

The identification of cracks using boundary elements and evolutionary algorithms

T. Burczynski^{a,b,*}, W. Beluch^a

^aDepartment for Strength of Materials and Computational Mechanics, Silesian University of Technology, Konarskiego 18a, PL-44-100 Gliwice, Poland

^bInstitute of Computer Modelling, Cracow University of Technology, Warszawska 24, 31-455 Cracow, Poland

Abstract

This paper is devoted to the identification problems for structures which contain cracks. The problem of crack identification is formulated as the minimization of the difference between the measured and computed values of displacements or stresses at selected boundary nodes. The coupling of the dual boundary element method and evolutionary algorithms is proposed to solve the problem. The identification of single cracks of different shapes is presented. The multiple crack identification is also considered. The problem of the identification of unknown number of cracks is formulated by introducing a special kind of chromosome. The influence of random errors in experimentally measured displacements on a convergence of the evolutionary identification is examined. A hybrid evolutionary approach based on sensitivity information of the fitness function is tested. Several numerical examples are presented. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Boundary element method; Fracture mechanics; Crack identification; Evolutionary algorithms

1. Introduction

The Boundary Element Methods (BEMs) are powerful numerical techniques in analysis of cracked solids [1,22]. The original and pioneering contribution to formulation and application of the BEM to analysis of fracture mechanics problems was made by T.A. Cruse and co-workers [17–21,42].

In the direct problems one assumes that the shape and positions of cracks are known. Unfortunately, such assumption is idealistic and the analysis problem should be preceded by the identification of cracks.

The identification of cracks can be classified as topics which belong to *inverse problems*. Generally speaking, inverse problems deal with the determination of the mechanical system — with unknown material properties, geometry, sources or boundary conditions — from the knowledge of the responses to given excitations on its boundary [4]. From the mathematical point of view, such problems are ill-posed and have to be overcome by the developments of new computational methods, the introduction of new objective functionals into the optimization algorithms, new sensitivity analysis methods, new regularization techniques, new experimental procedures, etc.

Among many kinds of inverse problems, topics of the identification of cracks are especially well suited for boundary element treatment and belong to geometrical inverse problems.

The methods of crack identification based on sensitivity information and the BEM have found broad applications [3,9–15,24–29,31–39,41,43,44]. Unfortunately, approaches based on sensitivity information have some drawbacks: an objective function must be continuous, the shape variation of the boundary defect should be regular, the hessian of the objective function should be positive definite, there is a large probability of convergence to a local optimum, computation starts from a single point narrowing the search domain and the choice of the starting point may influence the convergence.

Because of these difficulties there is a demand for checking other methods, free from the restrictions mentioned above. Genetic algorithms are one of them. Genetic algorithms are stochastic algorithms whose search methods model natural phenomena: genetic inheritance and Darwinian strife for survival. Classical genetic algorithms are based only on the fitness value information and coded chromosomes. They work on populations of solutions and use binary operators of crossover and mutation and the probability of operators is constant.

Evolutionary algorithms are modified and generalized classical genetic algorithms in which populations of chromosomes are not coded binary and *floating point*

* Corresponding author. Fax: +48-3237-1267.

E-mail address: burczyns@polsl.gliwice.pl (T. Burczynski).

representation is used. They use modified crossover (simple, arithmetical and heuristic) and mutation (uniform, boundary and non-uniform) operations. The selection is performed in the form of the ranking selection or the tournament selection and the probability of operators can be variable.

This paper is devoted to the application of the BEM and evolutionary algorithms to identification of cracks in solids described in the framework of elastostatics.

2. The formulation of the crack identification problem

Consider the problem of the identification of the internal cracks in an elastic body. An elastic body occupying a domain Ω and having a boundary $\Gamma \equiv \partial\Omega$ is presented in Fig. 1. There are two fields prescribed on the boundary Γ : the field of displacements $u^0(x)$, $x \in \Gamma_u$, and the field of tractions $p^0(x)$, $x \in \Gamma_p$, and $\Gamma_u \cup \Gamma_p = \Gamma$ and $\Gamma_u \cap \Gamma_p = \emptyset$. The domain contains an internal traction-free crack C . Displacements are allowed to jump across C : $[u] \equiv u^+ - u^- \neq 0$.

The set of experimental data \hat{q}^m (e.g. displacements or stresses) measured at the boundary points x^m , $m = 1, \dots, M$ and the set of theoretical data $q(x^m)$ computed at the points x^m are given.

In order to identify the crack the functional J_0 representing norm between theoretical and computed values of displacements is constructed:

$$J_0 = \frac{1}{2} \sum_{m=1}^M [q(x^m) - \hat{q}^m]^2 = \int_{\Gamma} \varphi(q) d\Gamma \tag{1}$$

where

$$\varphi(q) = \frac{1}{2} \sum_{m=1}^M [q(x) - \hat{q}^m]^2 \delta(x - x^m) \tag{2}$$

The size and the shape of the crack are described by a set of the shape parameters $\mathbf{a} = (a_r)$, $r = 1, \dots, R$. In order to solve the identification problem one should find the vector \mathbf{a} , which minimizes the functional J_0 .

To have a physical meaning, the crack specified by the vector \mathbf{a} , must be completely inside the body. This condition

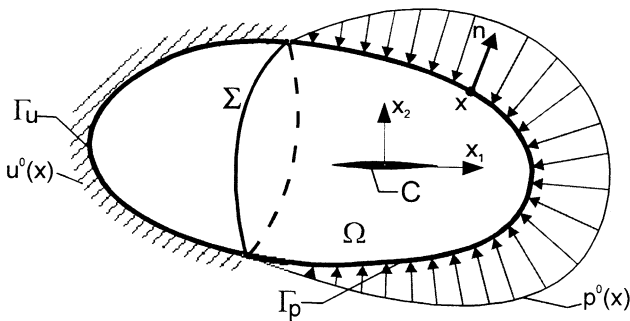


Fig. 1. A body containing crack.

is ensured through the imposition of the geometrical constrains.

In this paper the BEM is proposed to solve the boundary problem (calculate the objective function) and evolutionary algorithms are applied as the identification method.

3. Dual BEM in fracture mechanics

BEM turns out to be an appropriate numerical technique for solving foregoing problem. This method seems to be the most suitable for the problems with cracks because it is capable of accurately modeling the high stress gradients near the crack tip and discretization of the inside of the body is not required (the crack is a part of the boundary).

If the body is not subjected to body forces, the displacement of a point x can be represented by the following boundary displacement integral equation:

$$c(x)u(x) = \int_{\Gamma} \mathbf{U}(x,y)p(y) d\Gamma(y) - \int_{\Gamma} \mathbf{P}(x,y)u(y) d\Gamma(y), x \in \Gamma \tag{3}$$

where $\mathbf{U}(x,y)$ and $\mathbf{P}(x,y)$ are fundamental solutions of elastostatics, $c(x)$ is a constant which depends on the position of the collocation point x , y is the boundary point.

If Eq. (3) is applied on both crack surfaces, then two identical equations would be formed, so the resulting set of algebraic equations is singular.

In order to overcome this difficulty the dual boundary integral equation method [16,40] can be used. In this technique two different boundary equations are applied on two sides of the crack, namely Eq. (3) and an additional hyper-singular tractions integral equation in the form:

$$\frac{1}{2}p(x) = \mathbf{n} \left[\int_{\Gamma} \mathbf{D}(x,y)p(y) d\Gamma(y) - \int_{\Gamma} \mathbf{S}(x,y)u(y) d\Gamma(y) \right], x \in \Gamma \tag{4}$$

where $\mathbf{D}(x,y)$ and $\mathbf{S}(x,y)$ are the third-order fundamental solution tensors obtained from suitable differentiation of $\mathbf{U}(x,y)$ and $\mathbf{P}(x,y)$ with respect to the source point x and application of the Hooke's law, \mathbf{n} is the unit outward normal vector at the collocation point x .

Modeling of the cracked structures using the dual BEM proceeds in the following manner:

- the displacements integral equation is applied on one side of each crack;
- the tractions integral equation is applied on the opposite side of each crack;
- the displacements integral equation is applied on the remaining boundary.

4. The classical methods of identification

Classical identification (optimization) methods base on the algorithm which can be presented as follows:

$$a_{i+1} = a_i + \beta_i \mathbf{h}_i \quad (5)$$

where β_i is the numerical factor determining the length of the step in direction \mathbf{h}_i and \mathbf{h}_i the vector determining the direction from the a_i to a_{i+1} :

$$\mathbf{h}_i = \mathbf{h}_i \left(\frac{DJ_0^*}{Da_r} \right) \quad (6)$$

The method of choosing the vector \mathbf{h}_i determines the general rate of the convergence of the optimization process. Most of the presently known methods (the steepest descent method, the conjugate gradient method, the variable metric method and others) use the information about the gradient of the augmented objective function to estimate the vector \mathbf{h}_i . These methods can be called the gradient methods because all of them are based on the sensitivity analysis information and optimum searching direction depends on gradient of the objective function. The gradient methods are very fast, but they have limitations, referring to the continuity of the objective function, the gradient (hessian) of the objective function and the substantial probability of getting a local optimum (strongly dependent on the choosing of the starting point). This causes that for some optimization problems the optimal solution is either very difficult or quite impossible to obtain.

In the present paper the application of the evolutionary algorithms for the problems of the crack identification is presented.

5. The evolutionary methods of identification

5.1. Evolutionary algorithms

Evolutionary algorithms [30] stem from the classical genetic algorithms, which map the evolutionary process of the nature to adapt an individual to the environment it lives in.

Genetic algorithms are procedures to search the feasible space of solutions starting from a population of individuals (chromosomes) randomly generated from the feasible solution domain. Chromosomes evolve towards better solutions by applying genetic operators: selection, mutation and crossover modeled on the genetic processes occurring in the nature. After applying the genetic operators the new population should have a better fitness, so the population undergoes the evolution in the form of the natural selection. An objective function with constraints plays the role of the environment.

Genetic algorithms, opposed to classical methods, need only information about the value of the objective function and are able to find the optimal solution satisfying

constraints without calculations of the gradient of the hessian of the objective function.

The classical genetic algorithms work only on binary chains with simple crossover and simple mutation, but many problems need the more flexible approach. There are two ways: either the problem is modified in such a way that the genetic algorithms can be applied directly or the algorithm is modified under the problem. The modified genetic algorithms are called evolutionary algorithms [7,11]. They use modified crossover (simple, arithmetical and heuristic) and mutation (uniform, boundary and non-uniform) operations. The selection is performed in the form of the ranking selection or the tournament selection and the probability of operators can be variable.

The chromosome which represents the design vector can be expressed in the form:

$$\mathbf{ch} = \langle s_1, s_2, \dots, s_i, \dots, s_N \rangle \quad (7)$$

where restrictions on genes, s_j , are introduced in the form:

$$s_{iL} \leq s_i \leq s_{iR}, \quad (8)$$

5.1.1. Simple crossover

This operator produces two offspring from two parents. The simple crossover may produce an offspring outside the design space. To avoid this, a parameter $\alpha \in [0,1]$ is applied. For randomly generated crossing parameter i it works as follows (chromosomes \mathbf{ch}_1 , \mathbf{ch}_2 are parents in the vector form):

$$\mathbf{ch}_1 = \langle s_1, s_2, \dots, s_i, \dots, s_N \rangle, \quad \mathbf{ch}_2 = \langle s_1, s_2, \dots, s_i, \dots, s_N \rangle \quad (9)$$

$$\mathbf{ch}'_1 = \langle s_1, \dots, s_i, S_{i+1}\alpha + s_{i+1}(1 - \alpha), \dots, S_N\alpha + s_N(1 - \alpha) \rangle$$

and

$$\mathbf{ch}'_2 = \langle S_1, \dots, S_i, s_{i+1}\alpha + S_{i+1}(1 - \alpha), \dots, s_N\alpha + S_N(1 - \alpha) \rangle \quad (10)$$

where ' (prime) denotes the offspring.

5.1.2. Arithmetical crossover

This operator produces two offspring being a linear combination of two parents \mathbf{ch}_1 and \mathbf{ch}_2 :

$$\mathbf{ch}'_1 = \alpha \mathbf{ch}_1 + (1 - \alpha) \mathbf{ch}_2, \quad \mathbf{ch}'_2 = \alpha \mathbf{ch}_2 + (1 - \alpha) \mathbf{ch}_1 \quad (11)$$

5.1.3. Heuristic crossover

This operator produces a single offspring from two parents:

$$\mathbf{ch}_3 = r(\mathbf{ch}_2 - \mathbf{ch}_1) + \mathbf{ch}_2 \quad (12)$$

where r is a random value from the range $[0,1]$ and $J(\mathbf{ch}_2) \leq J(\mathbf{ch}_1)$.

Mutation operators such as the uniform mutation, the

boundary mutation and the non-uniform mutation are used.

$$\mathbf{ch}_1 = \langle s_1, s_2, \dots, s_i, \dots, s_N \rangle, \quad \mathbf{ch}'_1 = \langle s_1, s_2, \dots, s'_i, \dots, s_N \rangle \quad (13)$$

5.1.4. Uniform mutation

Offspring are allowed to move freely within the feasible domain and the gene s'_i takes any arbitrary value from the range $[s_{iL}, s_{iR}]$.

5.1.5. Boundary mutation

The chromosome can take only boundary values of the design space, $s'_i = s_{iL}$ or $s'_i = s_{iR}$. The boundary mutation works very well when the solution lies either on or near the boundary of the feasible search space.

5.1.6. Non-uniform mutation

This mutation operator depends on the generation number and is used to tune of the system

$$s'_i = \begin{cases} s_i + \Delta[t, s_{iR} - s_i] & \text{if a random digit is 0} \\ s_i - \Delta[t, s_i - s_{iL}] & \text{if a random digit is 1} \end{cases} \quad (14)$$

where the function Δ takes value from the range $[0,y]$, t is a current generation number.

All the operators were applied dynamically in the identification procedure according to their probability. Parameters α and r have been randomly generated.

In the present paper the following problems are considered:

- the identification of the single cracks;
- the identification of the multiple cracks — number of the cracks is known;
- the identification of the multiple cracks — number of the cracks is unknown;
- the identification of the cracks with stochastic disturbance of the measured quantities.

5.2. Evolutionary identification algorithm

The proposed method of crack identification [2] consists of two main steps. The first one is to define an identification problem, to prepare a correct model and to choose design variables. In the next step the evolutionary algorithm is applied as the optimization module. All necessary values (especially fitness function values) are obtained from the analysis of the structure (for each individual in the population) using the dual BEM. For each chromosome in the population the new shapes and positions (and number, if necessary) of the cracks are generated and the structural analysis is performed. The flow chart of the proposed approach of the evolutionary identification for cracked structures is presented in Fig. 2.

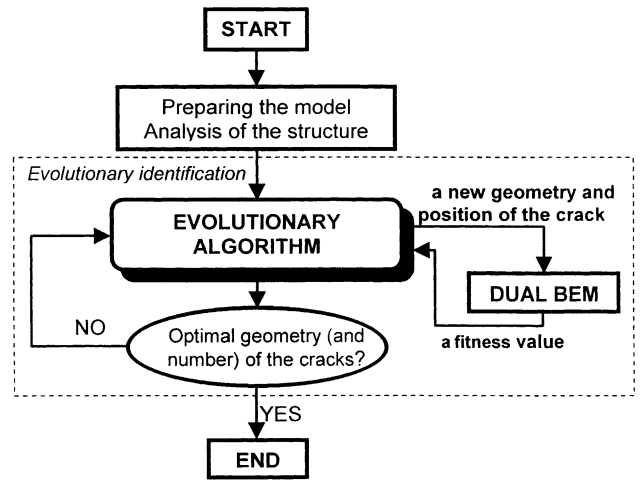


Fig. 2. Algorithm of the evolutionary identification.

5.3. Chromosome structure

The chromosome, representing the shape and the position of the crack, is constructed in a different way (depending on the problem).

5.3.1. The identification of the single crack and multiple cracks — the number of the cracks is known

For the linear cracks each chromosome \mathbf{ch}_i contains four genes s_1, \dots, s_4 determining coordinates x_1 and x_2 of the crack tips.

In case of the segmental-straight crack two methods are proposed.

In the first method one assumes, that the shape of the crack is known and the chromosome is constructed in the way similar to the linear crack — each chromosome \mathbf{ch}_i contains four genes s_1, \dots, s_4 determining coordinates x_1 and x_2 of the crack tips.

In the second method two genes s_1 and s_2 of each chromosome determine coordinates x_1 and x_2 of the first crack tip and the subsequent genes describe the length and the rotation angle of each segment. The number of genes i_s in chromosome depends on the number of the crack segments L_s :

$$i_s = 2L_s + 2 \quad (15)$$

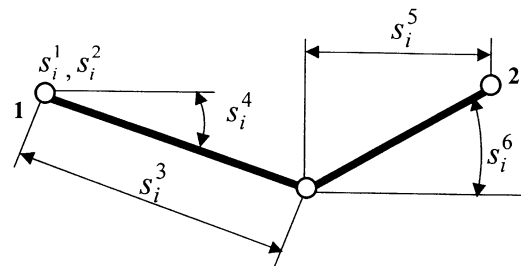


Fig. 3. The method of the parameterization of the segmental-straight crack.

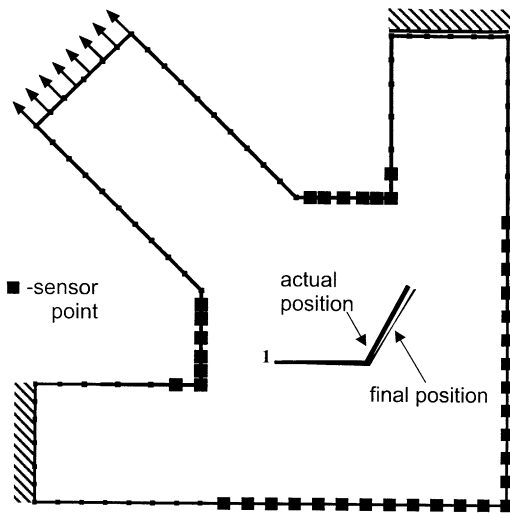


Fig. 4. The identification of one crack — numerical example 1.

The method of the unique description (assuming limitations: $-\pi/2 \leq s_i^n \leq \pi/2, n = 4, 6, \dots, 2L_s$) of the crack composed of two segments is presented in Fig. 3.

The number of sensor points has been chosen depending of the complexity of the task.

5.3.1.1. Numerical example 1. A structural element containing a single crack is considered. The aim is to find the shape and the position of the crack having measured displacements at 39 boundary (sensor) points. It has been assumed that the crack consists of two linear segments and the second method of the crack description has been used. The actual and final positions of the crack are shown in Fig. 4. The parameters of the evolutionary algorithm and the numerical results are presented in Table 1.

5.3.1.2. Numerical example 2. A structural element containing two rectilinear cracks is considered. One should find the positions of the cracks having measured displacements at 95 boundary (sensor) points. The actual and final positions of the cracks are shown in Fig. 5. The parameters of the evolutionary algorithm and the numerical results are presented in Table 2.

Table 1
Numerical example 1 — the parameters of the evolutionary algorithm (generation number, 2000; population size, 100) and the results

Coordinates		
	Actual	Final
(1)	0.00; -0.04	0.000; -0.041
(2)	0.04; -0.04	0.04; -0.041
	0.06; -0.01	0.06; -0.01
Generation of the optimal solution		1951
Fitness function value		0.165

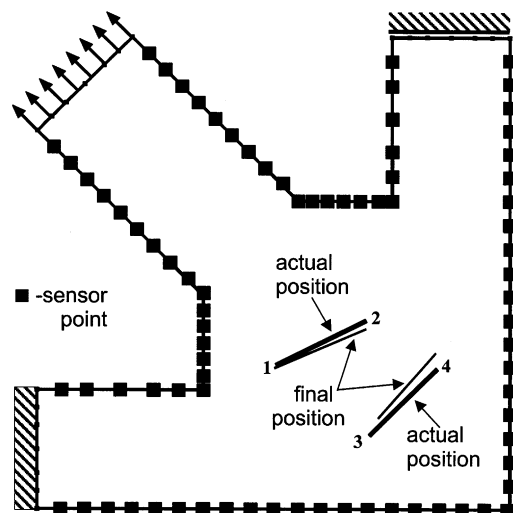


Fig. 5. The identification of two cracks — numerical example 2.

5.3.2. The identification of the multiple cracks — the number of the cracks is unknown

For the sake of simplicity only linear cracks are considered. One assumes that the unknown number of cracks is included in the interval $[1, N]$, where N is the maximum number of the cracks inside the body. Each chromosome s_i is composed of $4N + 1$ genes. The first gene represents the number $N_s \in \langle 1, \dots, N \rangle$ denoting the actual number of cracks for a given chromosome. The other genes $s_i^2, \dots, s_i^{4 \times N + 1}$ specify coordinates x_1 and x_2 of each crack tip for N cracks. Only first $1 + 4N_s$ genes (the first one and ones specifying the position of the N_s cracks) are considered.

5.3.2.1. Numerical example 3. One assumes that there are $1 \leq N_s \leq 5$ linear cracks of unknown lengths and positions inside the structure. The aim is to find the number and positions of the cracks having measured displacements at 95 boundary (sensor) points. The real number of the cracks is two. The actual and final positions of the cracks are presented in Fig. 6. The parameters of the evolutionary algorithm and the numerical results are presented in Table 3.

Table 2
Numerical example 2 — the parameters of the evolutionary algorithm (generation number, 3500; population size, 100) and the results

Coordinates		
	Actual	Final
(1)	0; -0.04	0.0; -0.041
(2)	0.04; -0.02	0.04; -0.023
(3)	0.04; -0.07	0.044; -0.062
(4)	0.07; -0.04	0.069; -0.033
Generation of the optimal solution		3494
Fitness function value		0.132

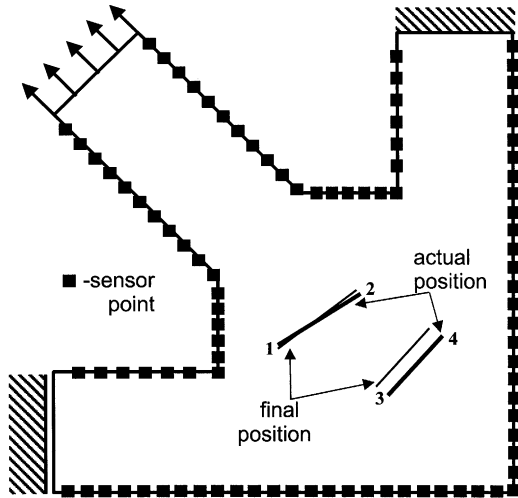


Fig. 6. The identification of 1–5 cracks — numerical example 3.

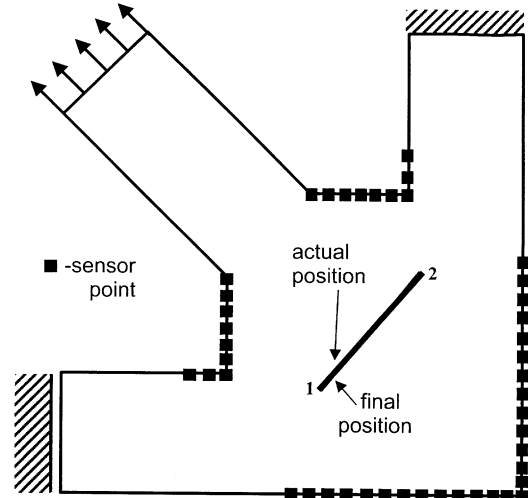


Fig. 7. The identification of the crack with the stochastic disturbance of the measurements — numerical example 4.

5.4. The identification of the cracks with stochastic disturbance of the measured quantities

Measurements are never totally precise (measurement error). In this part the influence of the stochastic disturbance of the measured quantities on the identification of the crack is considered. The identification task becomes a non-deterministic problem.

It is assumed that the distribution of the measured stresses is normal and the expected value $E[\hat{\sigma}^m]$ and the standard variation $D[\hat{\sigma}^m]$ are known.

5.4.1. Numerical example 4

It is assumed that the standard variation equals 1/30 of the expected value of the measured quantity. The equivalent stresses in 45 sensor points are measured. The actual and final positions of the cracks are presented on Fig. 7. The parameters of the evolutionary algorithm and the numerical results are presented in Table 4.

Table 3
Numerical example 3 — the parameters of the evolutionary algorithm (generation number, 3500; population size, 200) and the results

Coordinates		
	Actual	Final
(1)	0; -0.04	0.01; -0.041
(2)	0.04; -0.02	0.04; -0.019
(3)	0.04; -0.07	0.042; -0.073
(4)	0.07; -0.04	0.072; -0.043
Generation of the optimal solution		3487
Fitness function value		0.449

6. Hybrid methods of identification

By coupling of classical gradient and evolutionary methods into the hybrid identification methods one can avoid disadvantages of the both approaches [5]. The main advantages of the hybrid methods are the lower cost of calculation than using only the evolutionary optimization and the higher probability of finding the global optimum than using only gradient methods. Two types of hybrid algorithms can be proposed.

6.1. The linear model of the hybrid algorithm

In this model advantages of each single approach in any part of calculation are considered. The evolutionary algorithm is used first since it is very often able to generate a strong chromosome. After that the evolutionary algorithm slowly converges, so the gradient method is used to finish calculation. The scheme of this model is presented in Fig. 8a.

6.2. The parallel model of the hybrid algorithm

In the second model of the hybrid optimization a new

Table 4
Numerical example 4 — the parameters of the evolutionary algorithm (generation number, 3500; population size, 100) and the results

Coordinates		
	Actual	Final
(1)	0.05; -0.01	0.05; -0.011
(2)	0.01; -0.05	0.0101; -0.0494
Generation of the optimal solution		3498
Fitness function value		2.016

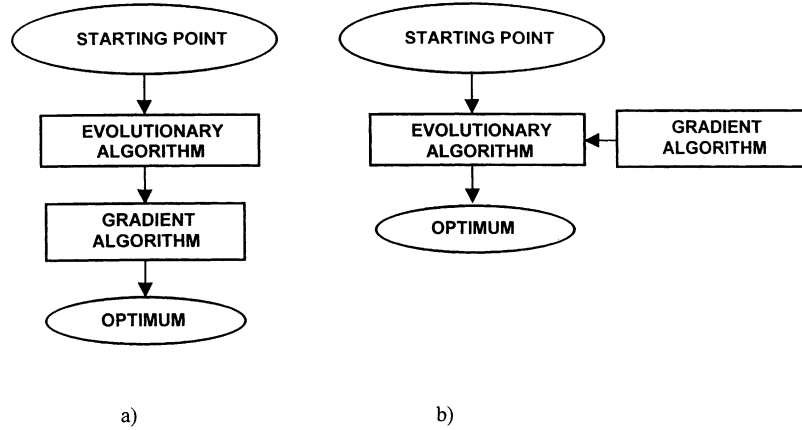


Fig. 8. Models of coupling of evolutionary and gradient algorithms: (a) linear model, (b) parallel model.

operator is introduced. This operator — the gradient mutation — changes any (especially the best in generation) chromosome using information about the gradient of the fitness function. A new chromosome

$$\mathbf{ch}'_1 = \langle s'_1, s'_2, \dots, s'_i, \dots, s'_N \rangle \quad (16)$$

is obtained after mutation of all its genes as follows:

$$s'_i = s_i + \Delta s_i, \quad \Delta s_i = \alpha \xi(\mathbf{ch}) \quad (17)$$

where α is a coefficient determining increment of a step in the search direction ξ , which depends on gradient of the fitness function.

The scheme of this model is presented in Fig. 8b.

In both models information about the gradient of the fitness function is taken from the sensitivity analysis.

Only the linear model of the hybrid algorithm has been numerically tested in this phase of the work.

6.3. Sensitivity analysis with respect to the shape and position of the crack

Consider the problem of the sensitivity analysis of the functional (1) with respect to shape and position of the crack [6]. The shape variation of the crack is introduced by a special kind of shape transformation:

- translation (T), by prescribing variations δb_k , $k = 1, 2$, where b_k is a translation parameter (Fig. 9a);
- rotation (R), by prescribing $\delta \omega_1$, where ω_1 is a rotation parameter (Fig. 9b);

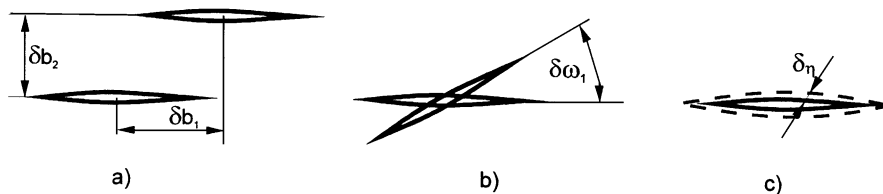


Fig. 9. Special shape transformation of the crack: (a) translations, (b) rotation, (c) expansion.

- scale change (expansion or contraction) (E) of the crack by prescribing $\delta \eta$, where η is a scale change parameter (Fig. 9c).

For 2D problems the vector of shape parameters a contains four parameters:

$$\mathbf{a} = \text{col}\{a_1, a_2, a_3, a_4\} \equiv [b_1, b_2, \omega_1, \eta]^T \quad (18)$$

One can prove that derivatives DJ_0/Da_q are expressed by path-independent integrals along an arbitrary closed surface (contour) S enclosing the crack [23]:

$$\frac{DJ_0}{Da_q} = \int_S Z_L^q(\sigma, \epsilon, u, \sigma^a, \epsilon^a, u^a) dS, \quad (19)$$

$$L = T, R, E; \quad q = 1, 2, 3, 4.$$

Integrands Z_L^q depend on state fields of primary and adjoint solutions and have the form:

- for translation ($k = 1, 2$):

$$Z_T^k = (\sigma_{ij} u_{i,k}^a + \sigma_{ij}^a u_{i,k} - \sigma_{il} \epsilon_{il}^a \delta_{kj}) n_j, \quad (20)$$

- for rotation:

$$Z_R^3 = e_{k3l} (-\sigma_{iq} \epsilon_{iq}^a x_l \delta_{kj} + \sigma_{ij} u_{i,k}^a x_l + \sigma_{ij} u_k^a + \sigma_{lj}^a u_k + \sigma_{ij}^a u_{i,k} x_l) n_j, \quad (21)$$

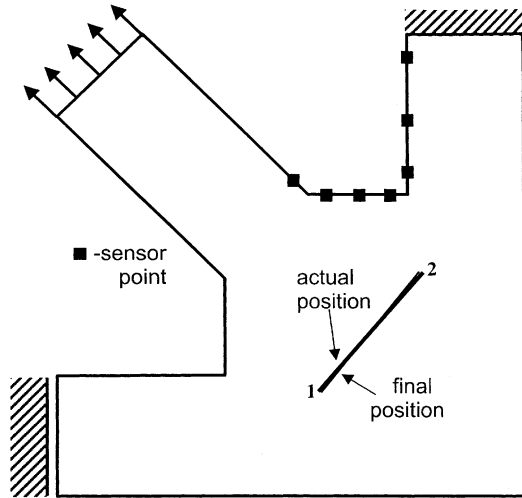


Fig. 10. The identification of the cracks using evolutionary algorithm — numerical example 5.

- for expansion or contraction:

$$Z_E^A = [2\sigma_{ij}^a u_i + 2\sigma_{ij}^a u_i^a + x_k \sigma_{ij}^a u_{i,k} - x_k \sigma_{ip} u_{i,p}^a \delta_{jk} + x_k \sigma_{ij}^a u_{i,k}^a] n_j, \quad (22)$$

where state fields u^a , ϵ^a , p^a are calculated for an adjoint system for which boundary conditions depend on the function ϑ :

$$u^{a0} = -\frac{\partial \varphi(u, p)}{\partial p} = 0 \text{ on } \Gamma_u \quad (23)$$

$$p^{a0} = \frac{\partial \varphi(u, p)}{\partial u} = 2 \sum_{i=1}^M [u(x) - \hat{u}^m] \delta(x - x^m) \text{ on } \Gamma_p. \quad (24)$$

The constitutive law for the adjoint system has the same form as for the primary cracked system.

6.3.1. Numerical example 5

The linear hybrid algorithm is tested. The structure

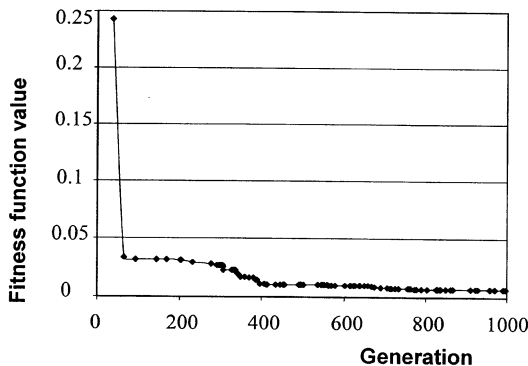


Fig. 11. History of the fitness function — numerical example 5.

Table 5

Numerical example 5 — the parameters of the evolutionary algorithm (generation number, 1000; population size, 100) and the results

Coordinates		
	Actual	Final
(1)	0.05; -0.01	0.0486, -0.0859
(2)	0.01; -0.05	0.0121, -0.05177
Generation of the optimal solution		997
Fitness function value		0.00727164

contains a linear crack with an unknown position (for the simplicity one assumes that the length of the crack is known). The vertical components of the displacements are measured in the seven sensor points. In the first step only the evolutionary algorithm is tested. Results are presented in Fig. 10. The history of the fitness function for the best chromosome is presented in Fig. 11. The parameters of the evolutionary algorithm and the numerical results are presented in Table 5. Optimal solution has been found in 15,040 s. In the second step computations using the evolutionary algorithm are finished if the improvement of the objective function is less than the assumed value. Computations are continued using the steepest descent method. The gradient is evaluated using the sensitivity analysis of the displacement functional with respect to the shape and the position modification of the crack. Due to the assumption of the known length of the crack only information about the sensitivity of the boundary functional with respect to translation and rotation of the crack is taken.

The evolutionary algorithm has been stopped in the 406 generation. The precise solution has been found using the classical identification algorithm in six steps. Optimal solution has been found in 7188 s. The reduction of the evaluation time is over 52%.

No instability in the gradient has been observed in this case, so there was no necessity to employ any regularization technique.

7. Summary

Several numerical examples of crack identification have been presented. Coupling of the BEM and the evolutionary methods appear to be very convenient and useful in such problems. The solution is very stable and in case of identification problems it is not necessary to modify the fitness function by adding regularization terms. Errors in the experimentally measured displacements (or stresses) do not cause a significant difference in the computed defect geometry. Evolutionary algorithms are especially suitable when the number of cracks is unknown. In this case by the special structure of the chromosome one can obtain the numerical procedure of identification which are based on the concept of active and sleeping genes.

A disadvantage of the evolutionary methods is the time consuming calculation because in order to achieve a satisfactory solution one should produce many generations.

In order to omit this drawback, two hybrid approaches have been proposed: linear and parallel models. Both models are based on using the additional information about the gradient fitness function. The application of the linear model has appeared very effective.

The evolutionary identification of cracks, presented in the paper, was applied to elastostatics problems. It is possible to develop this approach to the evolutionary identification of cracks to elastodynamics problems [7,8].

Acknowledgements

This paper is dedicated to Prof. Thomas A. Cruse to honor him on the occasion of his retirement and for his great influence on the development of the boundary element method.

This research has been carried out in the framework of the KBN grant 8T11F00316.

References

- [1] Aliabadi MH, Rooke DP. Numerical fracture mechanics. Computational mechanics publications. Southampton/Dordrecht: Kluwer Academic, 1991.
- [2] Beluch W. Sensitivity analysis and evolutionary optimization of cracked mechanical structures. PhD thesis, Mechanical Engineering Faculty, Silesian University of Technology, Gliwice, 2000 (in Polish).
- [3] Bonnet M, Burczynski T, Nowakowski M. Shape sensitivity analysis for identification of internal cracks or voids using BIE and adjoint variable approach. *Int J Solid Structures* (submitted).
- [4] Bui HD. Inverse problems in the mechanics of materials: an introduction. Boca Raton: CRC Press, 1994.
- [5] Burczynski T, Orantek P. Coupling of genetic and gradient algorithms. *Potok Zloty: Proceedings of the Conference on Evolutionary Algorithms and Global Optimization*, 1999.
- [6] Burczynski T, Beluch W, Kuhn G. Sensitivity analysis of cracked structures using boundary elements. XIII Polish Conference on Computer Methods in Mechanics, Poznan, vol. 1, 1997;213–20.
- [7] Burczynski T, Beluch W, Dlugosz A, Orantek P, Nowakowski M. Evolutionary methods in inverse problems of engineering mechanics. Nagano: Proceedings of the International Symposium on Inverse Problems in Engineering Mechanics — ISIP 2000. London: Elsevier, 2000.
- [8] Burczynski T, Bonnet M, Fedelinski P, Nowakowski M. Sensitivity analysis and identification of materials defects in dynamical systems. *System Analysis Model Simul* 2001 (in press).
- [9] Burczynski T, Polch EZ. Path-independent and boundary integral approach to sensitivity analysis and identification of cracks, Inverse problems in engineering mechanics. Rotterdam: Balkema, 1994 (355–361).
- [10] Burczynski T. 3D and 2D boundary elements in stochastic shape sensitivity analysis, *Computational stochastic mechanics*. Rotterdam: Balkema, 1995. p. 489–95.
- [11] Burczynski T, Beluch W, Kokot G. Optimization of cracked structures using boundary elements and evolutionary computation. *Boundary element*. London, 1999.
- [12] Burczynski T. Boundary element method in stochastic shape design analysis and identification of uncertain elastic solids. *Engng Analysis Boundary Elements* 1995;15:151–60.
- [13] Burczynski T, Habarta M, Kokot G. Coupling of boundary elements and path-independent integrals in generalized shape optimization and defect identification. *Proceedings of the 2nd International Conference on Inverse Problems*, Le Croisic, France, 1996.
- [14] Burczynski T, Habarta M, Polch EZ. Boundary element formulation for sensitivity analysis and detection of cracks and voids, *Structural and multidisciplinary optimization*. Oxford: Pergamon, 1995. p. 537–542.
- [15] Burczynski T. Stochastic boundary elements to analysis, shape design sensitivity and identification problems, *Computational stochastic mechanics*. CMP, 1993 p. 25.
- [16] Hong H-K, Chen JT. Derivation of integral equation in elasticity. *J Engng Mech ASCE* 1988;114:1028–44.
- [17] Cruse TA, Polch EZ. Elastoplastic BIE analysis of cracked plates and related problems. Part 1: formulation, Part 2: numerical results. *Int J Numer Analysis Engng* 1986;23:429–37 (see also p. 439–52).
- [18] Cruse TA, Van Buren W. Three-dimensional elastic stress analysis of a fracture specimen with an edge crack. *Int J Fracture Mech* 1971;7:1–15.
- [19] Cruse TA. Numerical evaluation of elastic stress intensity factors by the boundary integral equation method, *The surface crack: physical problems and computational solutions*. New York: ASME, 1972. p. 153–70.
- [20] Cruse TA. Two- and three-dimensional problems of fracture mechanics, *Developments in boundary element method*, 1. London: Applied Science, 1979. p. 97–119.
- [21] Cruse TA. Two-dimensional BIE fracture mechanics. *Appl Math Model* 1978;2:287–93.
- [22] Cruse TA. Boundary element analysis in computational fracture mechanics. Dordrecht: Kluwer Academic, 1988.
- [23] Dems K, Mróz Z. On a class of conservation rules associated with sensitivity analysis in linear elasticity. *Int Solids Struct* 1986;22:737–58.
- [24] Kobayashi S. Inverse analysis by boundary element method, *Boundary element method*, 16. CMP, 1994. p. 141–8.
- [25] Kubo S. Inverse problems related to the mechanics and fracture of solids and structures. *JSME Int J* 1988;31:157–66.
- [26] Mellings SC, Aliabadi MH. Crack identification using inverse analysis, *Boundary element method XV*. CMP, 1993. p. 261–73.
- [27] Mellings SC, Aliabadi MH. Dual boundary element formulation for inverse potential problems in crack identification. *Engng Analysis Boundary Elements* 1993;12:275–81.
- [28] Mellings SC, Aliabadi MH. Flaw identification using the boundary element method. *Int J Numer Methods Engng* 1995;38:399–419.
- [29] Mellings SC, Aliabadi MH. Three-dimensional flaw identification using sensitivity analysis, *Boundary element method XIV*. CMP, 1994. p. 149–56).
- [30] Michalewicz Z. Genetic algorithms + data structures = evolutionary programs, AI series. New York: Springer, 1992.
- [31] Nishimura N, Kobayashi S. A boundary integral equation method for an inverse problem related to crack detection. *Int J Numer Methods Engng* 1991;32:1371–87.
- [32] Nishimura N, Kobayashi S. Crack determination in time domain with collocation BIEM. In: Atluri SN, Yagawa G, Cruse TA, editors. *Computational mechanics '95*, 2. Berlin: Springer, 1995. p. 2862–7.
- [33] Nishimura N, Kobayashi S. Determination of cracks having arbitrary shapes with the boundary integral equation method. *Engng Analysis Boundary Elements* 1995;15:189–95.
- [34] Nishimura N, Kobayashi S. Further applications of regularised integral equations in crack problems, *Boundary integral methods*. Berlin: Springer, 1991 p. 400–9.
- [35] Nishimura N, Kobayashi S. Regularised BIE for crack shape determination problems, *Boundary elements XII*. CMP, Springer, 1990 p. 425–34.
- [36] Nishimura N. Applications of boundary integral equation method to

- various crack determination problems, *Dynamic fracture mechanics*, 7. CMP, 1995. p. 285–311.
- [37] Nishimura N, Furukawa A, Kobayashi S. Regularised boundary integral equations for an inverse problem of crack determination in time domain, *Boundary element methods*. Berlin: Springer, 1992 p. 252–61.
- [38] Nishimura N. Regularised integral equations for crack shape determination problems, *Inverse problems in engineering sciences*. Tokyo: Springer, 1991. p. 59–65.
- [39] Ohtsu M. Source inversion of acoustic emission based on the boundary element formulation, *Boundary elements X*, 1. CMP, 1988. p. 593–602.
- [40] Portela A, Aliabadi MH, Rooke DP. The dual boundary element method: effective implementation for crack problems. *Int J Numer Methods Engng* 1992;33:1269–87.
- [41] Sakagami T, Kubo S, Ohji K. Crack identification by the electric potential CT inverse analyses incorporating optimization techniques. *Engng Analysis Boundary Elements* 1990;7:59–65.
- [42] Snyder MD, Cruse TA. Boundary integral equation analysis of cracked anisotropic plates. *Int J Fracture* 1975;11:315–28.
- [43] Tanaka M, Nakamura M. Identification of cracks or defects by means of the elastodynamic BEM, *Boundary integral methods*. Berlin: Springer, 1991. p. 470–9.
- [44] Tanaka M, Nakamura M, Nakano T. Detection of cracks in structural components by the elastodynamic boundary element method, *Boundary elements XII*. CMP, 1990. p. 413–24.