A linear theory for wave scattering by double slotted barriers in weak steady currents[1]

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ABSTRACT

Recent experimental results have shown that the presence of a steady current can significantly reduce the energy of transmitted waves. In this paper, a theory is developed to study the wave scattering by single or double vertical slotted barriers in the presence of a weak uniform current. The quasi-linear theory is based on an eigenfunction expansion method. Comparisons with existing experimental results for both single slotted barrier and double slotted barriers show satisfactory agreements. For waves propagating in a weak current it is found that the friction factor used to characterize the head loss at the slotted barrier depends on both the geometry of the slotted barrier and the strength of the steady current.

Key words: *Surface waves; Wave scattering; Breakwaters; Wave-current interactions*

1. Introduction

Slotted or perforated breakwaters have several advantages over traditional rubble mound breakwaters: (1) slotted or perforated breakwaters allow the exchange of seawaters between harbors and open seas, especially the water exchange forced by tidal currents and the wind-driven currents; (2) the gaps or holes in the slotted/perforated walls may also allow fish to pass through, reducing the adverse impact of traditional breakwaters on the ecosystem inside the harbor; (3) In deep water, the reduced construction cost is another advantage of this type of breakwaters.

In the past, most theoretical studies of wave scattering by slotted or perforated barriers focused on pure waves. The methods used fall into two groups: (1) eigen-function expansion methods and (2) boundary element methods (BEM)/integral equation method. The eigen-function methods can provide analytical analysis, but can be used only for vertical

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barriers. The BEM is more versatile, but the resulting singular integral equation needs to be solved numerically. In the category of eigen-function expansion methods, Hayashi et al.(1966) first developed a linear theory for the scattering of long waves by a slotted wall. For a similar problem, Mei et al. (1974) performed a nonlinear analysis, and examined the effects of the higher harmonics on the scattering of long waves by a single slotted barrier (see also, Mei (1989)).Bennett et al.(1992) extended the theory of Mei et al. (1974) to surface waves in water of intermediate depth. Kriebel (1992) developed a simple expression for regular wave transmission through a single slotted wall. Kakuno and Liu (1993) studied, with a matched asymptotic expansion method, the linear scattering of water waves by an array of vertical cylinders. Isaacson et al. (1998) and Isaacson et al. (1999) studied the linear wave scattering by single slotted barrier and double slotted barriers, respectively. Zhu and Chwang (2001) studied the slotted seawalls. Recently, Li et al. (2003) and Teng et al. (2004) studied the interaction of oblique waves with perforated breakwaters/seawalls. Huang (2007b) revisited the problem of the wave scattering by two closely-spaced rectangular cylinders. In the category of boundary element method/integral equation method, Liu and Abbaspour (1982) studied the wave interaction with perforated/slotted structures. Hagiwara (1984) derived a set of integral equations for wave scattering by multiple slotted structures. Chen et al. (2004) examined the wave scattering by a thin surface-piercing impermeable barrier. All the studies mentioned above did not consider the effects of currents.

Recent experimental results (Huang (2007a),Huang (2007c)) showed that the presence of a steady current can significantly reduce the energy of transmitted waves. For long surface waves, Huang and Ghidaoui (2007) provided a theory to study the effects of current on wave scattering by a single slotted barrier. This paper is an extension of the long wave theory by Huang and Ghidaoui(2007) to intermediate waves interacting with double slotted barriers. A linear theory based on eigen-function expansion method is presented in section (2).Comparisons with existing theoretical and experimental results are provided in section (3). Discussion and conclusions are given in section (4).

2 .Theoretical analysis

Let the x-coordinate be pointing in the direction of wave propagation, and the horizontal velocity in the x-direction be denoted by u. The z-coordinate points vertically upwards with its origin at the still water level. In this coordinate system, the bottom is located at $z = -h$, and the moving surface is described by $z = \eta(x, t)$ with t being the time. A wave barrier, in the form of double slotted walls of porosity ε, extends from the bottom to the surface. The distance between these two slotted walls is B. Considered here are regular waves propagating in a current, which can be represented by a uniform current V, as shown in figure (1). The regular waves are assumed to be normal to the double slotted walls, which are located at $x = 0$ and B, respectively.

The total horizontal velocity and the total surface displacement may be written as

$$u = V + \tilde{u},\ \ \eta = \overline{\eta} + \tilde{\eta} \quad , \tag{1}$$

where the over-bar represents the time-mean component, and the tilde represents the fluctuating component. The wave-induced set-up or set-down, which contribute to $\bar{\eta}$, normally is small and will be ignored in this analysis. Therefore, the mean surface displacement $\bar{\eta}$ is due purely to the loss of the energy associated with the mean flow through the barrier. The wave-induced secondary currents are also ignored in this study, thus the time-mean velocity V is the same as that in the absence of waves. For the case where the porosity is not very small and the time-mean velocity is not too large, the variation of the time-mean current along the flume can be neglected. In this paper, a time-mean current V, which is uniform over the depth and along the flume, will be used, which implies that $\bar{\eta} = 0$.

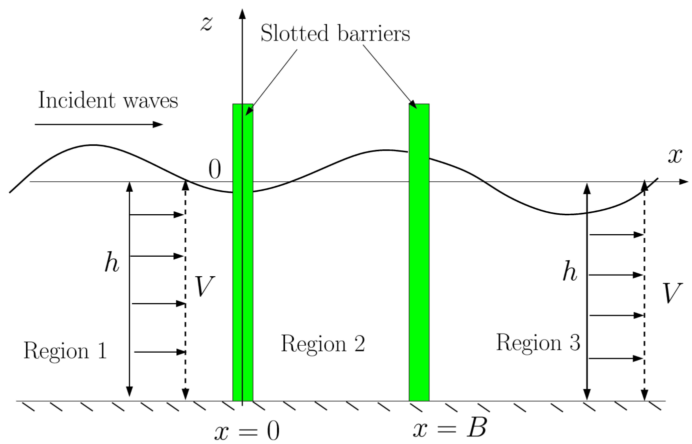

Fig. 1. Definition sketch for the interaction between a slotted barrier and surface waves riding on a following current

We study monochromatic incident waves whose surface displacement $\tilde{\eta}$ can be written as

$$\tilde{\eta} = a\,\mathrm{Re}(e^{i(\omega t-\gamma kx)}) \quad , \tag{2}$$

where the operator Re means taking the real part of its argument. ω is the wave angular frequency, k the wave number, and a the wave amplitude. The wave phase angle is set to zero by adjusting the origin of the time t. For waves propagating in the positive x-direction $\gamma=1$, otherwise $\gamma=-1$.

When this train of linear incident waves interacts with the slotted barriers, part of wave energy will be reflected back, while part of wave energy will be transmitted through the barriers. The amount of wave energy that is dissipated by the slotted barrier is the difference between the incident wave energy and sum of the reflected wave energy and transmitted wave energy. When there is no current, it can be shown that the evanescent waves have only trivial solution (see Fugazza and Natale (1992) for example). When a current is presented in the wave field, the left-going waves and right-going waves will have different wave lengths. To match the velocities and pressures at the slotted barrier, evanescent waves must have non-trivial solutions. In the following, it is assumed that the mean current is weak so that the change of the mean water level across the slotted barriers can be ignored. In their study of the interaction between waves and the slotted/perforated barrier without the presence of currents, Mei et al. (1974) showed that the higher harmonics, owing to the nonlinear matching condition at the barrier, contribute little to the wave reflection and transmission coefficients of the first harmonic. Therefore, the higher harmonics generated by the barrier will be ignored here.

2.1 Formal linear wave solutions

It is assumed that away from the slotted walls wave motions in various regions can be approximately viewed as being irrotational to the first order in wave slope. Referring to Figure (1), the surface displacement, the horizontal component of wave orbital velocity and wave-induced pressure in various regions can be written as

$$\tilde{\eta}_j = a\,\mathrm{Re}\left(\sum_{n=0}^{\infty} T_{n,j} e^{i(\omega t - k_n^+ x)} + \sum_{n=1}^{\infty} R_{n,j} e^{i(\omega t + k_n^- x)}\right), \tag{3}$$

$$\tilde{u}_j = \omega a\,\mathrm{Re}\left(\sum_{n=0}^{\infty} T_{n,j} U_n^+(z) e^{i(\omega t - k_n^+ x)} + \sum_{n=1}^{\infty} R_{n,j} U_n^-(z) e^{i(\omega t + k_n^- x)}\right), \tag{4}$$

$$\tilde{p}_j = \rho\omega^2 ha\,\mathrm{Re}\left(\sum_{n=0}^{\infty} T_{n,j} P_n^+(z) e^{i(\omega t - k_n^+ x)} + \sum_{n=0}^{\infty} R_{n,j} P_n^-(z) e^{i(\omega t + k_n^- x)}\right), \tag{5}$$

where $n = 0$ is taken for progressive waves and $n > 1$ is taken for the nth evanescent waves. $R_{n,j}$ and $T_{n,j}$ are the reflection and transmission coefficients of nth wave mode in the jth region. For our problem, $T_{0,1} = 1$, $R_{0,3} = 0$, and $T_{n,1} = 0$, $R_{n,3} = 0$ for $n > 0$. Other values of $R_{n,j}$ and $T_{n,j}$ are unknowns. For the slotted barriers, the overall reflection R and transmission coefficient T are defined by

$$R = R_{0,1}, \qquad T = T_{0,3}\,. \tag{6}$$

In equations (3)-(5), the shape functions, $U_n^+(z)$, $U_n^-(z)$, $P_n^+(z)$, $P_n^-(z)$, are defined by

$$U_n^+(z) = (1 - \vartheta k_n^+ h)\frac{\cosh(k_n^+(h+z))}{\sinh(k_n^+ h)}, \tag{7}$$

$$U_n^-(z) = -(1 + \vartheta k_n^+ h)\frac{\cosh(k_n^-(h+z))}{\sinh(k_n^- h)}, \tag{8}$$

$$P_n^+(z) = \frac{(1 - \vartheta k_n^+ h)^2}{k_n^+ h}\frac{\cosh(k_n^+(h+z))}{\sinh(k_n^+ h)}, \tag{9}$$

$$P_n^-(z) = \frac{(1 + \vartheta k_n^- h)^2}{k_n^- h}\frac{\cosh(k_n^-(h+z))}{\sinh(k_n^- h)}, \tag{10}$$

where $\vartheta = V/\omega h$. The wave number k_n^+ or k_n^- can be obtained by solving the following dispersion equations

$$\mu^2(1 - k_n^+ h\vartheta)^2 = k_n^+ h \tanh(k_n^+ h), \tag{11}$$

$$\mu^2(1 + k_n^- h\vartheta)^2 = k_n^- h \tanh(k_n^- h), \tag{12}$$

where $\mu = \omega h/\sqrt{gh}$. Dispersion equations (11) and (12) have infinite number of complex solutions. The solution method for wave number kh is detailed in Appendix (A).

2.2 Matching conditions at the slotted barriers

The wave velocity and pressure in various regions are matched asymptotically at each slotted barrier, as long as the thickness of the barrier is much smaller than wave length and water depth.

The conservation of mass is ensured by requiring

$$\tilde{u}_1 = \tilde{u}_2, \quad \text{at} \quad x = 0. \tag{13}$$

$$\tilde{u}_2 = \tilde{u}_3, \quad \text{at} \quad x = B. \tag{14}$$

When water flows through the gaps of the slotted barriers, there will be head loss due to turbulent dissipation. The fluctuating head losses across the slotted barriers at $x = 0$ and $x = B$ can be written, respectively, as

$$\frac{\tilde{p}_1 - \tilde{p}_2}{\rho} = \frac{f_1}{2}|V + \tilde{u}_2|(V + \tilde{u}_2) + \rho\lambda_1 \frac{\partial \tilde{u}_2}{\partial t} - \frac{f_1}{2}\overline{|V + \tilde{u}_2|(V + \tilde{u}_2)} \tag{15}$$

$$\frac{\tilde{p}_2 - \tilde{p}_3}{\rho} = \frac{f_2}{2}|V + \tilde{u}_3|(V + \tilde{u}_3) + \rho\lambda_2 \frac{\partial \tilde{u}_3}{\partial t} - \frac{f_2}{2}\overline{|V + \tilde{u}_3|(V + \tilde{u}_3)} \tag{16}$$

where the quadratic friction factor f_j and the length of jet flow λ_j need to be determined experimentally. In their study of wave reflection by a slotted seawall, Zhu and Chwang (2001) showed that the jet length λ might slightly affect the location of the minimum reflection coefficient. For our double slotted barriers, experiments showed that transmission coefficients does not change significantly with the chamber width B, therefore, in the following, the jet length λ_j was set to zero in all calculations. But for completeness, we have kept all terms related to λ_j in the theory.

For motion of the first harmonic waves, the matching conditions (15) and (16) can be linearized by

$$\tilde{p}_1 - \tilde{p}_2 = \rho\sqrt{gh}\beta_1(z)\tilde{u}_2 + \rho\lambda_1 \frac{\partial \tilde{u}_2}{\partial t} \tag{17}$$

$$\tilde{p}_2 - \tilde{p}_3 = \rho\sqrt{gh}\beta_2(z)\tilde{u}_3 + \rho\lambda_2 \frac{\partial \tilde{u}_3}{\partial t}. \tag{18}$$

The linear dissipation coefficient β_j needs to be determined in such a way that losses of the wave energy predicted by the linearized equations (17) and (18) and the non-linear equations (15) and (16) are equivalent. To achieve an equivalent energy loss, β_j is determined by

$$\beta_1(z) = \frac{f_1}{2}\frac{\overline{|V + \tilde{u}_2|(V + \tilde{u}_2)\tilde{u}_2}}{\sqrt{gh}\overline{\tilde{u}_2\tilde{u}_2}}, \beta_2(z) = \frac{f_2}{2}\frac{\overline{|V + \tilde{u}_3|(V + \tilde{u}_3)\tilde{u}_3}}{\sqrt{gh}\overline{\tilde{u}_3\tilde{u}_3}}, \tag{19}$$

with the over-bar indicating the time-average over one wave period. Equation (19) is called Lorentz's principle of equivalent work (see, e.g., Sollitt and Cross (1972), Madsen (1974), Mei et al. (1974), Mei (1989), Zhu and Chwang (2001), etc.). Note that the terms related to λ_j in equations (17) and (18) do not dissipation energy, but will induce a change in the wave phase angles(see also Zhu and Chwang (2001)).

2.3 Reflection and transmission coefficients

To determine the reflection and transmission coefficients, we first multiply appropriate orthogonal functions on both sides of the four matching conditions (equations (13), (14), (17), and (18)), and then integrate the resulting equations from $z = -h$ to $z = 0$ to obtain the following set of algebraic equations:

$$\int_{-h}^{0} U_m^-(z)\tilde{u}_1 dz = \int_{-h}^{0} U_m^-(z)\tilde{u}_2 dz, \quad \text{at} \quad x = 0. \tag{20}$$

$$\int_{-h}^{0} U_m^-(z)\tilde{u}_2 dz = \int_{-h}^{0} U_m^-(z)\tilde{u}_3 dz, \quad \text{at} \quad x = B. \tag{21}$$

$$\int_{-h}^{0} P_m^-(z)(\tilde{p}_1 - \tilde{p}_2)dz = \rho\sqrt{gh}\int_{-h}^{0} P_m^-(z)\beta_1(z)\tilde{u}_2 dz + \rho\lambda_1\int_{-h}^{0} P_m^-(z)\frac{\partial \tilde{u}_2}{\partial t}dz, \quad \text{at} \quad x=0 \tag{22}$$

$$\int_{-h}^{0} P_m^+(z)(\tilde{p}_2 - \tilde{p}_3)dz = \rho\sqrt{gh}\int_{-h}^{0} P_m^+(z)\beta_2(z)\tilde{u}_3 dz + \rho\lambda_2\int_{-h}^{0} P_m^+(z)\frac{\partial \tilde{u}_3}{\partial t}dz, \quad \text{at} \quad x=B \tag{23}$$

where $m = 0, 1, 2, \ldots N$ with N being the number of evanescent wave mode considered in the solution. After substituting the expressions for u_j and p_j given in equations (4) and (5) into equations (20)-(23), the time factor exp(iωt) can be dropped automatically from the equations, resulting in a system of algebraic equations of the form

$$\boldsymbol{MX} = \boldsymbol{b} \tag{24}$$

where $\boldsymbol{M}$ is a $4(N+1)\times4(N+1)$ matrix; both $\boldsymbol{X}$ and $\boldsymbol{b}$ are one dimensional array of length $4(N+1)$. The array $\boldsymbol{b}$ contains information of incident waves and the array $\boldsymbol{X}$ contains the reflection coefficients R, $R_{1,n}$, $R_{2,0}$, $R_{2,n}$ and transmission coefficients T, $T_{2,0}$, $T_{2,n}$, $T_{3,n}$, where $n = 1, 2, \ldots, N$.

3. Applications to single and double slotted wave barriers

3.1 Code verification

The code was first checked with available theoretical and experimental results. For cases of pure waves, the code was checked with the analytical expressions for the reflection and transmission coefficients for double slotted barriers reported in Huang (2007b). For cases of regular surface waves propagating in an opposing current, the code was checked with the measured and predicted reflection and transmission coefficients for single slotted barrier reported in Huang (2007a) and Huang and Ghidaoui (2007) for long waves. In all these comparisons, good agreements are found and the figures showing the comparisons are omitted here.

3.2 Comparison with experiments for double slotted barriers

To understand the effects of current on wave scattering by single or double slotted barriers, a series of experiments were conducted in a wave-current flume located in the Hydraulics Laboratory at HKUST, Hong Kong, China. The glass-walled wave flume is 15.0m in total length, 0.3m in width and 0.5m in depth. The section with the glass wall is 12.5m long. In our experiments, the still water depth was fixed at h = 0.3m and the porosity of the barriers was fixed at $\varepsilon = 0.21$ for all our tests. Experimental results for a single slotted barrier were reported in (Huang (2007a)) and the experimental results for double slotted barriers are reported in Huang (2007c). In both aforementioned papers, experimental results for strong currents are included. As we have ignored in the present theory the change of the mean water level across the slotted barrier, we will apply the theory only to those cases where the currents are weak. In the following, the friction factor for $V = 0$ is calculated by the empirical formula given in Huang (2007b) and the friction factor for $V \neq 0$ is obtained by fitting the predictions with experimental results.

3.3 Single slotted barrier

The theory of Huang and Ghidaoui (2007) can only be applied for long waves, thus the variation of reflection and transmission coefficients with wave period was not studied. Now we apply the theory developed in the paper to intermediate water waves. For single slotted barrier, we set $B = 0$ and $f_l = 0$. Figure (2) shows the measured and predicted reflection and transmission coefficients as functions of wave period for a single slotted wave barrier with a porosity of 0.21.

The best fitting between predictions and experiments can be achieved with friction factor $f = 23.5$ for $V = 0$ and $f = 12.5$ for $V=-0.1$m/s (or $V/\sqrt{gh} \approx -0.058$). Experiments showed that reflection coefficients were slightly increased by the opposing current and the transmission coefficients were reduced by the same opposing current; these trends are well predicted by the theory as shown in Figure (2). It was also found through numerical experiments that reflection coefficients were in general slightly over-predicted no matter what friction factor f was used. Therefore, the optimal friction factor $f = 12.5$ for $V = -0.1$m/s was chosen by minimizing the error between the measured and predicted transmission coefficients[3]. We have also tried to apply the present theory to the cases of a single slotted barrier in a strong current of $V=-0.15$m/s (or $V/\sqrt{gh} \approx -0.087$), the agreement between the predictions and measurements is, however, not satisfactory, and thus not reported here.

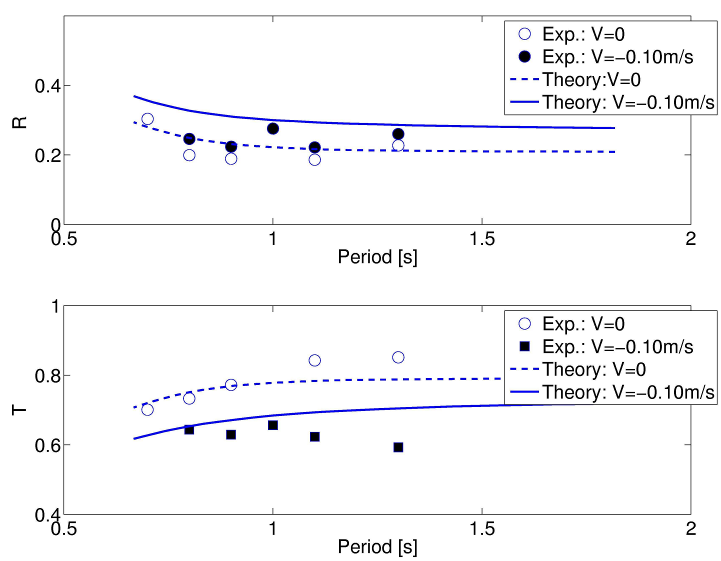

Fig. 2. Comparison with experimental results for single slotted barrier ($a = 0.021m$, $h = 0.3m$)

3.4 Double slotted barriers

Now we apply the theory to wave scattering by double slotted barriers in the presence

[3] In Huang and Ghidaoui (2007), who studied long waves, f=12.5 was used for both V=0 and V=–0.1m/s.

of a weak opposing current. Huang (2007c) reported the reflection and transmission coefficients for two non-zero currents: V=−0.075m/s (or $V/\sqrt{gh}\approx-0.044$) and V=−0.125m/s (or $V/\sqrt{gh}\approx-0.073$). Here comparisons between the theoretical predictions and the experimental results for V=−0.075m/s are reported.

Shown in Figure (3) are the measured and predicted reflection and transmission coefficients for waves of amplitude a = 0.02m and period T_w = 1.1s. The friction factor f = 23.5 was used for V = 0 and f = 17 for V=−0.125m/s. In the figure, the wave length L was determined by averaging the wave numbers of left-going and right-going waves in the current. For this set of experiments, transmission coefficients are well-predicted by the present theory, but the reflection coefficients for B/L≈0.25 are somewhat over-predicted. Experiments showed that the variation of the transmission coefficient with chamber B is mild, which can be well-captured by the present theory. We have also tried to apply the present theory to the cases of double slotted barriers in a strong current of V=−0.125m/s, the agreement between the predictions and measurements is, however, not satisfactory, and thus not reported here.

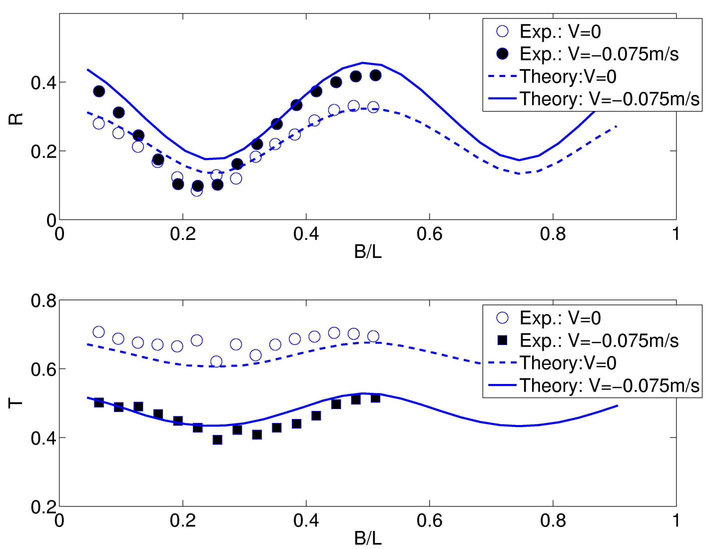

Fig. 3. Comparison with experimental results for double slotted barriers for a = 0.02m and T_w = 1.1s with B varying from 0.1m to 0.8m

Shown in Figure (4) are the measured and predicted reflection and transmission coefficients for waves of amplitude a = 0.02m and chamber width B = 0.5m. Again, the friction factor f = 23.5 was used for V = 0 and f = 17 for V=−0.075m/s. In this case, the transmission coefficients are somewhat over-predicted by the theory for relatively long waves, while the reflection coefficients are somewhat over-predicted by the theory for relatively short waves (large B/L), suggesting that the friction factor may have also a weak dependence on wave conditions when steady currents are present. We hypothesize that the nonlinear wave-current interaction may have contributed to the discrepancy shown in Figure (4). The nonlinear wave-current interaction will make the friction factor have a complicated dependency on the wave frequency. Also in this simple model, the energy exchange between waves and current is not considered, which may also contribute to the under-estimate of the energy dissipation due to slotted barriers to certain extend.

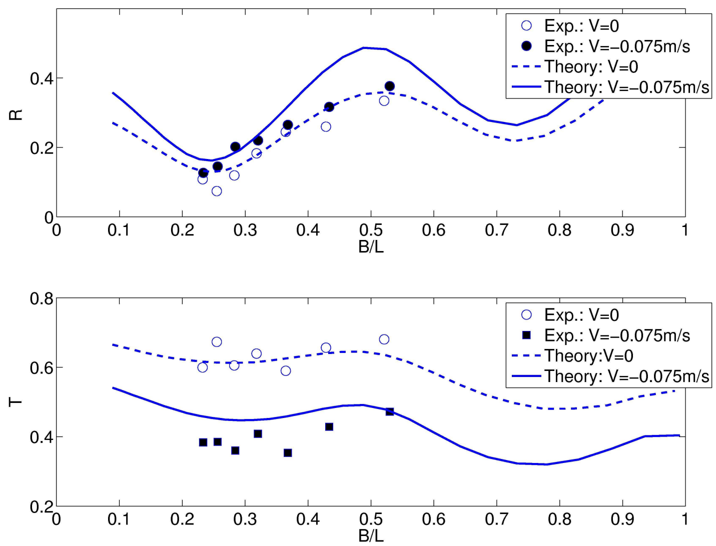

Fig. 4. Comparison with experimental results for double slotted barriers with B = 0.5m. Wave amplitude is fixed at a = 0.02m

3.5 Friction factor f and dissipation coefficient β

It is known that in the absence of current, the friction factor f increases with decreasing barrier porosity (see Mei (1989) and Huang (2007b)). Large fraction factor will result in small transmission coefficient (in the limit of $f \rightarrow \infty$, the transmission coefficient becomes zero). However, when currents are present, the optimized friction factor f is actually reduced, as shown by the results in the previous section. To understand the reason why currents can still reduce the transmission coefficient with a reduced friction factor f, we examine the dissipation coefficient β defined by equation (19).

Shown in Figure (5) are the dissipation coefficients, β_1 and β_2, for V = 0 and V=−0.075m/s, where waves have a period of T_w = 1.1s and an amplitude a = 0.02m. The chamber width B = 0.65m for all cases. It can be seen that the current can considerably increase the energy dissipation coefficient β even though the friction factor f is reduced by the same current. When currents are present, the reduced transmission coefficient is the direct results of an increased dissipation coefficient β rather than an increased friction factor f. Another interesting feature to note is that when currents are present the value of β is almost constant throughout the depth, suggesting that waves have only weak effects on the dissipation coefficient.

When currents are strong relative to water waves, strong nonlinear wave-current interaction may occur. This is especially true for wave opposing current, where the high current velocity due to the flow through barrier gaps may block waves so that part of wave energy can be transferred to current. The nonlinear energy exchange between waves and current are not considered in the classic derivation of the friction factor f (see Mei et al. (1974), who showed that f could be determined by the porosity of the slotted barrier in the absence of currents). Therefore, the friction factor f for combined wave-current flow will be different from that for pure waves, and has to be determined by fitting with experimental data at present.

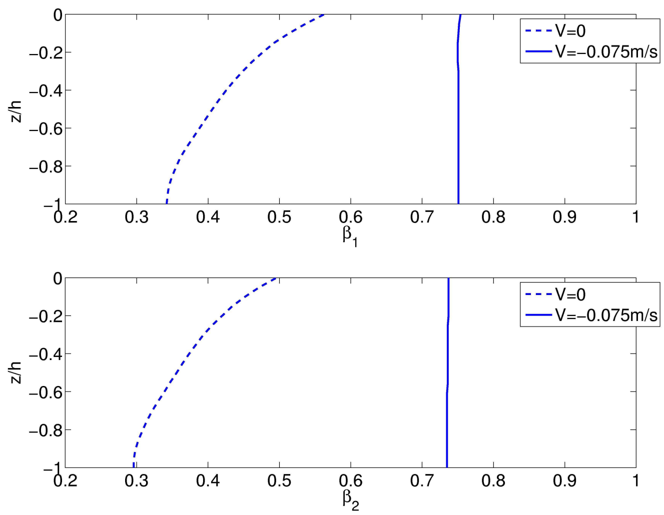

Fig. 5. Effects of current on the dissipation coefficient β. f = 23.5 for V = 0 and f = 17 for V=-0.075m/s

4. Concluding remarks

A linear theory is developed to study the wave scattering by single or double slotted barriers in the presence of a steady current. The theory is based on the eigen-expansion method with evanescent waves being included. Comparisons between predicted and measured reflection/transmission coefficient revealed that the friction factor f depends not only the geometry of the slotted barriers but also the current strength. With optimized friction factors f for different current velocities, the minor increase in the reflection coefficients and the considerable reduction in the transmission coefficients found in previous experiments can be satisfactorily predicted by the theory. The present theory is valid for weak current (say, $V/\sqrt{gh}<0.06$) where the change of mean surface elevation across the slotted barriers can be ignored. Further theoretical research is needed in order to further understand wave scattering by slotted barriers in the presence of relatively strong currents.

Appendix

A Solution of dispersion equation

We use equation (11) as an example to show the solution methods for the complex wave numbers *kh* for both the progressive waves and evanescent waves.

A.1 Perturbation solution

If $\vartheta << 1$, the wave number *kh* can be written in the following perturbation series

$$kh = k_0 h + \vartheta k_1 h + \vartheta^2 k_2 h + \cdots \tag{A.1}$$

Substituting equation (A.1) into the dispersion equation (11) gives

$$\begin{aligned} &\mu^2 (1 - \vartheta(k_0 h + \vartheta k_0 h + \vartheta^2 k_2 h + \cdots))^2 \\ &= (k_0 h + \vartheta k_1 h + \vartheta^2 k_1 h + \cdots) \tanh(k_0 h + \vartheta k_1 h + \vartheta^2 k_2 h + \cdots) \end{aligned} \tag{A.2}$$

After collecting terms of same power of the small parameter ϑ, a series of equations for $k_j h$, j = 0, 1, ... can be obtained. The expressions for the solutions at each order are long, but can be easily managed by algebraic software like MAPLE or MATHEMATICA.

A.2 Numerical method

To find a numerical solution for *kh,* we denote the complex solution of *kh* by $kh = \xi_r + i\xi_i$, where both ξ_r and ξ_i are real. It then follows that the dispersion equation (11) can be written, in terms of ξ_i, ξ_r, as

$$\mu^2 (1 - \vartheta(\xi_r + i\xi_i))^2 = (\xi_r + i\xi_i) \tanh(\xi_r + i\xi_i) \tag{A.3}$$

Equating the real and imaginary parts, we obtained the following two equations

$$f_1(\xi_r, \xi_i; \mu, \vartheta) = 0,\ f_2(\xi_r, \xi_i; \mu, \vartheta) = 0, \tag{A.4}$$

where

$$\begin{aligned} f_1(\xi_r, \xi_i; \mu, \vartheta) &= (\sinh(\xi_r)^2 + \cos(\xi_i)^2)((\mu - \mu\vartheta\xi_r)^2 - \mu^2\vartheta^2\xi_i^2) \\ &\quad - \frac{\xi_r \sinh(2\xi_r) - \xi_i \sinh(2\xi_i)}{2} \end{aligned} \tag{A.5}$$

$$\begin{aligned} f_2(\xi_r, \xi_i; \mu, \vartheta) &= -2\mu^2\vartheta\xi_i(\sinh(\xi_r)^2 + \cos(\xi_i)^2)(1 - \xi_r) \\ &\quad - \frac{\xi_r \sinh(2\xi_i) + \xi_i \sinh(2\xi_r)}{2} \end{aligned} \tag{A.6}$$

These two nonlinear algebraic equations can be solved by an iteration procedure with the following initial conditions: (i) $\xi_r = \pm k_0 h$, $\xi_i = 0$ for left and right-going progressive modes; and (ii) $\xi_r = 0$, $\xi_i = \pm m\pi$ for the m*th* evanescent wave mode that decays spatially in the left/right direction.

As an example, we take μ = 1.099, ϑ = 0.106. In the absence of current, the wave numbers of the progressive modes and the first three evanescent modes are

$$kh = [\pm 1.3734, \pm 2.7243i, \pm 6.0873i, \pm 9.2956i] \tag{A.7}$$

Table A.1 Wave numbers by various methods for μ = 1.099 and ϑ = 0.106

Modes/Method	Newton method	Pert-1	Pert-8
$k_0 h$	(1.1436, -1.8102)	(1.0780, -1.6688)	(1.1437, -1.7975)
$k_1 h$	-0.2629 ± 2.7315i	-0.2477±2.7243i	-0.2628±2.7315i
$k_2 h$	-0.2712 ± 6.1633i	-0.2544± 6.0873i	-0.2713±6.1632i

k_3h	0.2697 ± 9.4246i	-0.2553± 9.2956i	-0.2698± 9.4247i

In the presence of a current with ϑ= 0.106, the wave numbers for progressive waves and evanescent waves are listed in Table (A.1), where Pert-1 means the first order perturbation solution and Pert-8 means the eighth order perturbation solution. As can be seen, Newton's method based on equations (A.5) and (A.6) can produce correct wave numbers with desired accuracy.

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