The Use of the Tangential Differential Operator in the Boundary Integral Equation for Stresses and the Dual Boundary Element Method

L Palermo Jr[1], LPCPF Almeida[1], PC Gonçalves[1]

[1] Universidade Estadual de Campinas, Faculdade de Engenharia Civil, Arquitetura e Urbanismo, Brazil, leandro@fec.unicamp.br.

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Abstract. Accurate values for stresses at the boundary may be evaluated with the stress boundary integral equation (BIE) [1]. Nevertheless, the differentiation of the kernels of integrals in the displacement equation to obtain a BIE for stresses increases the order of the kernel singularity and an additional care is necessary to treat the improper integrals. The use of the tangential differential operator (TDO) in the stress BIE is an interesting procedure when Kelvin type fundamental solutions are employed. The order of the kernel singularity is reduced with this strategy [2, 3, 4] and the Cauchy principal value sense or the first order regularization can be used in the resultant BIE.

Several strategies have been employed to analyze crack problems such as the displacement discontinuity method [5, 6], the crack Green's function method [7, 8], the subregion (or subdomain) method [9] and the dual boundary element method (DBEM) [10, 11, 12]. The dual equations of the method are the displacement and the traction BIEs. When the displacement BIE is applied to one of the crack surfaces and the traction equation to the other, general mixed-mode crack problems can be solved with a single domain formulation. Although the integration path is still the same for coincident points on the crack surfaces, the respective boundary integral equations are now distinct. The collocation point position to perform the traction boundary integral equation and the strategy used to treat improper integrals are the essential features of the formulation.

Plane problems containing an internal or an edge crack are studied in this paper. Linear shape functions were employed to approximate displacements and efforts in the boundary elements. The same shape function was used for conformal and non-conformal interpolations with nodal parameters positioned at the ends of the elements. The collocation points were shifted to the interior of the element at a distance of a six part of its length from the end. The number of collocation points in the element was defined by the computer code according to the condition of the last node, which means that elements with the discontinuity at the first node had one collocation point. The present numerical implementation was studied in [13] for the dual formulation with traction BIE containing a hypersingularity of order r^{-2}. The main feature shown in that study was the use of conformal interpolations along the crack surfaces without losing the accuracy of the dual formulation.

The dual formulation with TDO in the traction BIE is analyzed in the present study. The derivatives of the adopted shape function for displacements (linear functions) are used in the traction equation as required for the TDO. The use of non-conformal interpolations required that the effect of the ends was introduced in the integration by parts to obtain the TDO. The near-tip displacement extrapolation was used to obtain stress intensity factors as explained in [10, 13].

The numerical results obtained were close to literature values. A minimum difference was noted in the results presented in [10, 13] and those obtained using the TDO, which have the benefit of the reduction of the order of the singularity. The use of derivatives of the adopted shape function for displacement without using other interpolation for TDO was an interesting alternative. It is important to note that constant values were obtained as derivatives of the linear shape function and the results were not degraded. Regarding the present numerical implementation, a conformal interpolation on the crack surface can be used without losing the accuracy of the dual formulation even when the BIE for tractions employs the TDO on low order elements.

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