BEM-FEM ACOUSTIC-STRUCTURAL COUPLING FOR SPACECRAFT STRUCTURE INCORPORATING TREATMENT OF IRREGULAR FREQUENCIES

Harijono Djojodihardjo*
Universiti Sains Malaysia
14300 Nibong Tebal, Pulau Pinang, Malaysia
harijono@djojodihardjo.com

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Harijono Djojodihardjo
Professor, Universitas Al-Azhar Indonesia and
Professor, Universiti Sains Malaysia
harijono@djojodihardjo.com

ABSTRACT

Acoustic disturbances and the resulting structural vibration is a very significant problem in aerospace engineering. The magnitude of the acoustic loads transmitted to the payload is a function of the external acoustic environment as well as the design of the spacecraft structure and its sound absorbing treatments. The high intensity acoustic fields produced during a launch of a Space Shuttle or an Expendable Launch Vehicle (ELV) can easily damage a spacecraft's mission critical flight hardware, such as its avionics, antennas, solar panels and optical instruments. The loads transmitted to the spacecraft structure from the launch vehicle (LV) in the first few minutes of flight are far more severe than any load that a payload experiences on orbit. The severity of the environment affects the design for withstanding higher launch loads, hence the cost of placing the payload into orbit.

With such motivation and following earlier work, structural-acoustic interaction is modeled and analyzed using boundary and finite element coupling. The analysis is founded on the idealization of the problem into three parts; the calculation of the acoustic radiation from the vibrating structure, the finite element formulation of structural dynamic problem, and the calculation of the acousto-elasto-mechanic fluid-structure coupling using coupled BEM/FEM techniques. The computational scheme developed for the calculation of the acoustic radiation as well as the structural dynamic response of the structure using coupled BEM/FEM has given satisfactory results for acoustic disturbance in the low frequency range, which was the range of particular interest in many practical applications. However, for larger frequency range, it is well known that while the solution to the original boundary value problem in the exterior domain to the boundary is perfectly unique for all wave numbers, this is not the case for the numerical treatment of integral equation formulation, which breaks down at certain frequencies known as irregular frequencies or fictitious frequencies. Although such phenomenon is completely nonphysical since there are no discrete eigenvalues for the exterior problems, a method known as CHIEF (Combined Helmholtz Interior integral Equation Formulation) can be utilized to overcome such problem. Applications of CHIEF method to a spherical shell geometry has given excellent results.

Introduction

Structural-acoustic interaction, which is a significant issue found in many applications, including modern new and relatively lighter aircrafts operating at very high speed and altitudes, vibration problems can severely and adversely affect spacecraft structures and their payloads, and other engineering applications [1-6]. A series of work has been carried out by the author and colleagues in addressing the vibration of structures due to sound waves [8-10]. By modeling structural-acoustic interaction using boundary and finite element coupling, it is possible to couple the boundary element method and the finite element method to solve the structure-acoustic interaction problem [11-13]. In the Structural-Acoustic Analysis for Aerospace Structure Design problems, it is recognized that Computational Structural Mechanics (CSM) is efficient for structural-acoustic prediction in low-frequency ranges. Analysis of the detailed behavior of individual modes is possible using finite-element and classical methods.

The work carried out thus far is focused on the formulation of the basic problem of acoustic excitation and vibration of elastic structure in a coupled fluid-elastic-structure interaction. The approach consists of three parts. The first is the formulation of the acoustic field governed by the Helmholtz equation subject to the Sommerfeld radiation [14-18] condition for the basic acoustic problem without solid boundaries. The interface between the acoustic domain and the surface of the structure poses a particular boundary condition. Boundary element method will be utilized for solving the governing Helmholtz equation subject to the boundary conditions for the calculation of the acoustic pressure on the interface boundary. The second part deals with the structural dynamic problem, which is formulated using finite element approach. The third part involves the calculation of the acoustic-structure coupling, which is formulated using coupled BEM/FEM techniques. The acoustic loading on the structure is calculated on the part of the boundary of the acoustic domain, which coincides with the structural surface as defined by the problem.

Helmholtz Integral Equation for the Acoustic Field

For an exterior acoustic problem, as depicted in Fig. 1, the problem domain V is the free space V_{ext} outside the closed surface S. V is considered enclosed in between the surface S and an imaginary

surface Λ at a sufficiently large distance from the acoustic sources and the surface S such that the boundary condition on Λ satisfies Sommerfeld's acoustic radiation condition as the distance approaches infinity[14-18]. Green's first and second theorem provide the basis for transforming the differential equations in the problem domain V and the boundary conditions on the surface S into an integral equation over the surface S.
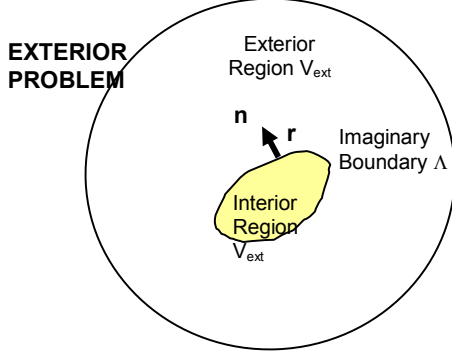

Fig 1. Exterior problem for homogeneous Helmholtz equation

For time-harmonic acoustic problems in fluid domains, the corresponding boundary integral equation is the Helmholtz integral equation [14-16]

$$cp(R) = \int_S \left(p(R)\frac{\partial g}{\partial \hat{n}_0} - g(|R-R_0|)\frac{\partial p}{\partial \hat{n}_0} \right) dS \quad (1)$$

where $\hat{n}_0$ is the surface unit normal vector, and the value of c depends on the location of R in the fluid domain, and where g the free-space Green's function. R_0 denote a point located on the boundary S, as given by

$$g(|R-R_0|) = \frac{e^{-ik|R-R_0|}}{4\pi|R-R_0|} \quad (2)$$

To solve Eq. (1) with g given by Eq. (2), one of the two physical properties, acoustic pressure and normal velocity, must be known at every point on the boundary surface. The specific normal impedance, which describes the relationship between pressure and normal velocity, can also serve as a boundary condition. At the infinite boundary Λ, the Sommerfeld radiation condition in three dimensions can be written as [14-16]

$$\lim_{|R-R_0|\to\infty} r\left(\frac{\partial g}{\partial r} + ikg\right) \Rightarrow 0 \text{ as } r \Rightarrow \infty, r = |R-R_0| \quad (3)$$

which is satisfied by the fundamental solution.
In the present work, the problem formulation also considers acoustic scattering pressure, which along with the incident acoustic pressure; give the total pressure which serves as an acoustic excitation on the structure.

For scattering problems, Eq. (1) only gives the solution to the scattered wave [17]. The boundary conditions are however given in terms of the total wave,

which is the sum of the incident and the scattered waves. It is necessary to modify the equation to include the incident wave. For exterior scattering problems, the modification can be carried out by adding to the scattered wave integral equations the result of applying the interior Helmholtz equations to the incident wave p_{inc}. Then the integral equation for the total wave is given by

$$cp(R) - p_{inc}(R) = \int_S \left[p(R)\frac{\partial g(R-R_0)}{\partial \hat{n}_0} - \frac{\partial p(r_q)}{\partial \hat{n}_0}g(R-R_0) \right] dS \quad (4)$$

where $p = p_{inc} + p_{scatter}$, and where

$$c = \begin{cases} 1 & , R \in V_{ext} \\ 1/2 & , R \in S \\ \Omega/4\pi & , R \in S \quad (\text{non smooth surface}) \\ 0 & , R \in V_{int} \end{cases} \quad (5)$$

BE Discretization of the Helmholtz Integral Equation

The first step in discretizing the Helmholtz equation is dividing the continuous system into a discrete one with N number of elements. Following the standard procedure in defining the elements on the boundary surface S, the discretized boundary integral equation becomes,

$$cp - p_{inc} - \sum_{j=1}^{N} \int_{S_j} p\frac{\partial g}{\partial \hat{n}}dS = -\sum_{j=1}^{N} \int_{S_j} g\frac{\partial p}{\partial \hat{n}}dS \quad (6)$$

where i indicates field point, j source point and S_j surface element j. For simplicity, introduce

$$\bar{g} \equiv \frac{\partial g}{\partial n} \quad (7)$$

Then Eq. (6) can be written as

$$cp_i - p_{inc} - \sum_{j=1}^{N} \int_S p\bar{g}dS = i\rho_0\omega\sum_{j=1}^{N} \int_S gvdS \quad (8)$$

For constant element, p and v in Eq.(8) are assumed to be constant over each element, and therefore they can be taken out of the integrals, and denoted by p_j and v_j, respectively, for any surface element j. By placing the node of the surface elements on the smooth part of the surface, $c = \frac{1}{2}$. Let

$$\bar{H}_{ij} = \int_{S_j} \bar{g}dS \quad (9)$$

$$G_{ij} = \int_{S_j} gdS \quad (10)$$

then Eq. (8) becomes,

$$\frac{1}{2}p_i - p_{inc} - \sum_{j=1}^{N} \bar{H}_{ij}p_j = i\rho_0\omega\sum_{j=1}^{N} G_{ij}v_j \quad (11)$$

Introducing

$$H_{ij} = \frac{1}{2}\delta_{ij} - \bar{H}_{ij} \quad (12)$$

where δ_{ij} is the Kronecker's delta, Eq.(11) reduces to

$$\sum_{j=1}^{N} H_{ij} p_j = i\rho_0\omega \sum_{j=1}^{N} G_{ij} v_j + p_{inc} \qquad (13)$$

The discretized equation forms a set of simultaneous linear equations, which relates the pressure p_i at field point i due to the boundary conditions p and v at surface S_i of the source element i and the incident pressure p_{inc}. In matrix form:

$$\mathbf{H}p = i\rho_0\omega \mathbf{G}v + p_{inc} \qquad (14)$$

where, $\mathbf{H}$ and $\mathbf{G}$ are two NxN matrices of influence coefficients, while p and v are vectors of dimension N representing total pressure and normal velocity on the boundary elements. This matrix equation can be solved if the boundary condition $\partial p/\partial n$ and the incident acoustic pressure field p_{inc} are known.

In the discretization carried out in this work, quadrilateral boundary elements will be utilized throughout. For spherical acoustic source, special care is taken on those elements at the poles, as elaborated in [8]. To facilitate solution of Eq. (1) and its discretized forms Eqs. (13)-(14), monopole free-space Green's fundamental solution has been utilized for g. Accordingly eq. (10) becomes:

$$G_{ij} = \int_{S_j} g dS = \int_{S_j} g\left(|R_j - R_i|\right) dS = \int_{S_j} \frac{e^{-ik|R_j - R_i|}}{4\pi |R_j - R_i|} dS \qquad (15)$$

In Cartesian coordinate system, this equation can be expressed as

$$G_{ij} = \int_{S_j} \frac{e^{-ik\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}}}{4\pi \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}} dS \qquad (16)$$

where R_j is the midpoint of element j and R_i is the location of the node i. For $i = j$, and to avoid singularity in calculating the value of the Green's function in the integral, Gaussian quadrature has been utilized, and in particular, four Gauss points are used on element j, instead of at the midpoint. These Gauss points in the isoparametric system are defined as[19-21]:

$$\left(\xi_1, \eta_1\right) = \frac{1}{\sqrt{3}}(1,-1) \qquad \left(\xi_2, \eta_2\right) = \frac{1}{\sqrt{3}}(1,1)$$

$$\left(\xi_3, \eta_3\right) = \frac{1}{\sqrt{3}}(-1,1) \qquad \left(\xi_4, \eta_4\right) = \frac{1}{\sqrt{3}}(-1,-1) \qquad (17)$$

and G_{ii} is then given by

$$G_{ij}\big|_{i=j} = G_{ii} = \frac{G_{ij}(\xi_1,\eta_1) + G_{ij}(\xi_2,\eta_2) + G_{ij}(\xi_3,\eta_3) + G_{ij}(\xi_4,\eta_4)}{4}\bigg|_{R_i = R_j} \qquad (18)$$

To calculate $\overline{H}_{ij}$ in Eq.(9), the derivative $\overline{g} = \dfrac{\partial g}{\partial n}$ is evaluated as follows

$$\overline{H}_{ij} = \int_{S_j} \overline{g} dS = \int_{S_j} \frac{\partial g}{\partial \hat{n}} dS = \int_{S_j} (\nabla g)\bullet\hat{n} dS \qquad (19)$$

where $\hat{n}$ is the local normal at surface element S_j and ∇g is the gradient of g. When $i \neq j$, $H_{ij} = \overline{H}_{ij}$. When $i = j$, the $\overline{H}_{ii}$ terms are identically zero and therefore $H_{ii} = 1/2$.

Four-node iso-parametric quadrilateral elements are utilized. Then the pressure p and the normal velocity v at any position on the element can be defined by their nodal values and linear shape functions, i.e.,

$$v(\xi,\eta) = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \qquad (20)$$

$$p(\xi,\eta) = N_1 p_1 + N_2 p_2 + N_3 p_3 + N_4 p_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \qquad (21)$$

where the linear shape functions are well known [20-21]. The evaluation of $\mathbf{H}$ and $\mathbf{G}$ can be carried out in similar fashion and is elaborated in reference [8]. The four node quadrilateral elements can have any arbitrary orientation in the three-dimensional space.
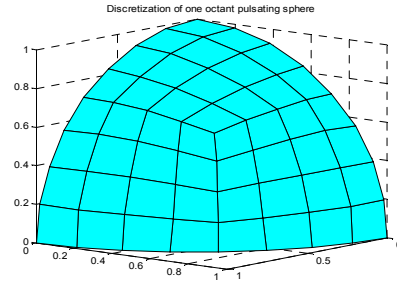

Fig 2. Discretization of one octant pulsating sphere using iso-parametric quadrilateral elements

For a pulsating sphere an exact solution for acoustic pressure a at a distance r from the center of a sphere with radius a pulsating with uniform radial velocity U_a is given by

$$p(r) = \frac{a}{r} U_a \frac{iz_0 ka}{1 + ika} e^{-ik(r-a)} \qquad (22)$$

where z_0 is the acoustic characteristic impedance of the medium and k is the wave number. Fig. 2 shows the discretization of the surface elements of an acoustics pulsating sphere representing a monopole source. Excellent agreement of the BEM calculation for scattering pressure from acoustic monopole source with exact results is exhibited in Fig. 3. The calculation was based on the assumption of f=10 Hz, ρ=1.225 Kg/m^3, and c=340 m/s. The excellent agreement of these results with exact calculation serves to validate the developed MATLAB® program for further utilization.
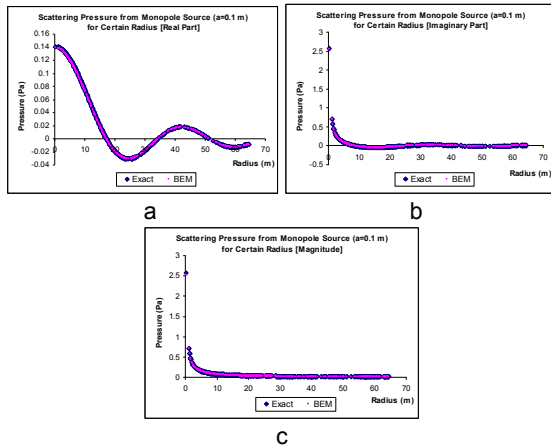
Fig 3. Comparison of monopole source exact and BE scattering pressure results (a-b Real and Imaginary parts, c magnitude)

Fig. 4 shows the convergence trend of the computational scheme as a function of the grid size on the calculation of the sound pressure level on the pulsating sphere for various frequencies of the pulsating sphere..
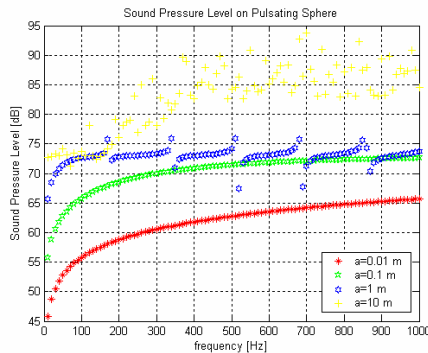

Fig 4. Convergence trend of the computational scheme as a function of the grid size on the calculation of the sound pressure level on the pulsating sphere

Spherical Shell BEM-FEM Simulation

This example is worked out to validate the BEM-FEM fluid-structure coupling modeling by applying the computational scheme for a constant thickness spherical shell subject to concentrated load and submerged in water. Numerical examples are presented by selecting a constant thickness spher**e** under concentrated external forces. The material data for the spherical shell are as follows: the radius of the shell is 1 m, the thickness of the shell is 003 m, Young's modulus is 2.07×10^{11} Pa (N/m^2), the Poisson ratio is 0.3, the density of the shell and water are 7669 and 1000 kg/m^3, respectively, and the sound speed of the water is 1524 m/s. An external concentrated alternating radial force is exerted at one point on the shell of the sphere. Table 1 exhibits the frequencies for the first four modes of FEM simulation using the present MATLAB® based computational

routine and quadrilateral elements, which are compared to PATRAN [20] results, using triangular and quadrilateral elements.
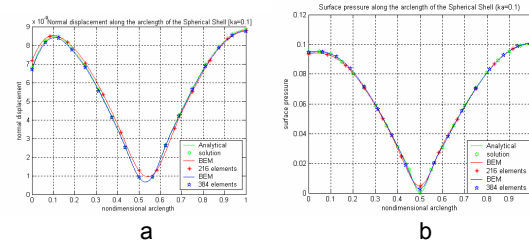

Fig. 5 a. Normal displacement and b. Surface pressure distribution, on a completely flexible sphere modeled using BEM-FEM Coupling, and subjected to external normal force at a point on the surface; $k_a = 0.1$

Table 1 FEM Simulation

Mode	Frequency (Hz)		
	PATRAN®		MATLAB®
	Tria	Quad	Quad
1	585.35	576.69	605.44
2	586.91	578.80	607.48
3	592.39	585.91	609.08
4	601.43	597.33	610.30

The normal displacement and surface pressure along the meridian line on the sphere, as exhibited in Fig. 5a and b, respectively, compares very well with the analytical solutions given in Junger and Feit [22].
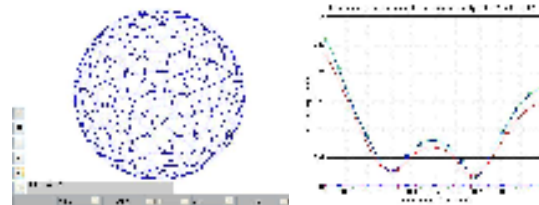

Fig 6 a. Discretization of the spherical shell
b. Normal displacement along a meridian line on the spherical shell, for $k_a = 1.6$
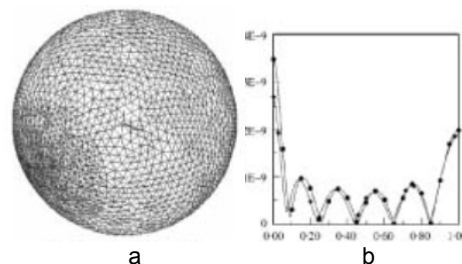

Fig 7 a. Discretization of the spherical shell
b. Normal displacement along a meridian line on the spherical shell, for $k_a = 1.6$, Chen et al [23]

Combined Helmholtz Interior integral Equation Formulatio (CHIEF) method

A major problem with the boundary element method is that, although the solution to the original differential equation and boundary conditions is unique, the transformed integral equation fails to give a unique solution at certain frequencies - the characteristic frequencies. In the case of the formulation of the exterior problem, these frequencies corresponds to the natural frequencies of acoustic resonances in the interior region. When the interior region resonates, the pressure field inside the interior region has non-trivial solution. Since the interior problem and the exterior problem shares similar integral operators, the exterior integral equation may also break down.
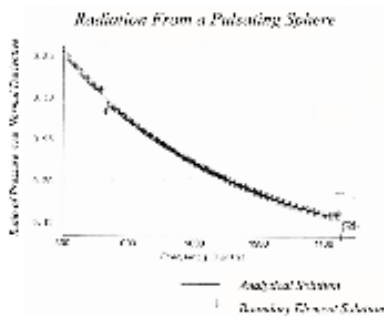

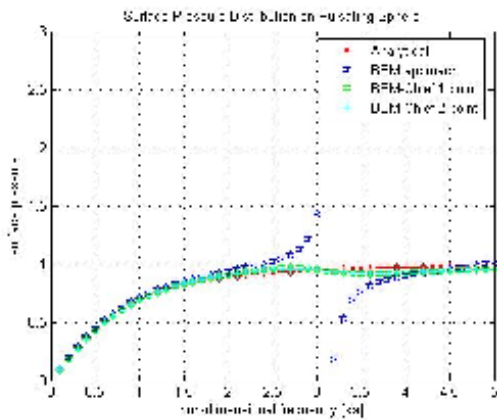Fig 8 Radiation from a puylsating sphere (reproduced from [17])

Fig. 9 Surface pressure distribution on pulsating sphere for analytical, BEM, and BEM-CHIEF solution for one and two CHIEF point

One method that has exhibited simplicity and effectiveness to overcome the problem of non-unique solution at a characteristic frequency is the CHIEF (Combined Helmholtz Interior integral Equation Formulation) method, suggested by Schenck [24] and Burton and Miller[25], which has als0 been applied rlsewher [26-27], is applied here.

In this method, the integral equation for r , S is combined with the additional constraint provided by the homogeneous integral equation for r , V_{int} for the exterior problem (or that for r , V_{ext} for the interior problem). Upon

discretisation, an over-determined matrix equation is formed which can be solved by the least square method. This method is very simple to implement and effective when the internal points chosen do not coincide with the nodal points of the interior resonant field. The main difficulty is how to choose suitable number and positions of the interior points. Applications of CHIEF method to a spherical shell geometry has given excellent results, as illustrated in Fig.9.

BEM-FEM Acoustic-Structural Coupling

Two approaches can be followed to treat the structural-acoustic coupling on the boundary element and finite element interface region [19-21]. In the first approach, the BE region is treated as a super finite element and its stiffness matrix is computed and assembled into the global stiffness matrix and is identified as the coupling to finite elements. In the second approach, the FE region is treated as an equivalent BE region and its coefficient matrix is determined and assembled into the global coefficient matrix and is identified as the coupling to boundary elements. In the present study, the first approach is considered. The state of affairs is schematically depicted in Fig. 10.
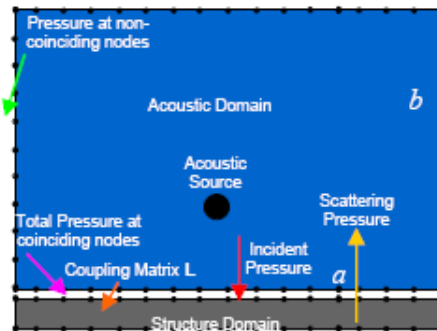

Fig 10. Schematic of Fluid-Structure Interaction Domain

The FEM leads to a system of simultaneous equations which relate the displacements at all the nodes to the *nodal forces*. In the BEM, on the other hand, a relationship between nodal displacements and *nodal tractions* is established.

The elastic structure can be represented by FE model. For a dynamic structure, noting that no structural damping has been assumed throughout, the equation of motion is given by

$$[\mathbf{M}]\{\ddot{x}\} + [\mathbf{C}]\{\dot{x}\} + [\mathbf{K}]\{x\} = \{\mathbf{F}\} \qquad (23)$$

where **M**, **C**, and **K** are structural mass, damping and stiffness, respectively, which are expressed as matrices in a FE model. **F** is the given external forcing function vector, and *{x}* is the structural displacement vector. Taking into account the acoustic pressure *p* on the structure at the fluid-structure interface as a separate excitation force, the acoustic-structure

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problem can be obtained from Eq.(23) by introducing a fluid-structure coupling term given by **L** p [12,17]. It follows that

$$[\mathbf{M}]\{\ddot{x}\}+[\mathbf{K}]\{x\}+[\mathbf{L}]\{p\}=\{\mathbf{F}\} \qquad (24)$$

where **L** is a coupling matrix of size $M \times N$ in the BEM/FEM coupling thus formulated. M is the number of FE degrees of freedom and N is the number of BE nodes on the coupled boundary. For the BE part of the surface at the fluid-structure interface a, Eq.(14) can be rewritten as

$$[\mathbf{H}]\{p\}=i\rho_0\omega[\mathbf{G}]\{v\}+\{p_{inc}\} \qquad (25)$$

The global coupling matrix **L** transforms the acoustic fluid pressure acting on the nodes of boundary elements on the entire fluid-structure interface surface a, to nodal forces on the finite elements of the structure. Hence **L** consists of n assembled local transformation matrices **L$_e$**, given by

$$L_e = \int_{S_e} N_F^T n N_B \, dS \qquad (26)$$

in which N_F is the shape function matrix for the finite element and N_B is the shape function matrix for the boundary element. It can be shown that:

$$N_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}[N_i] \qquad (27)$$

The rotational parts in N_F are neglected since these are considered to be small in comparison with the translational ones in the BE-FE coupling, consistent with the assumptions in structural dynamics as, for example, stipulated in [28].

For the normal fluid velocities and the normal translational displacements on the shell elements at the fluid-structure coupling interface a relationship has to be established which takes into account the velocity continuity over the coinciding nodes [12,17]:

$$v = i\omega(\mathbf{T}.x) \qquad (28)$$

Similar to **L**, **T** $(n \times m)$ is also a global coupling matrix that connects the normal velocity of a BE node with the translational displacements of FE nodes obtained by taking the transpose of the boundary surface normal vector n. The local transformation vector **T$_e$** can then be written as:

$$\mathbf{T}_e = n^T \qquad (29)$$

The normal fluid velocities of the acoustic problem and the normal translation of the structural surface-fluid interface have to satisfy a certain relationship which takes into account the velocity continuity over the coinciding nodes.

The presence of an acoustic source can further be depicted by Fig.11. Two regions are considered, i.e. a and b ; region a is the afore mentioned fluid-structure interface region, where FEM mesh and BEM mesh coincide and region b is the region where all of the boundary conditions (pressure or velocity) are known.
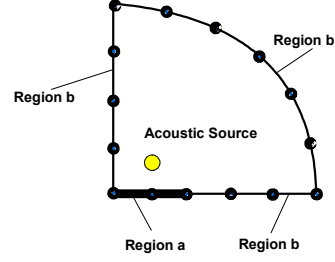

Fig 11. Schematic FE-BE problem representing quarter space problem domains

For the coupled FEM-BEM regions, Eq. (25) and Eq. (29) apply at $\{x\} \in$ a. Based on regional division, BEM equation can now be written as:

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix}\begin{Bmatrix} p_a \\ p_b \end{Bmatrix} = i\rho_0\omega\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix}\begin{Bmatrix} v_a \\ v_b \end{Bmatrix} + \begin{Bmatrix} p_{inc_a} \\ p_{inc_b} \end{Bmatrix}$$

$$(29)$$

Using Eq.(32) for v_a, the BEM Eq. (34) can be modified accordingly as:

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix}\begin{Bmatrix} p_a \\ p_b \end{Bmatrix} = i\rho_0\omega\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix}\begin{Bmatrix} i\omega(\mathbf{T}x) \\ v_b \end{Bmatrix} + \begin{Bmatrix} p_{inc_a} \\ p_{inc_b} \end{Bmatrix}$$

$$(30)$$

or:

$$\mathbf{H}_{11}p_a + \mathbf{H}_{12}p_b = -\rho_0\omega^2\mathbf{G}_{11}\mathbf{T}x + i\rho_0\omega\mathbf{G}_{12}v_b + p_{inc_a}$$

$$\mathbf{H}_{21}p_a + \mathbf{H}_{22}p_b = -\rho_0\omega^2\mathbf{G}_{21}\mathbf{T}x + i\rho_0\omega\mathbf{G}_{22}v_b + p_{inc_b}$$

$$(31)$$

If the velocity boundary condition on b (v_b) and the incident pressure on a and b are known, reorganizing the unknowns to the left side, Eq.(31) become:

$$\rho_0\omega^2\mathbf{G}_{11}\mathbf{T}x + \mathbf{H}_{11}p_a + \mathbf{H}_{12}p_b = i\rho_0\omega\mathbf{G}_{12}v_b + p_{inc_a}$$

$$\rho_0\omega^2\mathbf{G}_{21}\mathbf{T}x + \mathbf{H}_{21}p_a + \mathbf{H}_{22}p_b = i\rho_0\omega\mathbf{G}_{22}v_b + p_{inc_b}$$

$$(32)$$

Since the pressure p on FEM equation lies in region a, Eq. (24) can be written as

$$[\mathbf{M}]\{\ddot{x}\}+[\mathbf{K}]\{x\}+[\mathbf{L}]\{p_a\}=\{\mathbf{F}\} \qquad (33)$$

Following the general practice in structural dynamics, solutions of Eq.(33) are sought by considering synchronous motion with harmonic frequency ω. Correspondingly, Eq. (33) reduces to:

$$\left[\mathbf{K}+\omega^2\mathbf{M}\right]\{\bar{x}\}+[\mathbf{L}]\{\bar{p}_a\}=\{\mathbf{F}\} \qquad (34)$$

where

$$x = \bar{x}e^{i\omega t} \; ; \; p_a = \bar{p}_a e^{i\omega t} \qquad (35)$$

or, dropping the bar sign for convenience, but keeping the meaning in mind, Eq. (34) can be written as

$$\left[\mathbf{K}+\omega^2\mathbf{M}\right]\{x\}+[\mathbf{L}]\{p_a\}=\{\mathbf{F}\} \qquad (36)$$

Combining Eq. (32) and Eq. (36), the coupled BEM-FEM equation can then be written as:

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$$\begin{bmatrix} \mathbf{K}+\omega^2\mathbf{M} & \mathbf{L} & 0 \\ \rho_0\omega^2\mathbf{G}_{11}T & \mathbf{H}_{11} & \mathbf{H}_{12} \\ \rho_0\omega^2\mathbf{G}_{21}T & \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{Bmatrix} x \\ p_a \\ p_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ i\rho_0\omega\mathbf{G}_{12}v_b \\ i\rho_0\omega\mathbf{G}_{22}v_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ p_{inc_a} \\ p_{inc_b} \end{Bmatrix} \qquad (37)$$

This equation forms the basis for the treatment of the fluid-structure interaction in a unified fashion. The solution vector consisting of the displacement vector of the structure and total acoustic pressure on the boundaries of the acoustic domain, including the acoustic-structure interface, is represented by $\{x\ p_a\ p_b\}^T$, while $\{0\ p_{inca}\ p_{incb}\}^T$ is the acoustic excitation vector. Due consideration should be given to $\mathbf{L}$, where $\mathbf{L}$ is a coupling matrix of size $m \times n$, m is the number of FE degrees of freedom and n is the number of BE nodes on the coupled (interface) boundary a.

The detail of the Finite Element Formulation of the Structure has been elaborated in previous work [8-9], and will not be repeated here. It suffices to mention that two types of elements have been considered to be utilized; these are an eight node hexahedral for solid modeling and four node quadrilateral elements for shell modeling.

Flexible Structure Subject to Harmonic External Forces in Acoustic Medium

Several cases are considered to validate the program as well as to evaluate its performance. An example is carried out for a box with vibrating membrane as shown in Fig.12. The structure consists of five hard walls and one flexible membrane on the top and has the dimension a × b x c = 304.8 × 152.4 × 152.4 mm. The flexible structure, which consists of a 0.064 mm thick undamped aluminum plate, is modeled with coupled boundary and finite elements.
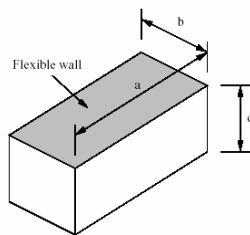

Fig. 12 Generic flexible structure typical of space-structure subjected to harmonic external forces in acoustic medium

The frequency response due to the application of arbitrary harmonic excitation forces to the membrane following the present method using BEM-FEM Coupling is shown in Fig. 13, and is compared to results obtained using FEM-FEM approach [12]. Comparable agreement has been indicated and serves to validate the present method.
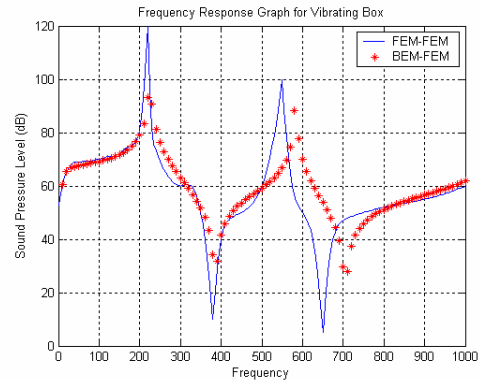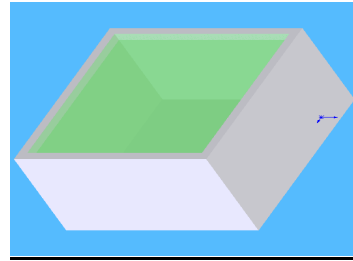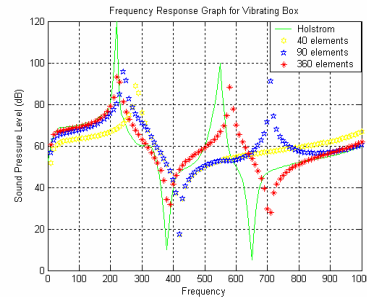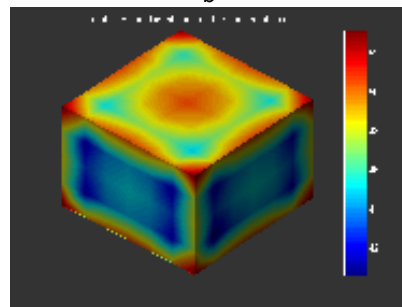

Fig. 13 Comparison between FEM-FEM and BEM-FEM approach for acoustic pressure [dB] on plate center

a

b

c

Fig 14. b. Convergence trend of the sound pressure level frequency response of a vibrating top membrane of an otherwise rigid box due to monopole source at the center of the box a studied in [12] as a function of frequency calculated using present computational scheme for various grid size; c. Total pressure distribution on the surfaces of the same box. Top surface is modeled as BE-FE, others as BE.

8

The cnvergence trend of the sound pressure level frequency response of a vibrating top membrane of an otherwise rigid box due to monopole source at the center of the box as studied in [12] is exhibited in Fig.14

Flexible Structure Subject to Acoustic Excitation in a Confined Medium

Application of the method to another example is carried out for a box shown in Fig. 6 with a dimension of a × b x c = 450 × 450 × 270 mm.. Five walls of the structure are modeled as shell elements and the bottom of the box is modeled as a rigid wall. Each of the flexible walls is assumed to be made of 1 mm aluminum plate, and is modeled as coupled boundary and finite elements.

This box structure is subjected to an acoustic monopole source at the center of the box and the acoustic medium is air with the following properties: density, ρ =1.21 kg/m^3 and sound velocity c =340 m/s. The discretization of the box is also depicted in Fig.15.
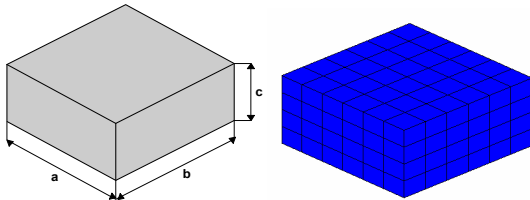

Fig. 15 Generic flexible structure typical of space-structure subjected to monopole acoustic source in an acoustic medium

a. Normal Mode Analysis

The result of the modal analysis of the box to obtain the first three normal modes using finite element program developed in MATLAB$^®$ is shown in Fig. 16. The first five eigen frequencies for the shell elements obtained using the present routine are compared to those obtained using commercial package NASTRAN$^®$. As exhibited in Table 2, good agrrement is obtained.
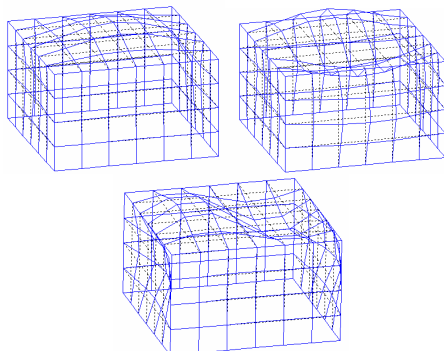

Fig.16 First three normal modes for box modeled with five flexible walls using shell elements

b. Acoustic Excitation

Acoustic excitation due to an acoustic monopole source with initial frequency, f = 10 Hz, is applied at the center of the box. No other external forces are applied. The resulting distribution of the incident acoustic pressure is shown in Fig. 17.

Table.2 First five Eigen frequencies for box modeling with shell element

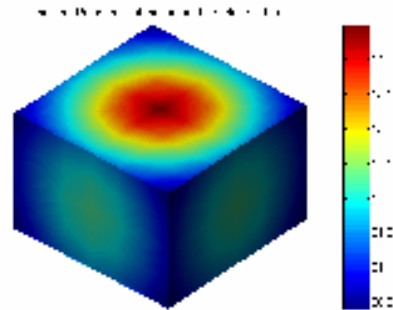| Mode | Natural Frequency (Hz) | |
	MATLAB$^®$	NASTRAN$^®$
1	17.644	16.190
2	36.026	35.986
3	41.072	41.520
4	44.585	44.787
5	51.231	51.689

Fig.17 Incident acoustic pressure distribution on the surface of box due to monopole acoustic excitation

The total pressures on the surface of the box obtained from the computational results are shown in Fig.18. The frequency response for the incident pressure and total pressure on the center top surface of the box computed using Eq. (42) is shown in Fig. 19.
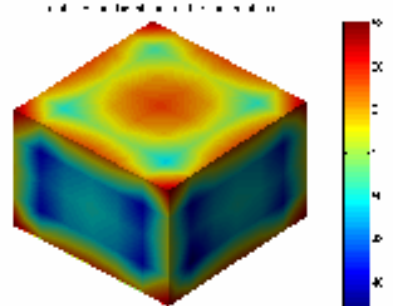

Fig. 18 Total acoustic pressure distribution on the surface of box due to monopole acoustic excitation
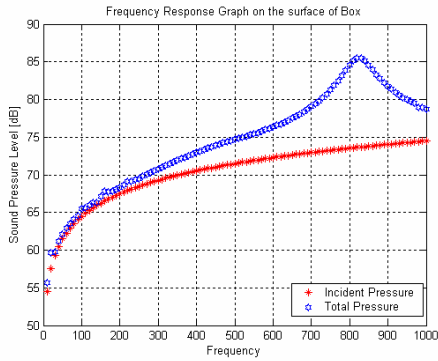
Fig. 19 Incident and total acoustic pressure distribution on the center top surface of box as a function of frequency

Fig.21 Incident pressure distribution [dB] due to monopole acoustic source as an acoustic excitation on flexible structure

Numerical Simulation of Flat Plate Subject to Acoustic Excitation in an Infinite Medium

The present method is then applied to a flat plate spacecraft structure. For the purpose of this study, the dimension of the flexible structure is 450 x 150 x 5 mm. The material properties for flat plate made of AISI 4130 Steel is as follows: Modulus of Elasticity, $E = 29 \times 10^6$ N/m^2, Shear Modulus, $G = 11 \times 10^6$ N/m^2, Poisson's Ratio $\upsilon = 0.32$ and density, $\rho = 7.33145 \times 10^{-4}$ Kg/m^3

The flexible structure is now modeled with Finite Element and the surrounding boundary is represented by a quarter space is modeled using Boundary Elements; typical boundary element discretization of the surface, and the finite element discretization of the plate, is exhibited in Fig. 20.
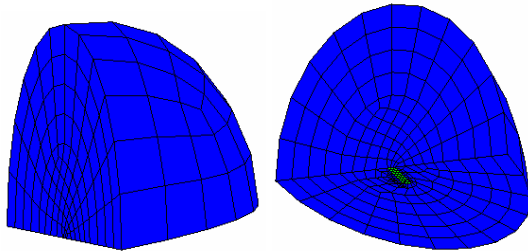

Fig.20 FEM-BEM discretization

The monopole acoustic source is placed at the center of the plate, at a distance 0.1 m above it.

For illustrative purposes, the frequency of the monopole source is assumed to be 10 Hz and the fluid medium is air with density $\rho = 1.21$ kg/m^3 and sound velocity c =340 m/s. The incident pressure distributions is depicted in Fig.21 and the total acoustic pressure distribution is shown in Fig.22.

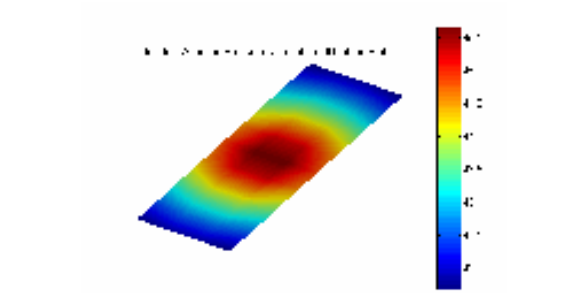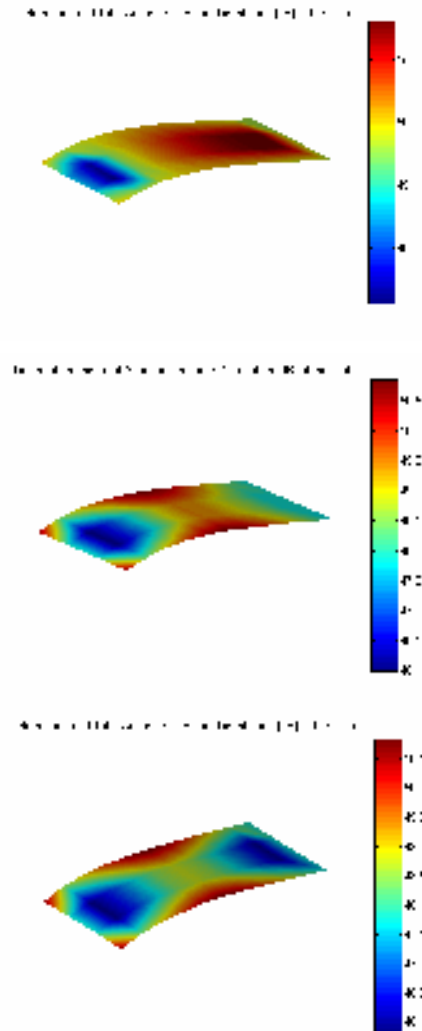Fig. 22 Deformation and total acoustic pressure response [dB] on flexible structure for plate thickness 5, 10 & 20 mm (top to down)

From the results of these case studies, some fundamental aspects of the computational procedure can be summarized:

1. The computational procedure for solving combined excitation due to acoustic and external forces on structural problem formulated as coupled FEM-BEM equation has given good results.
2. The solution of Eq.(42) as a result of the effect of acoustic disturbance to a structure is given as the pressure loading response on the structure as well as the pressure field in the fluid medium, given as total pressure. This allows the calculation of the scattering pressure due to the incident pressure.

Conclusions

A method and computational procedure for BEM-FEM Acousto-Elasto-Mechanic Coupling has been developed. The method is founded on three generic elements; the calculation of the acoustic radiation from the vibrating structure, the finite element formulation of structural dynamic problem, and the calculation of the acousto-elasto-mechanic fluid-structure coupling using coupled BEM/FEM techniques.

The procedure developed has been validated by reference to classical examples or others available in the literature. The applicability of the method for analyzing typical problems encountered in space structure has been demonstrated. Refinement of the method for fast computation and for complex geometries are in progress. Such steps may be useful for acoustic load qualification tests as well as structural health monitoring.

References

[1] Edson, J.R.,"Review of Testing and Information on Sonic Fatigue,"Doc. No. D-17130, Boeing Co., 1957.

[2] Powell, C.A. and Parrott, T.L., "A summary of Acoustic Loads and Response Studies, "Tenth National Aero-Space Plane Technology Symposium, Paper No. 106, April, 1991.

[3] Eaton, D.C.G., An Overview of Structural Acoustics and Related High-Frequency-Vibration Activities. http://esapub.esrin.esa.it /bulletin/bullet92/b92eaton.htm.

[4] Renshaw, A. A., D'Angelo, C., and Mote, C. D., Jr., 1994, ''Aeroelastically Excited Vibration of a Rotating Disk,'' J. Sound Vib., **177**~5!, pp. 577–590.

[5] Kang, Namcheol and Raman, Arvind, Aeroelastic Flutter Mechanisms of a Flexible Disk Rotating in an Enclosed Compressible Fluid, Transactions of the ASME, Vol. 71, January 2004, pp 120-130.

[6] Tandon, N. , Rao, V.V.P. and Agrawal, V.P. , Vibration and noise analysis of computer hard disk drives, Measurement 39 (2006) 16–25, www.elsevier.com/locate/measurement

[7] Djojodihardjo, H. and Tendean,E., A Computational Technique for the Dynamics of Structure Subject

To Acoustic Excitation, ICAS 2004, Vancouver, October 2004.

[8] Djojodihardjo, H. and Safari, I., Further Development of the Computational Technique for Dynamics of Structure Subjected to Acoustic Excitation, paper IAC-05-C2.2.08, presented at the 56th International Astronautical Congress / The World Space Congress-2005, 17-21 October 2005/Fukuoka Japan.

[9] Djojodihardjo, H and Safari, I., Unified Computational Scheme For Acoustic Aeroelastomechanic Interaction, paper IAC-06-C2.3.09, presented at the 57th International Astronautical Congress / The World Space Congress-2006, 1-6 October 2006/Valencia Spain.

[10] Harijono Djojodihardjo and Irtan Safari, BEM-FEM COUPLING FOR ACOUSTIC EFFECTS ON AEROELASTIC STABILITY OF STRUCTURES, keynote paper, ICCES 07, paper ICCES0720060930168, Miami Beach, Florida, 3-6 January 2007

[11] Chuh Mei, and Carl S. Pates, III, *Analysis Of Random Structure-Acoustic Interaction Problems Using Coupled Boundary Element And Finite Element Methods*, NASA-CR-195931, 1994.

[12] Holström, F., Structure-Acoustic Analysis Using BEM/FEM; Implementation In MATLAB[®], *Master's Thesis,* Copyright © 2001 by Structural Mechanics & Engineering Acoustics, LTH, Sweden, Printed by KFS i Lund AB, Lund, Sweden, May 2001, Homepage: http://www.akustik.lth

[13] Meddahi, S., Marquez, A. and Selgas, V. , "A new BEM–FEM coupling strategy for two-dimensional fluid–solid interaction problems", Journal of Computational Physics 199 pp. 205–220, 2004.

[14] Dowling, A.P. and Ffowcs Williams, J.E., Sound and Sources of Sound, Ellis Horwood Limited, Chichester – John Wiley & Sons, New York Brisbane Chichester Toronto, 1983, ©1983, A.P.Dowling and J.E. Ffowcs Williams.

[15] Wrobel, L.C., The Boundary Element Methods, Vol.1, Applications in Thermo-Fluids and Acoustics, John Wiley & Sons, Chichester, West Sussex PO19 IUD, Englenad, 2002

[16] Norton, M.P., *Fundamentals of Noise and Vibration Analysis for Engineers*, Cambridge University Press, Cambridge, New York, Melbourne, Sydney,1989.

[17] -, Acoustic Boundary Element Method 1. Basic Theory,Acoustics.salford.ac.uk/student_area/bsc 3/.../BEM2005.pdf

[18] S,Langdon, Approximation of high frequency acoustic wave surface integrals,http:// www.personal.rdg.ac.uk/~sms03snc/ smart_ numerics .html

[19] Felippa C. A. and Clough R.W., "The Finite element method in solid mechanics", American Mathematical Society, Providence, R.I., 210–252, 1969.

[20] Weaver, W. and Johnston, P. R., "Structural Dynamics by Finite Elements", Prentice-Hall, Inc. Englewood Cliffs, New Jersey. 1987.

[21] Beer, G. and Watson, J.O., "Introduction to Finite and Boundary Element Methods for Engineers". John Wiley & Sons Ltd, Baffins Lane, Chichester, England, 1992.

[22] Junger, M. C. and Feit, D., "Sound, structures and their interaction," M.I.T. Press, 1986.

[23] Chen, P.-T., Ju, S.-H. and Chia, K.-C., A Symmetric Formulation Of Coupled BEM/FEM In Solving Responses Of Submerged Elastic Structures For Large Degrees Of Freedom, Journal of Sound and Vibration (2000) 233(3), 407-422, available online at http://www.idealibrary.com on

[24] H. A. Schenck, Improved integral formulation for acoustic radiation problems, J. Acoust. Soc. Am. 44(1) (1976) 41–58.

[25] J. Burton and G. F. Miller, The application of integral equation methods to numerical solutions of some exterior boundary value problem, Proc. Royal Society London Ser. A 323 (1971) 201–210.

[26] J.T. Chen, I. L. Chen and M. T. Liang, 2001, "Analytical Study and Numerical Experiments for Radiation and Scattering Problems using the CHIEF Method", Journal of Sound and Vibration, 248(5), pp.809-828.

[27] J.T. Chen, I. L. Chen, S. R. Kuo and M. T. Liang, 2001, "A New Method for True and Spurious Eigensolutions of arbitrary cavities using the CHEEF Method", J. Acoustic Society of America, 109(3), [DOI:10.1121/1.1349187].

[28] R. L. Bisplinghoff, H. Ashley, and R. L. Halfman, *Aeroelasticity*, Addison-Wesley Publishing Company, Inc., 1955.