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Neural-Network Solution of the Nonuniqueness Problem in Acoustic Scattering Using Wavelets

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The solution of Helmholtz integral equation for acoustic scattering is confronted with a nonuniqness issue at the characteristic wave numbers. In this paper, a neural network-based solution to this problem is proposed. To start with, the moment matrix resulting from the discretized Helmholtz integral equation is sparsified using appropriate wavelet techniques. This sparse matrix is, further, analyzed to obtain unique patterns that characterize its structure. As a result, a proper training set of these patterns is constructed and utilized to train a back propagation neural network. The trained network is capable of predicting the scattered acoustic field for the wave numbers at which the problem doesn't suffer any nonuniqueness. Moreover, the network can also be used to obtain the scattered field even for such wavenumbers at which the nonuniqueness occurs. Comparing the neural network outputs with the exact solutions demonstrates the validity and efficiency of the proposed method.

Keywords Aacoustic Scattering, Helmholtz Integral Equation, Neural Networks, Daubechies Wavelet Family, Nonuniqueness, Characteristic Wavenumber

1. INTRODUCTION

The solution of acoustic wave scattering problems by a closed body placed in a homogeneous medium reduces to the solution of the Helmholtz equation. The integral formulation of the problem is very attractive as it eliminates the need to consider the unbounded domain associated with scattering problems. Although the formulation is very simple and has been widely used, the computed solutions may include some interior resonant solutions that lead to nonuniqueness. In other words, the integral equation formulation on a scattering surface does not have a unique solution at the natural frequencies of an associated Dirichlet problem. Such complications arise due to the method of solution and not from the nature of the problem itself [1].

As long as the length scale of the scatterer is comparable to the used wavelength, standard moment-method approaches are well suited [2]. The method of moment (MoM) is essentially a discretization scheme whereby a general operator equation is transformed into a matrix equation that can be solved numerically. Accordingly, the moment matrix suffers from nonuniqueness problem at the characteristic wavenumbers of the corresponding acoustic scattering equation [1, 3]. The mechanism by which the modal participation factor may dominate the numerical instability at such frequencies is presented in [4].

Two main approaches have been proposed to resolve the nonuniqueness problem for acoustic scattering. Comparison between these two methods shows that they introduce their own complications [5]. A method to overcome nonuniqueness employs the addition of some constraints on internal fields at a finite number (M) of points ($M \ll N$ where N is the number of unknowns). This technique is known as CHIEF (Combined Helmholtz Integral Equation Formulation) method. Applying the discretization of the scattering surface to the CHIEF equations leads to an overdetermined system of equations for the surface field. A potential problem with this approach is the choice of appropriate interior points. The interior point must not be a nodal point of the corresponding interior eigenmode at the considered characteristic frequency [1]. Another approach for solving the nonuniqueness problem was introduced by Burton and Miller [6]. They formed a linear combination of the Helmholtz integral equation with its normal derivative. This formulation is valid for all wave numbers [1]. A major drawback of this approach is that it requires the evaluation of the hyper-singular integrals involving a double normal derivative of the free space Green's function [5]. Moreover, the reduction of the high computational burden involved in solving the nonuniqueness problem is tackled in [7] and [8].

The neural networks (NNs) are, recently, employed in improving the speed and accuracy of the solution of various mathematical problems. For example, NNs are used, efficiently, in approximating the mathematical expression of an acoustic scattering on a rigid sphere [9]. The NNs can also be used to solve the system of equations resulting from discretizing the integral equation of acoustic scattering. However, the NN size becomes intractable due to the large number of moment matrix elements.

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As a result, selection of the elements that characterize the moment matrix appears to be very crucial. The optimum selection of these elements is instrumental for reducing the size of the NN and, consequently, the training time. Data preprocessing and dimensionality reduction are essential for successful NN training [10]. The wavelet analysis proved to be very effective in alleviating the curse of dimensionality in the numerical solution of many modeling problems [11, 12].

In this work, the acoustic scattering field on the surface is expanded into Helmholtz integral equation in terms of wavelet basis functions. The substitution of such expansion into the integral equation results in a new matrix that can be thresholded appropriately to obtain a sparse matrix. This sparse matrix exhibits symmetrical patterns along its columns as a result of the symmetry of the wavelet transformation operator. NN-based solution of the problem is proposed based on the sparsified moment matrix. The sparse and symmetrical properties are exploited to reduce the number of inputs to the NN, and consequently, minimize the training computational load. As such, only subsets of the matrix elements are utilized to uniquely represent the whole matrix. This subset of moment matrix elements along with the incident field is the input patterns to the NN. Various patterns are constructed for different wavelengths of the incident field. The selected wavenumbers are different from the set of characteristic wavenumbers or the natural frequencies of the scattering problem. Every pattern is then paired with the corresponding analytical solution of the scattered acoustic field. This set of patterns is referred to as the training set since it is used to train a back-propagation NN. The trained NN is tested at different wavenumbers including characteristic numbers at which a nonuniqueness problem is encountered with traditional methods. The outputs of the trained NN are compared with the analytical solutions of the scattered field.

The developed NN-based method readily provides a solution of the scattering problem from the moment matrix without any further computations except that it is incurred by the NN. Accordingly, the high computational effort as well as the special treatment needed for solving the nonuniquenss problem, as presented in [1] and [8], are saved.

2. WAVELET ANALYSIS OF THE ACOUSTIC SCATTERING PROBLEM

Let V_i denote a bounded domain in $\Re^3$ with a boundary Σ which is a closed surface and ϕ is an acoustic field on Σ. The single and double layer operators $S\{.\}$ and $D\{.\}$ are defined as follows:

$$S\{\phi\} \equiv \int_\Sigma \phi(q)\, G(p,q)\, dS_q \tag{1}$$

$$D\{\phi\} \equiv \int_\Sigma \phi(q)\partial_{nq}\, G(p,q)\, dS_q \tag{2}$$

where ∂_ndenotes the derivative with respect to n, n is the outward normal to Σ, ds_q is a surface element on Σ and $G(x, x') = \frac{e^{-ikR}}{R}$ is the free space Green's function, $R = |p - q|$ and (p, q) denote a field point and an integration point, respectively. A field, ϕ^i is incident on the scatterer bounded by Σ. Applying Green's second identity, the following integral equation for the unknown acoustical field ϕ is obtained

$$\phi^i(p) + D\{\phi\} - S\{\upsilon\} = \begin{cases} \phi(p) & p \in V_o \\ \phi(p)/2 & p \in \sum \\ 0 & p \in V_i \end{cases} \tag{3}$$

where $\upsilon = \partial_n \phi$. The above equation is referred to as the Helmholtz integral formula.

The unknown field function, ϕ,, on the scatterer surface can be expanded in terms of wavelet bases functions. The approximation of $\phi(x) \in L^2(R)$ at a resolution of 2^{-j} can be defined as the projection on different wavelet functions as follows

$$\phi(x) = a_0 + \sum_j \sum_k a_{jk}\psi_{jk}(x) \quad j = 1, .., M \quad \text{and} \quad k = 1, .., N \tag{4}$$

where a_{jk} is the amplitude of each wavelet at different resolutions (scales) and locations while a_o represents a constant bias in the function. M is the discretization number of points on the scatterer surface while N is the number of levels for wavelet analysis.

Using (4), the Galerkin's discretized form of the integral equation takes the following form [7]

$$\mathbf{A}_{MxM}\mathbf{F}_{MxM}\mathbf{X}_M = \mathbf{Z}_M \tag{5}$$

Where **A** is the Galerkin's moment matrix, **X** is the unknown wavelet amplitudes vector, **Z** is the incident field vector defined at the surface points. The operator matrix **F** of a wavelet ψ_{mn}, consists of the mother wavelet for all possible & n in the domain.

The moment matrix, with the aid of (1), can also be expressed as follows

$$\mathbf{A} = \begin{bmatrix} G'(R11) & G'(R12)................ & G'(R1M) \\ G'(R21) & G'(R22)................ & G'(R2M) \\ . & . & \\ . & . & \\ . & . & \\ G'(RM1) & G'(RM2)................ & G'(RMM) \end{bmatrix} \tag{6}$$

Where $G'(RIJ)$ is the derivative of Green's function in (1) for the distance between the discretized points I and J.

The operator matrix **F** may take the following form if the Haar (Daubechies of order 2) wavelet is used

$$F^T = \begin{bmatrix} 1 \;\; 1 \;\; 1 \ldots. & & & 1 \;\; 1 \\ b_0 \; b_1 & & & \\ & b_0 \; b_1 & & \\ & & & .b_0 \; b_1 \\ b_0 \; b_0 \; b_1 \; b_1 & & & \\ & b_0 \; b_0 \quad b_1 \quad b_1 & & \\ & \ldots\ldots\ldots\ldots & & \\ b_0 \; b_0 \; b_0 \;\; . & b_0 \; b_0 \ldots\ldots\ldots\ldots\ldots & b_1 \; b_1 & b_0 \; b_1 \end{bmatrix} \tag{7}$$

where b_0 and b_1 are the Haar wavelet coefficients.

As such, (5) can be rewritten as

$$\mathbf{Q}_{MxM}\mathbf{X}_M = \mathbf{Z}_M \tag{8}$$

Each element in matrix **Q** can be set to zero if it does not exceed a prespecified sparsification threshold. Accordingly, **Q** will be highly sparse and exhibit some symmetrical properties similar to the operator matrix **F**.

Several families have proven to be useful in signal and matrix processing (i.e., Daubechies, Biorthogonal, Haar, Shanon, etc.). Each family has specific properties that make them suitable for certain applications. There are many orthonormal wavelets that give acceptable localizations both in time, and frequency [12, 13]. Recently, the use of Daubechies wavelet proves to be successful along a wide range of scattering problems [14–16].

3. NEURAL NETWORK FOR ACOUSTIC SCATTERING

The remarkable ability of NN to synthesize complicated nonlinear relationships through learning from examples is exploited to obtain a solution for (8) [17]. The input to the network will be some representative elements of the sparsified moment matrix (**Q**), and incident wave (excitation) **Z**, while the corresponding output is the scattered field ϕ on the scattering surface represented by the vector **X**. Training sets that characterize this relation are constructed.

Formally, the input vectors to the network are the unique representative patterns in the sparsified moment matrix **Q** in (8). These patterns can be attributed to the symmetry of the moment matrix **A** and the periodic structure of the operator **F** (circulant-like structure). Accordingly, only a unique subset of the repeated elements in **Q** is selected to represent the whole matrix. If Daubechies family of order 4 is adopted and the moment matrix size (M) is 32, an input vector of only 17 elements is constructed. Four elements from the first three levels of details in the matrix **Q** constitute the first 12 elements of the vector. Moreover, four elements from the summary level are added to this vector. The 17th element is the incident wave on the scatterer. All elements of **Q** are complex numbers. The magnitudes of the selected elements are used as inputs to the network. As a result, the dimension of the input vector to the network is

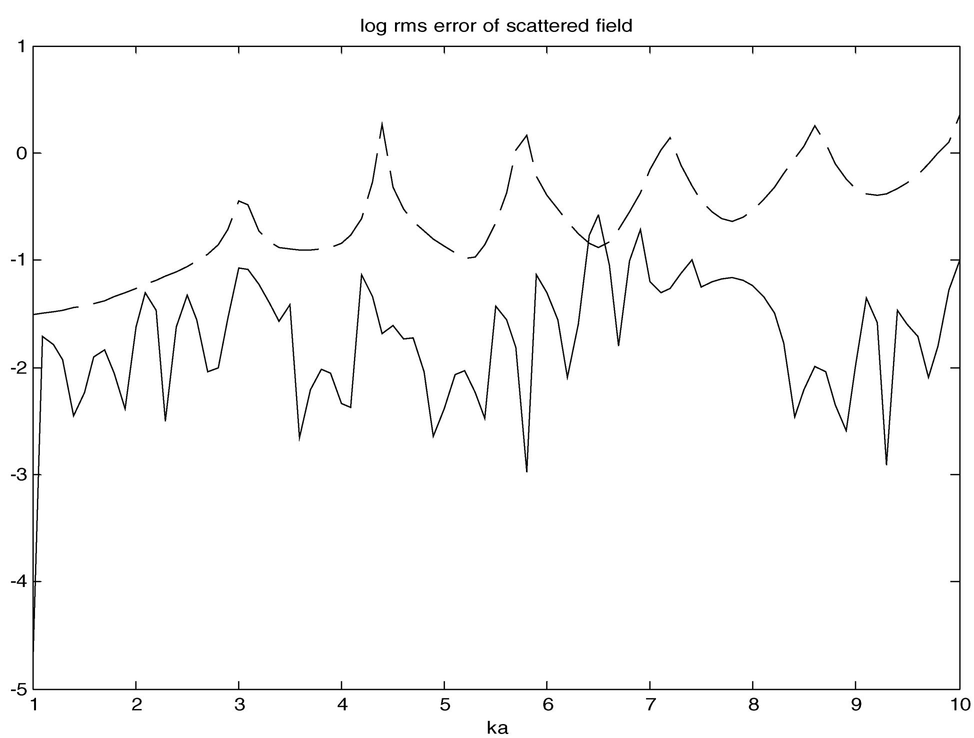

FIG. 1. Log RMS error between the analytical solution with both NN-based solution and tradition numerical solution versus ka (-) solid line represents the error with NN-based solution and (- -) dashed line represents the error with numerical solution.

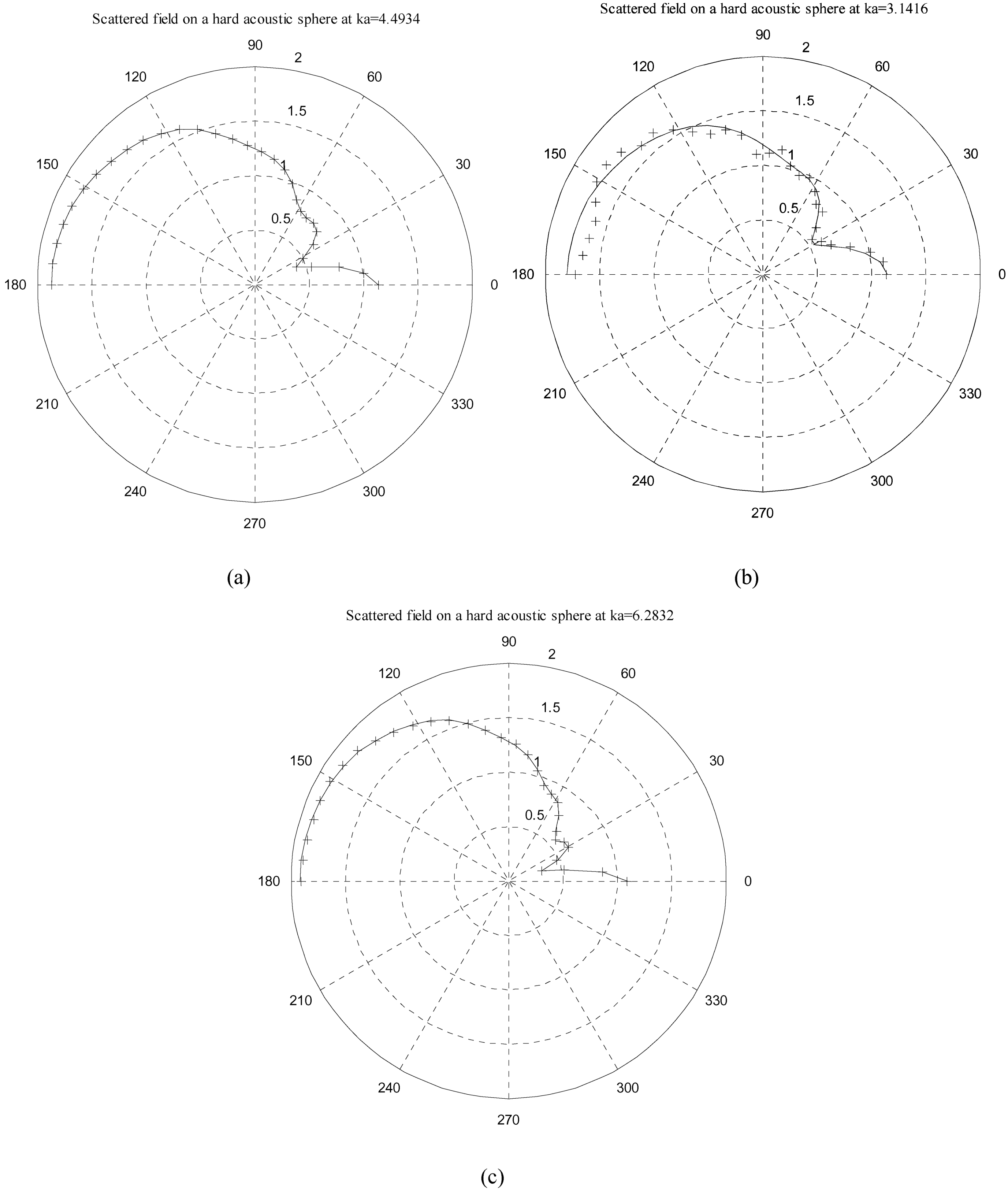

FIG. 2. A comparison between the analytical solution and NN-based solution. (-) solid line represents the analytical solution and (+) the NN-based solution.

reduced from the order of M^2 (i.e., $O(M^2)$) to $O(l_w^2)$ where MxM is the moment matrix size and l_w is the length of the wavelet filter.

To this end, for every specific wavenumber the 17 elements input vector is constructed, which is referred to as an input pattern. Each input pattern is paired with its corresponding output pattern, which is the magnitude of the analytical solution of the scattered field ϕ. A training set of several patterns are then collected for various wavenumbers within the noncharacteristic range of the scattering problem.

Before feeding the training set to the NN, all input patterns should be normalized. Data normalization is required to avoid large changes in the magnitude of the elements of the training patterns that may differ by several orders. These variations could represent an undesired domination, and should be removed through a normalization process. The zero mean and unit standard deviation normalization is adopted. Accordingly, undesired large changes in the element magnitude are avoided, and differences between input patterns are amplified. This allows fast convergence of the training phase.

The NN model used, in this research, is the back-propagation NN [18] with one hidden layer of hyperbolic tangent activation function.

4. EXPERIMENTAL RESULTS

The scattering surface of an acoustically hard sphere is considered. The incident plane wave travels toward the scatterer along the negative direction of z-axis. Accordingly, the exact form of the scattered field on the surface of this acoustic hard sphere is given by

$$\phi = \frac{i}{(ka)^2} \sum_{n=0}^{\infty} (-i)^n (2n+1) \frac{P_n(\cos \vartheta)}{h_n^{(1)'}(ka)} \tag{9}$$

where ϕ is the total field on the surface of an acoustically hard sphere of radius a and ϑ is the co-latitude angle. The incidence angle is taken to be π (in the direction of the negative z axis) in this application. P_n is the Legendre polynomial of order n, and h_n is the spherical hankel function [19].

Daubechies family of order 4 is used and the moment matrix size (M) is 32. The training set with input-output patterns is constructed as described in section 3.

The back-propagation NN topology is as follows: an input layer of 17 neurons (nodes), one hidden layer of 17 nodes with hyperbolic tangent activation function and an output layer of 32 nodes of linear activation function. The weights of the NN are initially randomized in the range of $[-1, 1]$. The learning rule is steepest descent with moment term and the objective function, to be minimized, is the mean square error. The accuracy is chosen to be 10^{-6} and the network converges within about 40 iterations.

Tests are conducted at different characteristic wavenumbers. The inputs to the NN are calculated at such wavenumbers; the corresponding output is denoted by ϕ_{nn}. These outputs are compared with the actual output ϕ_{ana} , which is obtained by (9). The employed simulator is the neural-network toolbox of MATLAB software [20].

Tests are also conducted at some wavenumbers at which nonuniqueness is encountered using traditional methods. The generalization ability of the trained NN is demonstrated in Fig. 1. The outputs of the NN (ϕ_{nn}) are compared with the exact field forms (ϕ_{ana}), which are obtained by (9). The root-mean square error (RMSE) is taken as a performance measure, which is defined as

$$RMSE_{nn} = \|\phi_{nn} - \phi_{ana}\| / \|\phi_{ana}\| \tag{10}$$

and

$$RMSE_{num} = \|\phi_{num} - \phi_{ana}\| / \|\phi_{ana}\| \tag{11}$$

where $RMSE_{nn}$ is the error incurred from NN outputs ϕ_{nn} while $RMSE_{num}$ is the error when the outputs of traditional numerical techniques ϕ_{num} is compared to the analytical solution ϕ_{ana}. Figure 1 shows the variation of log(RMSE) for different wavenumbers ranging from $ka = 1 \rightarrow 10$ with a step 0.35. The error is large at the characteristic wavenumbers if the conventional methods are used since it approaches the limit of zero log. These errors are suppressed using an NN, which is trained at noncharacteristic wavenumbers.

Figure 2 shows the analytical solution (ϕ_{ana}), and the corresponding NN-based solution for some wavenumbers at which solution nonuniqueness occurs. Specifically, $ka = 4.4934$ is considered since it is a characteristic wavenumber for both the Helmholtz integral equation and its normal derivative [5]. Also, the numbers $ka = \pi$ and 2π are tested. All results show accurate match between the NN-based solution and the exact solution while the traditional numerical methods fail at these characteristic wavenumbers.

5. CONCLUSIONS

The nonuniqueness problem arises in solving the integral equation formulation of acoustic scattering by a closed object requires an excessive computation through the solution of an overdetermined system of equations. In this research, this problem is tackled in a NN context. In order to avoid the excessive cost of treating the overdetermined system of equations, a simplified procedure is introduced. In particular, the wavelet transformation properties are used to sparsify the moment matrix. The sparsified matrix has some repeating patterns. These patterns are used to construct representative patterns of smaller size. These patterns are coupled with the analytical solution of the corresponding problem to constitute a training set for the NN. This set is, then, used to train a backpropagation NN to predict the solution of an acoustic scattering problem. The tests show an appreciated success of the network to predict the solution at the characteristic wavenumbers at which nonuniqueness is encountered using traditional methods. Consequently, the proposed approach ensures accurate and fast solution for the acoustic scattering problem, including the nonuniqueness cases.

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