}
\]

The Helmholtz equation than can be discretized by dividing the continuous system into a discrete one with \( N \) number of elements. Following the standard procedure in defining the elements on the boundary surface \( S \), the discretized boundary integral equation becomes,

\[
 cp - p_{\text{inc}} - \sum_{j} \left[ p \left. \frac{\partial g}{\partial n} \right|_{S_j} dS = \sum_{j} g \left. \frac{\partial p}{\partial n} \right|_{S_j} dS \right]
\]

where \( i \) indicates field point, \( j \) source point and \( S_j \) surface element \( j \). The discretized equation forms a set of simultaneous linear equations, which relates the pressure \( p_i \) at field point \( i \) due to the boundary conditions \( p \) and \( v \) at surface \( S_j \) of the source element \( j \) and the incident pressure \( p_{\text{inc}} \). In matrix form:

\[
 \mathbf{H} p = i \rho_0 \omega \mathbf{G} v + p_{\text{inc}}
\]

where, \( \mathbf{H} \) and \( \mathbf{G} \) are two \( N \times N \) matrices of influence coefficients, while \( p \) and \( v \) are vectors of dimension \( N \) representing total pressure and normal velocity on the boundary elements. This matrix equation can be solved if the boundary condition \( \partial p/\partial n \) and the incident acoustic pressure field \( p_{\text{inc}} \) are known.

### 3. BEM-FEM Acoustic-Structural Coupling

Two approaches can be followed to treat the structural-acoustic coupling on the boundary element and finite element interface region [10]. In the first approach, the BE region is treated as a super finite element and its stiffness matrix is computed and assembled into the global stiffness matrix and is identified as the coupling to finite elements. In the second approach, the FE region is treated as an equivalent BE region and its coefficient matrix is determined and assembled into the global coefficient matrix and is identified as the coupling to boundary elements. In the present study, the first approach is considered. The state of affairs is schematically depicted in Fig. 4.

The FEM leads to a system of simultaneous equations which relate the displacements at all the nodes to the nodal forces. In the BEM, on the other hand, a relationship between nodal displacements and nodal tractions is established.

The elastic structure can be represented by FE model. For a dynamic structure, the equation of motion is given by \([11-13]\)

\[
 \begin{bmatrix} \mathbf{M} & \mathbf{C} & \mathbf{K} \end{bmatrix} \{ \dot{x} \} + \begin{bmatrix} \mathbf{C} & \mathbf{K} \end{bmatrix} \{ x \} + \mathbf{K} \{ x \} = \{ F \}
\]

where \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are structural mass, damping and stiffness, respectively, which are expressed as matrices in a FE model. \( \mathbf{F} \) is the given external forcing function vector, and \( \{ x \} \) is the structural displacement vector. Taking into account the acoustic pressure \( p \) on the structure at the fluid-structure interface as a separate excitation force, the acoustic-structure problem can be obtained from Eq.(8) by introducing a fluid-structure coupling term given by \( \mathbf{L} p \) \([7]\). It follows that

\[
 \begin{bmatrix} \mathbf{M} & \mathbf{C} & \mathbf{K} \end{bmatrix} \{ \dot{x} \} + \begin{bmatrix} \mathbf{C} & \mathbf{K} \end{bmatrix} \{ x \} + \mathbf{K} \{ x \} + \mathbf{L} \{ p \} = \{ F \}
\]

where \( \mathbf{L} \) is a coupling matrix of size \( M \times N \) in the BEM/FEM coupling thus formulated. \( M \) is the number of FE degrees of freedom and \( N \) is the number of BE nodes on the coupled boundary. The global coupling matrix \( \mathbf{L} \) transforms the acoustic fluid pressure acting on the nodes of boundary elements on the entire fluid-structure interface surface \( a \), to nodal forces on the finite elements of the structure. Hence \( \mathbf{L} \) consists of \( n \) assembled local transformation matrices \( \mathbf{L}_{x} \), given by

\[
 \mathbf{L}_{x} = \left[ N_{x}^{T} n_{N_{x}} dS \right]
\]
in which $N_F$ is the shape function matrix for the finite element and $N_B$ is the shape function matrix for the boundary element. It can be shown that:

$$N_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} [N_i]$$ (11)

The rotational parts in $N_F$ are neglected since these are considered to be small in comparison with the translational ones in the BE-FE coupling, consistent with the assumptions in structural dynamics as, for example, stipulated in [13].

For the normal fluid velocities and the normal translational displacements on the shell elements at the fluid-structure coupling interface a relationship has to be established which takes into account the velocity continuity over the coinciding nodes:

$$v = i\omega (T x)$$ (12)

Similar to $L$, $T$ $(n \times m)$ is also a global coupling matrix that connects the normal velocity of a BE node with the translational displacements of FE nodes obtained by taking the transpose of the boundary surface normal vector $n$ [7,10]. The local transformation vector $T_e$ can then be written as:

$$T_e = n^T$$ (13)

The normal fluid velocities of the acoustic problem and the normal translation of the structural surface-fluid interface have to satisfy a certain relationship which takes into account the velocity continuity over the coinciding nodes.

The presence of an acoustic source can further be depicted by Fig.5. Two regions are considered, i.e. $a$ and $b$ ; region $a$ is the afore mentioned fluid-structure interface region, where FEM mesh and BEM mesh coincide and region $b$ is the region where all of the boundary conditions (pressure or velocity) are known.

![Fig 5. Schematic FE-BE problem representing quarter space problem domains](image)

For the coupled FEM-BEM regions, Eq. (9) and Eq. (7) apply at $\{x\} \in a$. Based on BEM-FEM model, BEM equation can now be written as:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} p_a \\ p_b \end{bmatrix} = i\rho_0\omega \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \begin{bmatrix} p_{inc} \\ p_{inc} \end{bmatrix}$$ (14)

Using Eq.(12) for $v_a$, the BEM Eq. (14) can be modified accordingly as:

$$\begin{cases} H_{11} p_a + H_{12} p_b = -\rho_0 \omega \dot{G}_{11} T x + i\rho_0\omega G_{12} v_b + p_{inc} \\ H_{21} p_a + H_{22} p_b = -\rho_0 \omega \dot{G}_{21} T x + i\rho_0\omega G_{22} v_b + p_{inc} \end{cases}$$ (15)

or:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} p_a \\ p_b \end{bmatrix} = -i\rho_0 \omega \begin{bmatrix} \dot{G}_{11} T x \\ \dot{G}_{21} T x \end{bmatrix} + \begin{bmatrix} i \rho_0\omega G_{12} v_b \\ i \rho_0\omega G_{22} v_b \end{bmatrix} + \begin{bmatrix} p_{inc} \\ p_{inc} \end{bmatrix}$$ (16)

If the velocity boundary condition on $b$ is known, reorganizing the unknowns to the left side, Eq.(16) become:

$$\rho_0 \omega \dot{G}_{11} T x + H_{11} p_a + H_{12} p_b = i\rho_0\omega G_{12} v_b + p_{inc}$$

$$\rho_0 \omega \dot{G}_{21} T x + H_{21} p_a + H_{22} p_b = i\rho_0\omega G_{22} v_b + p_{inc}$$

(17)

Since the pressure $p$ on FEM equation lies in region $a$, Eq. (9) can be written as:

$$\begin{bmatrix} \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} \mathbf{K} \end{bmatrix} \begin{bmatrix} \dot{x} \\ v \end{bmatrix} + \begin{bmatrix} \mathbf{L} \end{bmatrix} \begin{bmatrix} p_a \end{bmatrix} = \begin{bmatrix} \mathbf{F} \end{bmatrix}$$ (18)

Following the general practice in structural dynamics, solutions of Eq.(18) are sought by considering synchronous motion with harmonic frequency $\omega$. Correspondingly, Eq. (17) reduces to:

$$\begin{bmatrix} \mathbf{K} - i\omega \mathbf{C} + \omega^2 \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} \mathbf{L} \end{bmatrix} \begin{bmatrix} p_a \end{bmatrix} = \begin{bmatrix} \mathbf{F} \end{bmatrix}$$ (19)

where

$$x = \bar{x}e^{i\omega t} ; \quad p_a = \bar{p}_a e^{i\omega t}$$ (20)

or, dropping the bar sign for convenience, but keeping the meaning in mind, Eq. (19) can be written as:

$$\begin{bmatrix} \mathbf{K} - i\omega \mathbf{C} + \omega^2 \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \dot{v} \end{bmatrix} + \begin{bmatrix} \mathbf{L} \end{bmatrix} \begin{bmatrix} p_a \end{bmatrix} = \begin{bmatrix} \mathbf{F} \end{bmatrix}$$ (21)

Combining Eq. (16) and Eq. (21), the coupled BEM-FEM equation can then be written as:

$$\begin{bmatrix} \mathbf{K} - i\omega \mathbf{C} + \omega^2 \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 \\ \rho_0 \omega \dot{G}_{11} T x \\ \rho_0 \omega \dot{G}_{21} T x \end{bmatrix}$$

$$\begin{bmatrix} p_a \end{bmatrix} = \begin{bmatrix} i \rho_0\omega G_{12} v_b \\ i \rho_0\omega G_{22} v_b \end{bmatrix} + \begin{bmatrix} p_{inc} \\ p_{inc} \end{bmatrix}$$ (22)

This equation forms the basis for the treatment of the fluid-structure interaction in a unified fashion. The solution vector consisting of the displacement vector of the structure and total acoustic pressure on the boundaries of the acoustic domain, including the acoustic-structure interface, is represented by $\{x \ p_a \ p_b\}^T$, while $\{0 \ p_{inc} \ p_{inca}\}^{T\ T}$ is the acoustic excitation vector. Due consideration should be given to $\mathbf{L}$, where $\mathbf{L}$ is a coupling matrix of size $m \times n$, $m$ is the number of FE degrees of freedom and $n$ is the number of BE nodes on the coupled (interface) boundary $a$.

The detail of the Finite Element Formulation of the Structure has been elaborated in previous work [1][2][3], and will not be repeated here. It suffices to
mention that two types of elements have been considered to be utilized; these are an eight node hexahedral for solid modeling and four node quadrilateral elements for shell modeling.

The boundary integral equation of equation (4) fails at frequencies coincident with the interior cavity frequencies of homogeneous Dirichlet boundary conditions [14]. In the case of the formulation of the exterior problem, these frequencies correspond to the natural frequencies of acoustic resonances in the interior region. When the interior region resonates, the pressure field inside the interior region has non-trivial solution. Since the interior problem and the exterior problem shares similar integral operators, the exterior integral equation may also break down. The discretized equation of the [H] matrix in equation (14) becomes ill-conditioned when the exciting frequency is close to the interior frequencies, thus providing an erroneous acoustic loading matrix. This problem could be overcome by using the CHIEF[15][16], Burton Miller method[17][18], and a recent technique utilizing SVD and Fredholm alternative theorem[18] [19].

For the purpose of this study, such problem can be avoided by limiting the problem only for low frequency range; further treatment for this problem will be elaborated in a separate paper.

4. Case Study A-The Flexible Structure Subject to Harmonic External Forces in Acoustic Medium

Several cases are considered to validate the program as well as to evaluate its performance. An example is carried out for a box with vibrating membrane as shown in Fig. 6. The structure consists of five hard walls and one flexible membrane on the top and has the dimension \(a \times b \times c = 304.8 \times 152.4 \times 152.4 \text{ mm}\). The flexible structure, which consists of a 0.064 mm thick undamped aluminum plate, is modeled with coupled boundary and finite elements.

The frequency response due to the application of arbitrary harmonic excitation forces to the membrane following the present method using BEM-FEM Coupling is shown in Fig. 7, and is compared to results obtained using FEM-FEM approach [7]. Comparable agreement has been indicated and serves to validate the present method.

5. Case Study B-The Flexible Structure Subject to Acoustic Excitation in a Confined Medium

Application of the method to another example is carried out for a box shown in Fig. 8 with a dimension of \(a \times b \times c = 450 \times 450 \times 270 \text{ mm}\). Five walls of the structure are modeled as shell elements and the bottom of the box is modeled as a rigid wall. Each of the flexible walls is assumed to be made of 1 mm aluminum plate, and is modeled as coupled boundary and finite elements.

This box structure is subjected to an acoustic monopole source at the center of the box and the acoustic medium is air with the following properties: density, \(\rho = 1.21 \text{ kg/m}^3\) and sound velocity \(c = 340 \text{ m/s}\). The discretization of the box is also depicted in Fig. 8.

5.1. Normal Mode Analysis

The result of the modal analysis of the box to obtain the first three normal modes using finite element program developed in MATLAB is shown in Fig. 9.
The first five eigen frequencies for the shell elements obtained using the present routine are compared to those obtained using commercial package NASTRAN®. As exhibited in Table 1, good agreement is obtained.

Fig. 9 First three normal modes for box modeled with five flexible walls using shell elements

Table 1 First five Eigen frequencies for box modeling with shell element

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequency (Hz)</th>
<th>MATLAB®</th>
<th>NASTRAN®</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.644</td>
<td>16.190</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36.026</td>
<td>35.986</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>41.072</td>
<td>41.520</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>44.585</td>
<td>44.787</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>51.231</td>
<td>51.689</td>
<td></td>
</tr>
</tbody>
</table>

5.2. Acoustic Excitation

Acoustic excitation due to an acoustic monopole source with initial frequency, $f = 10$ Hz, is applied at the center of the box. No other external forces are applied. The resulting distribution of the incident acoustic pressure is shown in Fig. 10.

Fig. 10 Incident acoustic pressure distribution on the surface of box due to monopole acoustic excitation

The total pressures on the surface of the box obtained from the computational results are shown in Fig.11. The frequency response for the incident pressure and total pressure on the center top surface of the box computed using Eq. (22) is shown in Fig. 12.

Fig. 11 Total acoustic pressure distribution on the surface of box due to monopole acoustic excitation

Fig. 12 Incident and total acoustic pressure distribution on the center top surface of box as a function of frequency

6. Case Study C - Numerical Simulation of Flat Plate Subject to Acoustic Excitation in an Infinite Medium

The present method is then applied to a flat plate spacecraft structure. For the purpose of this study, the dimension of the flexible structure is 450 x 150 x 5 mm. The material properties for flat plate made of AISI 4130 Steel is as follows: Modulus of Elasticity, $E = 29 \times 10^6$ N/m$^2$, Shear Modulus, $G = 11 \times 10^6$ N/m$^2$, Poisson’s Ratio $\nu = 0.32$ and density, $\rho = 7.33145 \times 10^{-4}$ Kg/m$^3$

The flexible structure is now modeled with Finite Element and the surrounding boundary is represented by a quarter space is modeled using Boundary Elements; typical boundary element discretization of the surface, and the finite element discretization of the plate, is exhibited in Fig. 13.
The monopole acoustic source is placed at the center of the plate, at a distance 0.1 m above it.

For illustrative purposes, the frequency of the monopole source is assumed to be 10 Hz and the fluid medium is air with density $\rho = 1.21$ kg/m$^3$ and sound velocity $c = 340$ m/s. The incident pressure distributions are depicted in Fig.14 and the total acoustic pressure distribution is shown in Fig.15.

From the results of these case studies, some fundamental aspects of the computational procedure can be summarized:

1. The computational procedure for solving combined excitation due to acoustic and external forces on structural problem formulated as coupled FEM-BEM equation has given good results.
2. The solution of Eq.(22) as a result of the effect of acoustic disturbance to a structure is given as the pressure loading response on the structure as well as the pressure field in the fluid medium, given as total pressure. This allows the calculation of the scattering pressure due to the incident pressure.

3. Conclusions

A method and computational procedure for BEM-FEM Acousto-Elasto-Mechanic Coupling has been developed. The method is founded on three generic elements; the calculation of the acoustic radiation from the vibrating structure, the finite element formulation of structural dynamic problem, and the calculation of the acousto-elasto-mechanic fluid-structure coupling using coupled BEM/FEM techniques. The procedure developed has been validated by reference to classical examples or others available in the literature. The applicability of the method for analyzing typical problems encountered in space structure has been demonstrated.
Refinements of the method for fast computation and for complex geometries are in progress. Such steps may be useful for acoustic load qualification tests as well as structural health monitoring.

References


