Stable MFS Solution of Singular Cauchy Problems Associated with Two-Dimensional Helmholtz-Type Equations

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ABSTRACT

Helmholtz-type equations arise naturally in many physical applications related to wave propagation and vibration phenomena. They are often used to describe the vibration of a structure [1], the acoustic cavity problem [2], the radiation wave [3], the scattering of a wave [4] and the heat conduction in fins [5]. In many engineering problems governed by Helmholtz-type equations, boundary singularities arise when there are sharp re-entrant corners in the boundary, the boundary conditions change abruptly, or there are discontinuities in the material properties. It is well known that these situations give rise to singularities of various types and, as a consequence, the solutions to such problems and/or their corresponding derivatives may have unbounded values in the vicinity of the singularity. Singularities are known to affect adversely the accuracy and convergence of standard numerical methods, such as finite element (FEM), boundary element (BEM), finite-difference (FDM), spectral and meshless/meshfree methods. If, however, the form of the singularity is taken into account and is properly incorporated into the numerical scheme then a more effective method may be constructed [6, 7, 8].

The objective of this paper is to propose, implement and analyse a meshless method for the accurate and stable solution of Cauchy problem associated with two-dimensional Helmholtz-type equations in the presence of boundary singularities. More precisely, the governing equation and boundary conditions available on a part of the boundary (i.e. Cauchy data) are discretized by the method of fundamental solutions (MFS) [9]. Since the existence of boundary singularities affect adversely the accuracy and convergence of standard numerical methods, the solutions to such singular inverse problems and/or their corresponding derivatives, which are obtained by a straightforward inversion of the discretized MFS system, may have unbounded values in the vicinity of the singularity. Moreover, when dealing with inverse problems subjected to noisy data, the stability of solutions becomes a key issue [10] and this is usually accounted for by employing regularization methods. These inconveniences are overcome by combining the Tikhonov regularization method [11] with the subtraction from the original MFS solution the corresponding singular solutions, i.e. using the so-called singularity subtraction technique, without an appreciable increase in the computational effort and at the same time keeping the original MFS discretization. The choice of the optimal regularization parameter is realized by employing Hansen's L-curve method [12]. The proposed modified MFS is then implemented for the Cauchy problem associated with both the Helmholtz and the modified Helmholtz equations in two-dimensional domains with an edge crack or a V-notch, as well as L-shaped domain. The advantages of the proposed method over other methods, such as mesh refinement in the neighbourhood of the singularity, the use of singular BEMs

and/or FEMs etc., are the high accuracy which can be obtained even when employing a small number of collocation points and sources, and the simplicity of the computational scheme.

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