International Journal of Fracture **92:** 201–212, 1998.

The dual boundary contour method for two-dimensional crack problems

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Received 15 July 1998; accepted in revised form 19 January 1999

Abstract. This paper concerns the dual boundary contour method for solving two-dimensional crack problems. The formulation of the dual boundary contour method is presented. The crack surface is modeled by using continuous quadratic boundary elements. The traction boundary contour equation is applied for traction nodes on one of the crack surfaces and the displacement boundary contour equation is applied for displacement nodes on the opposite crack surface and noncrack boundaries. The direct calculation of the singular integrals arising in displacement BIEs is addressed. These singular integrals are accurately evaluated with potential functions. The singularity subtraction technique for determining the stress intensity factor K_{I}, K_{II} and the T-term are developed for mixed mode conditions. Some two-dimensional examples are presented and numerical results obtained by this approach are in very good agreement with the results of the previous papers.

Key words: Dual boundary contour method, boundary contour method, crack, stress intensity factor, boundary element method.

1. Introduction

The study of the events of catastrophic fracture failures in engineering structures has become increasingly important. Linear elastic fracture mechanics can be used for determining the damage tolerance of cracked structures. The stress intensity factors play a fundamental role in this work. The boundary element method (BEM) is a powerful numerical technique for computing the stress intensity factors. However, the BEM cannot be directly used in analyzing general crack problems because the coincidence of two crack surfaces gives rise to a singular system of equations. Some special techniques have been devised. The most important and efficient is the dual boundary element method (DBEM) (Hong and Chen, 1988).

In the dual boundary element method the problem of the singularity in the system of algebraic equations is overcome by using two different equations for collocation points on two crack surfaces. The displacement equation is applied to points on one crack surface and all noncrack boundaries, and the traction equation is applied to points on the other. As the result, crack problems can be solved in a single-region boundary element method. This method is developed for two-dimensional crack problems by Portela et al. (1992a) and for three-dimensional cases by Gray et al. (1990). It is also applied to dynamic load cases, thermoelastic crack problems, multiple crack and closed crack analyses and so on (Fedelinski et al., 1994; Prasad et al., 1994; Chen and Chen, 1995; Tuhkuri, 1997). However, in the dual boundary element method cracks are only modeled by using discontinuous boundary elements, since the numerical treatments of the singular integrals in the traction equation are difficult. Thus, this approach increases computing time. Moreover, the above works are based on the idea of the conventional boundary element method.

The conventional boundary element method requires numerical evaluation of line integrals for two-dimensional problems and surface integrals for three-dimensional cases. The displacement BIE contains strongly singular integrals and the traction BIE contains hypersingular integrals. The evaluation of these singular integrals is more difficult (Tanaka et al., 1994).

Recently, a novel variant of the boundary element method, called the boundary contour method (BCM), is proposed by Nagarajan et al. (1994; 1996). This method requires only numerical evaluation of potential functions at the ends of boundary elements for two-dimensional problems and the numerical evaluation of line integrals on the bounding contours of boundary elements for three-dimensional problems. The boundary contour methods with quadratic boundary elements are presented for two-dimensional problems by Phan et al. (1997) and for three-dimensional cases by Mukherjee et al. (1997). The boundary contour method for evaluating stresses is the subject of Zhou et al. (1997). The boundary contour method based on the equivalent boundary integral equation is presented for two-dimensional problems by Zhou et al. (1999b). They also propose the traction boundary contour method for linear elasticity (Zhou et al., 1999a). The hypersingular boundary contour method is presented by Phan et al. and Mukherjee et al. (1998).

In this paper the formulations of the dual boundary contour method (DBCM) are presented for solving two-dimensional crack problems. An alternative crack modeling strategy is given. The present method uses continuous quadratic boundary elements to model crack surfaces. The collocation of the displacement and the traction boundary contour equations is addressed. The evaluations of the singular integrals arising in boundary integral equations are carried out with potential functions when the source point is on the crack surface. The singularity subtraction technique (SST) for determining the K-values and T-term is proposed for mixed mode cracks. Finally, some examples are considered in order to demonstrate the accuracy of the present method.

2. The dual boundary contour method

2.1. Boundary integral equations

The present work starts from the standard boundary integral equations. The displacement BIE without body forces can be written in the form of the vector as

$$u_k(p) = \int_{\partial B} \left[U_{ik}(p,q)\sigma_{ij}(q) - \Sigma_{ijk}(p,q)u_i(q)\right] \mathbf{e}_j \cdot \mathrm{d}\mathbf{S}, \tag{1}$$

where p is the source point, q is the field point, σ_{ij} and u_i are stress tensor and displacement vector, respectively, $\mathbf{e}_j$ is a global Cartesian unit vector, d$\mathbf{S}$ is an infinitesimal boundary length vector, and U_{ik} and Σ_{ijk} are the fundamental solutions for displacements and stresses, respectively. The fundamental solutions for the plane strain cases are

$$U_{ik}(p,q) = \frac{-1}{8\pi(1-\nu)\mu}\{3-4\nu)\delta_{ik}\ln r + r_{,i}r_{,k}\}, \tag{2}$$

$$\Sigma_{ijk} = \frac{-1}{4\pi(1-\nu)r}[(1-2\nu)(\delta_{ik}r_{,j} + \delta_{jk}r_{,j} - \delta_{ij}r_{,k}) + 2r_{,i}r_{,j}r_{,k}], \tag{3}$$

where r is the distance between p and q, μ is the shear modulus, ν is the Poisson' s ratio, and δ_{ik} is the Kronecker delta.

By taking partial derivatives of (1) with respect to the source point p and using Hooke's law, the Somigliana stress identity can be formed as (Zhou et al., 1999a)

$$\sigma_{ij}(p) = \int_{\partial B} \left[D_{ijk}(p,q)\sigma_{k\beta}(q) - \boldsymbol{C}_{jkml}\Sigma_{ijk,L}(p,q)u_k(q) \right] \mathbf{e}_\beta \cdot \mathrm{d}\mathbf{S}, \tag{4}$$

where $\boldsymbol{C}_{jkml}$ is the elasticity tensor, a comma with the capital indicates partial derivatives at the source point. In the above equation, the integral kernel D_{ijk} is for plane strain problems as follows

$$D_{ijk} = \boldsymbol{C}_{jkml} U_{im,L} \tag{5}$$

Equations (1) and (4) have strongly singular and hypersingular integrals, respectively, as q approaches p. In traditional BEM, the free term needs to be separated off, and then CPV and HFP integrals are evaluated. However, evaluation of the singular integral of (1) can be avoided using rigid body translation (Nagarajan et al., 1994; Zhou et al., 1997) and (4) can be converted into a regular form for regular boundary points, which do not locate at the ends of boundary elements and corners in BCM (Zhou et al., 1999a). So here, (1) and (4) are expressed as the form without free terms.

By rewriting the vector in the integrand of (1) and (4) as $\mathbf{F}_k$ and $\mathbf{T}_{ij}$, respectively, these equations can be expressed as

$$u_k(p) = \int_{\partial B} \mathbf{F}_k \cdot \mathrm{d}\mathbf{S}, \tag{6}$$

$$\sigma_{ij}(p) = \int_{\partial B} \mathbf{T}_{ij} \cdot \mathrm{d}\mathbf{S}. \tag{7}$$

The traction boundary integral equation for the regular boundary points on the smooth boundary has the form

$$t_i = n_j(p) \int_{\partial B} \mathbf{T}_{ij} \cdot \mathrm{d}\mathbf{S}, \tag{8}$$

where n_j is a component of the outward normal of the surface ∂B at the source point. Equations (6) and (8) constitute the basis of the dual boundary contour method.

2.2. Formulations of the dual boundary contour method

When the displacement BIE and the traction BIE are transformed into the form of (6) and (8), the divergence of these vectors at the field point q is zero, that is

$$\nabla_q \cdot \mathbf{F}_k = 0, \tag{9}$$

$$\nabla_q \cdot \mathbf{T}_{ij} = 0, \tag{10}$$

everywhere except the source point, provided that u_i and σ_{ij} correspond to a vanishing body force elastostatic state with the same elastic constants as the fundamental solution, as shown in Nagarajan et al. (1994) and Zhou et al. (1999a).

For two-dimensional problems, the line integrals of the vectors with divergence free can be converted into the calculation of potential functions at the boundary element ends. When the whole boundary is discretized into **N** elements, Equations (6–8) are converted into the following form

$$u_k(p) = \sum_{n=1}^{N} \int_{\partial B_n} \mathbf{F}_k \cdot \mathrm{d}\mathbf{S} = \sum_{n=1}^{N} \{\Phi_k^n(Q_{n2}) - \Phi_k^n(Q_{n1})\}, \tag{11}$$

$$\sigma_{ij}(p) = \sum_{n=1}^{N} \int_{\partial B_n} \mathbf{T}_{ij} \cdot \mathrm{d}\mathbf{S} = \sum_{n=1}^{N} \{G_{ij}^n(Q_{n2}) - G_{ij}^n(Q_{n1})\}, \tag{12}$$

$$t_i = n_j(p) \sum_{n=1}^{N} \int_{\partial B_n} \mathbf{T}_{ij} \cdot \mathrm{d}\mathbf{S} = n_j(p) \sum_{n=1}^{N} \{G_{ij}^n(Q_{n2}) - G_{ij}^n(Q_{n1})\}, \tag{13}$$

in which Q_{n1} and Q_{n2} are the ends of the nth boundary element.

Since the line integrals on boundary elements are only dependent on potential function values at the ends of boundary elements, (11–13) are available for the regular boundary points. In BCM, the displacement shape functions and the stress shape functions must satisfy the equations of elasticity in order for the property (9–10) to be valid. As the element displacement shape functions and stress shape functions are chosen, the potential functions Φ_k and G_{ij} can be determined from the following equation

$$\mathbf{F}_k = F_{kj}\mathbf{e}_j = \frac{\partial \Phi_k}{\partial y}\mathbf{e}_1 - \frac{\partial \Phi_k}{\partial x}\mathbf{e}_2, \tag{14}$$

$$\mathbf{T}_{ij} = T_{ij\beta}\mathbf{e}_\beta = \frac{\partial G_{ij}}{\partial y}\mathbf{e}_1 - \frac{\partial G_{ij}}{\partial x}\mathbf{e}_2. \tag{15}$$

The element displacement shape functions for quadratic elements, as shown in Figure 1, in interior domain problems can be expressed with ten shape functions in the global coordinate (x, y). In the coordinate (ξ, η) centered at each source point, in which the ξ and η axes are parallel to the global x- and y-axes, the displacement shape functions are transformed into the form (Phan et al. 1997)

$$\begin{aligned} \begin{Bmatrix} u_1(p) \\ u_2(p) \end{Bmatrix} &= \begin{bmatrix} 1 & \xi & \eta & 0 & 0 & 0 & \xi^2 & \eta^2 & k_1\xi\eta & k_2\xi\eta \\ 0 & 0 & 0 & 1 & \xi & \eta & k_2\xi\eta & k_1\xi\eta & \xi^2 & \eta^2 \end{bmatrix} \{\widehat{\alpha}\}^n \\ &= [U(\xi,\eta)][B_m][T^n(x,y)]^{-1}\{b\}^n, \end{aligned} \tag{16}$$

in which $k_1 = -2(1-2\nu)$, $k_2 = -4(1-\nu)$, $\{\widehat{a}\}^n$ is a constant matrix, $[B_m]$ is the transformation matrix that depends only on the global coordinates of the mth source point, $[T^n(x, y)]$ is the matrix expressing physical variables with the artificial variables, $\{b\}^n$ is composed of 10 physical variables which are 6-node displacement components at 3 displacement nodes and 4-node traction components at 2 traction nodes. The stress shape functions can be gotten substituting (16) into Hooke's law.

The shape function (16) can be considered to be the linear combination of ten basic shape functions. For each one of these shape functions, let f_{ik} and t_{ijk} stand for the vector F_i and T_{ij}, and φ_{ij} and g_{ijk} denote the potential function Φ_k and $\mathbf{G}_{ij}$, respectively. These potential

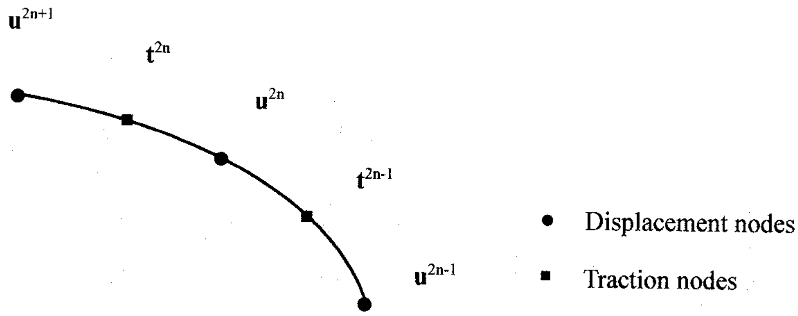

Figure 1. Quadratic boundary elements.

functions have been solved in Phan et al. (1994) and Zhou et al. (1999a). Thus, (11–13) can be expressed in the following form

$$\begin{aligned} u_k(p) &= \sum_{n=1}^{N} \int_{\partial B_n} \mathbf{f}_{k\beta}\widehat{a}_\beta \cdot \mathrm{d}\mathbf{S} = \sum_{n=1}^{N} \Delta\varphi_{k\beta}\widehat{a}_\beta \\ &= \sum_{n=1}^{N} [\Delta\varphi_{k1}, \Delta\varphi_{k2}, \ldots, \Delta\varphi_{k10}][B_m][T^n(x,y)]^{-1}\{b\}^n, \end{aligned} \tag{17}$$

$$\begin{aligned} \sigma_{ij}(p) &= \sum_{n=1}^{N} \int_{\partial B_n} \mathbf{t}_{ij\beta}\widehat{a}_\beta \cdot \mathrm{d}\mathbf{S} = \sum_{n=1}^{N} \Delta g_{ij\beta}\widehat{a}_\beta \\ &= \sum_{n=1}^{N} [\Delta g_{ij1}, \Delta g_{ij2}, \ldots, \Delta g_{ij10}][B_m][T^n(x,y)]^{-1}\{b\}^n, \end{aligned} \tag{18}$$

$$t_i = n_j(p) \sum_{n=1}^{N} [\Delta g_{ij1}, \Delta g_{ij2}, \ldots, \Delta g_{ij10}][B_m][T^n(x,y)]^{-1}\{b\}^n, \tag{19}$$

where $\widehat{a}_\beta$ is the βth component of the constant matrix and

$$\Delta\varphi_{k\beta} = \varphi_{k\beta}(Q_{n2}) - \varphi_{k\beta}(Q_{n1}), \tag{20}$$

$$\Delta g_{ij\beta} = g_{ij\beta}(Q_{n2}) - g_{ij\beta}(Q_{n1}). \tag{21}$$

Both the displacement and traction boundary contour method formulations can be independently used for analyzing problems (Nagarajan et al., 1994; 1996; Phan et al., 1997; Mukherjee et al., 1997; Zhou et al. 1997; 1999a). However, a mixed formulation can also be formed.

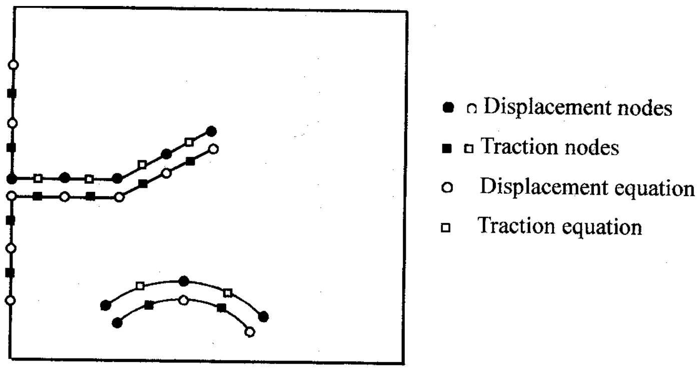

Figure 2. Crack modeling with continuous quadratic boundary elements.

3. Crack modeling and singular integral evaluation

3.1. Crack modeling

In the conventional dual BEM, discontinuous elements are used to model the crack surface in order to satisfy the continuous requirement of the Hadamard finite part integrals and the requirement of the smooth of geometry at collocation point in the traction BIE. The size of the system of equations generated by the mesh of discontinuous elements is larger than continuous elements. In the dual boundary contour method, the traction nodes are always the internal points of elements, as shown in Figure 1. It is ensured that all traction nodes are on the smooth boundary and the stresses at traction nodes are continuous by properly collocating elements. Therefore, continuous elements can be used to model crack surfaces, which are the same as on noncrack boundaries.

Here, the general modeling crack strategy, shown in Figure 2, is developed as follows:

- the crack surfaces are disrectized into the continuous quadratic elements that are the same as on noncrack boundaries;
- the displacement Equation (17) is applied for displacement nodes on one of the crack surfaces, crack tip for edge cracks (one of the crack tips for center cracks) and all noncrack boundaries; and
- the traction Equation (19) is applied for traction nodes on the other crack surfaces.

3.2. Evaluation of singular integrals

Since the middle point of the element is the regular boundary point, the displacement Equation (17) holds for these displacement nodes when the equation is applied on both noncrack boundaries and the locus of a crack. However, the potential functions of either φ_{k1} and φ_{k4} corresponding to the integrals of the vectors f_{k1} and f_{k4} are singular as the field point q approaches the source point p at one of the end nodes. When the source point p is not on the locus of a crack, evaluation of singular integrals or potential function singular values in (17) are avoided by using rigid body translation, as shown in paper (Nagarajan et al., 1994; Zhou et al., 1997).

The case where p is on the locus of a crack is somewhat more complicated, because the rigid body motion technique is not valid due to the presence of two surfaces. Without loss of generality let us consider the case where p is located at Q_2 point, as shown in Figure 3.

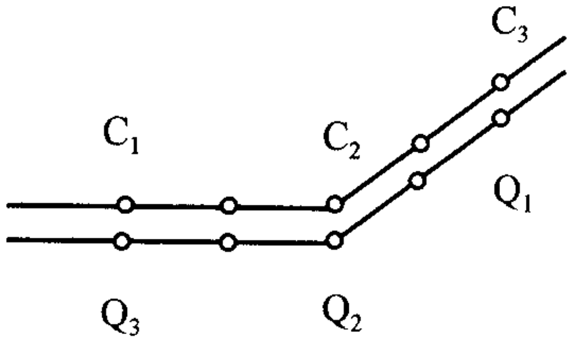

Figure 3. Source points on the crack surface.

In this case, the integrals of f_{k1} and f_{k4} over the right-element and left-element of Q_2 can be evaluated together, because the integrals of f_{k1} and f_{k4} over these two elements are only relation to the matrix coefficients of boundary variables $u_i(Q_2)$ in the system of equations (Zhou et al., 1997). Thus, these integrals can be calculated by the following equations

$$\int_{Q_1}^{Q_3} \mathbf{f}_{11} \cdot \mathrm{d}\mathbf{S} = \varphi_{11}(Q_3) - \varphi_{11}(Q_1), \tag{22}$$

$$\int_{Q_1}^{Q_3} \mathbf{f}_{21} \cdot \mathrm{d}\mathbf{S} = \varphi_{21}(Q_3) - \varphi_{21}(Q_1), \tag{23}$$

$$\int_{Q_1}^{Q_3} \mathbf{f}_{14} \cdot \mathrm{d}\mathbf{S} = \varphi_{14}(Q_3) - \varphi_{14}(Q_1), \tag{24}$$

$$\int_{Q_1}^{Q_3} \mathbf{f}_{24} \cdot \mathrm{d}\mathbf{S} = \varphi_{24}(Q_3) - \varphi_{24}(Q_1). \tag{25}$$

For the elements opposite to the lower crack surface, there is existence of the similar results. As a matter of fact, these analyses are available for the ends on noncrack-boundaries.

For the traction Equation (19), the source point is always located at the regular boundary point because traction nodes are the internal points of boundary elements. So (19) can be used for all traction nodes.

As the collocation point passes through all the nodal points, the DBCM transforms the boundary integral Equations (6) and (8) into a system of linear algebraic equations. Reordering the system of equations in accordance with the boundary conditions forms the following equation

$$[A]\{x\} = [B]\{y\}. \tag{26}$$

Here $\{x\}$ are unknown boundary variables and $\{y\}$ are known boundary variables.

4. Singularity subtraction technique

In order to avoid the numerical difficulties that arise from the presence of a singularity in the stress field, the singularity subtraction technique (SST) is used in DBEM (Portela et al., 1992b). As this technique regularize the singular field by means of a particular solution of the original problem, the boundary conditions are modified. The boundary conditions for the regularized problem become

$$t_i^R = t_i - t_i^S \tag{27}$$

on the traction boundary, and

$$u_i^R = u_i - u_i^S \tag{28}$$

on the displacement boundary. Here t_i and u_i are the boundary conditions of the original problem, t_i^S and u_i^S represent the components of a particular solution. When the boundary conditions (27) and (28) are introduced, Equation (26) can be written as

$$[A \quad C]\begin{Bmatrix} x^R \\ z \end{Bmatrix} = \{f\}. \tag{29}$$

In above equation $[C] = [B][E]$, $[E]\{z\}$ represents the components of the singular field, $\{z\}$ contains additional unknowns, and $f = [B]\{y\}$. When the stress intensity factors K_{I}, K_{II} are only included, $\{z\}$ contains two additional unknowns K_{I} and K_{II}. In order to get a unique solution of the regularized problem, the following extra conditions is applied at the crack tip

$$t_i^R = 0. \tag{30}$$

Recently studying works show that the T-term has a significant effect on the fracture process (Leevers and Radon, 1982; Eischen, 1987). The fracture process is governed by two parameters, that is, the K-values and the T-stress at the crack tip. However, very few articles are concerned with the determination of the T-term, especially with the BEM. Here, the singularity subtraction technique is developed for determining the K-values and the T-term. When the stress intensity factors K_{I}, K_{II} and the T-term are considered, the extra condition (30) is not enough because $\{z\}$ contains three additional unknowns. In order to get a unique solution, the following additional constraint condition is introduced

$$\sigma_{11}^R = 0, \tag{31}$$

where σ_{11} is the stress parallel to the crack flanks.

The condition (30) and (31) can be written as

$$[A_p \quad C_p]\begin{Bmatrix} x^R \\ z \end{Bmatrix} = \{f_p\}. \tag{32}$$

The additional Equations (32) are now coupled with the system of (29) to give final equation

$$\begin{bmatrix} A & C \\ A_p & C_p \end{bmatrix}\begin{Bmatrix} x^R \\ z \end{Bmatrix} = \begin{Bmatrix} f \\ f_p \end{Bmatrix}. \tag{33}$$

Equation (32) should be exactly located at the crack tip. In order to satisfy the collocation at the crack tip, a quadratic extrapolation procedure can be used (Portela et al., 1992b). A

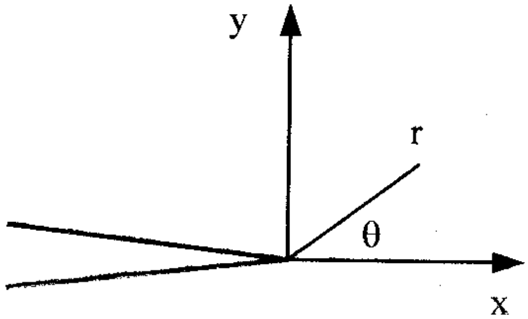

Figure 4. Coordinate reference system for edge cracks.

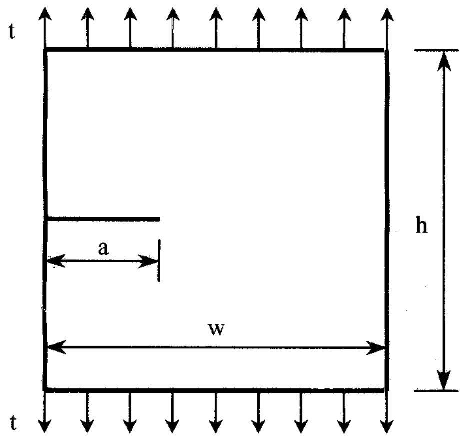

Figure 5. Square plate with a single edge crack.

particular solution representing the singular field around crack tips can be used for the SST analysis of crack problems. For the single edge cracks, a singular field is assumed to follow the first-order expansion series, see William (1957). In the coordinate reference system, as shown in Figure 4, the stress components are given by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \cos \tfrac{1}{2}\theta \begin{Bmatrix} 1 - \sin \frac{1}{2}\theta \sin \frac{1}{2}3\theta \\ 1 + \sin \frac{1}{2}\theta \sin \frac{1}{2}3\theta \\ \sin \frac{1}{2}\theta \cos \frac{1}{2}3\theta \end{Bmatrix} + \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin \frac{1}{2}\theta (2 + \cos \frac{1}{2}\theta \cos \frac{1}{2}3\theta) \\ \sin \frac{1}{2}\theta \cos \frac{1}{2}\theta \cos \frac{1}{2}3\theta \\ \cos \frac{1}{2}\theta (1 - \sin \frac{1}{2}\theta \sin \frac{1}{2}3\theta) \end{Bmatrix} + \begin{Bmatrix} T \\ 0 \\ 0 \end{Bmatrix}. \tag{34}$$

The displacement components are

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \frac{K_{\mathrm{I}}}{4\mu} \sqrt{\left(\frac{r}{2\pi}\right)} \begin{Bmatrix} (2k-1) \cos \frac{1}{2}\theta - \cos \frac{1}{2}3\theta \\ (2k+1) \sin \frac{1}{2}\theta - \sin \frac{1}{2}3\theta \end{Bmatrix} + \frac{K_{\mathrm{II}}}{4\mu} \sqrt{\left(\frac{r}{2\pi}\right)} \begin{Bmatrix} (2k-3) \sin \frac{1}{2}\theta - \sin \frac{1}{2}3\theta \\ (2k+3) \cos \frac{1}{2}\theta - \cos \frac{1}{2}3\theta \end{Bmatrix}, \tag{35}$$

Table 1. K_I and T-values for a single edge crack in a square plate ($h/w = 1$)

	$K = K_I/(t\sqrt{\pi a})$; $B = T\sqrt{\pi a}/K_I$							
	This paper		Paper (Portela et al., 1992)		Paper (Civelek and Erdogan, 1982)		Paper (Luvers and Radon, 1982)	
a/w	K	B	K	B	K	B	K	B
0.2	1.478	−0.361	1.483	–	1.488	–	–	−0.353
0.3	1.835	−0.168	1.841	–	1.848	–	–	−0.163
0.4	2.306	0.045	2.309	–	2.324	–	–	0.047
0.5	2.989	0.227	2.980	–	3.010	–	–	0.222
0.6	4.136	0.340	4.092	–	4.152	–	–	0.331

in which K_I and K_{II} are the stress intensity factors of the opening and sliding modes, respectively, k is equal to $(3 - 4\nu)$ for plain strain and $(3 - \nu)/(1 + \nu)$ for plain stress, and T is the T-stress.

5. Numerical results

5.1. Single edge crack

Consider a square plate with a single edge crack normal to the side of the vertical boundary, as shown in Figure 5. The crack length is denoted by a and the width of the plate is denoted by w. The plate is subjected to the action of a uniform tension t, acting in a direction perpendicular to the crack and applying symmetrically at the ends. The K-values of this problem are obtained by the conventional dual boundary element method in the paper (Portela et al., 1992b). Here, the K-values and the T-values are calculated using the present method. Convergence is achieved with 40 quadratic boundary elements, in which the crack is discretized with 6 quadratic continuous elements on each crack surface. The crack discretization is graded towards the tip with the ratios 0.4, 0.3, 0.2 and 0.1. The results compared against those presented in previous papers are shown in Table 1. The K-values obtained by the present method are in very good agreement with those of the paper (Portela et al., 1992b; Civelek and Erdogan, 1982). The T-stress is within an accuracy of 3 percent relating to reference result (Leevers and Radon, 1982).

5.2. Inclined edge crack

This example deals with a rectangular plate subject to uniform tension and penetrated by a crack, with a crack mouth starting at the left-hand side of the vertical boundary and extending upward at an angle of 45°. The width is 1, the height is 2 and the length of the crack is 0.5656, as shown in Figure 6. The material constants: Young's modulus is 1 and Poisson's ratio is 0.2. This problem is solved by using the present method with 40 quadratic boundary elements, in which the crack is discretized into 8 elements on each one crack surface. Numerical results are shown in Table 2. The results obtained with the presented method agree well with those obtained using the displacement regression at the crack tip (Eischen, 1987).

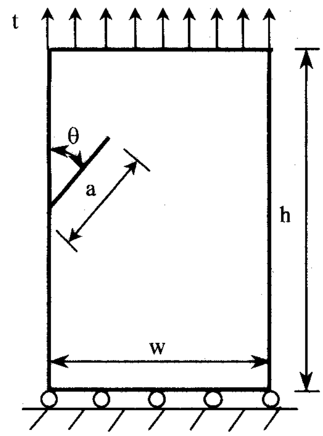

Figure 6. Plate with an inclined edge crack.

Table 2. K-values and T-values for an inclined edge crack

	$K_I/t\sqrt{\pi a}$	$K_{II}/t\sqrt{\pi a}$	T/t
This paper	1.425	0.622	0.770
Paper (Eischen, 1987)	1.437	0.613	0.783

6. Conclusion

In this work, a dual boundary contour method has been developed for two-dimensional crack problems. The present method uses continuous quadratic boundary elements to model the crack surface This approach requires only numerical evaluation of potential functions at the ends of boundary elements. The singular integrals can be evaluated with potential functions accurately. The singularity subtraction technique for the determination of the stress intensity factors K_I, K_{II} and the T-term in mixed mode conditions has been presented. Numerical results for illustrative examples demonstrate the efficiency of this approach for two-dimensional crack problems.

Acknowledgement

The authors are grateful to the Shandong Science Fund Council of People's Republic of China for financial support.

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