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Investigation on the spurious eigenvalues of vibration plates by non-dimensional dynamic influence function method

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ABSTRACT

This paper makes a numerical investigation on the spurious eigenvalues problem by the non-dimensional dynamic influence function (NDIF) method for plate vibration problems. We find that the NDIF spurious eigenvalues of plate vibration problems are often related to the eigenvalues of the corresponding membrane or plate, and vary with boundary conditions in the tested examples. It seems that we cannot simply identify the spurious eigenvalues of a practical problem. But, fortunately, through numerical experiments, we have obtained a few observations of the spurious eigenvalues in dealing with the plate vibration problems, which can be helpful in the practical application of the NDIF method.

1. Introduction

In recent years, the method of fundamental solutions (MFS) [1,2] has attracted a lot of attention in the research community thanks to its simplicity and high accuracy. However, the method has not become a popular numerical method in practical engineering simulations [3] because the method employs the singular fundamental solution and requires a controversial artificial boundary outside the physical domain. Despite hard work of many years, the fictitious boundary is still not easy to determine in a complex-shaped domain case.

In order to remedy the above-mentioned drawbacks, Kang and Lee [4,5] recently proposed a novel boundary-only meshless collocation technique, called the non-dimensional dynamic influence function (NDIF) method. Their numerical experiments show that the method is easy-to-program, efficient and accurate for vibration analysis of membranes and plates. Laura and Bambill [6] also conclude that the NDIF method is simple and attractive to simulate a variety of plate vibration problems.

On the other hand, it has been mathematically proved that the NDIF [7] and the MFS [8,9] yields spurious eigenvalues in the solution of circular plate vibration. However, to our best knowledge, no theoretical proof is available for non-circular plates nowadays. In this study, we will make a numerical investigation on the spurious eigenvalue issue of the NDIF method.

2. NDIF formulation of plate vibrations

The basis function of the NDIF method for plate vibration problems is

$$J_0(\lambda r) + I_0(\lambda r), \tag{1}$$

where $J_0(\lambda r)$ and $I_0(\lambda r)$ are respectively the zero-order Bessel and modified Bessel functions of the first kind, λ and r represent the frequency parameter and the Euclidian distance between the domain knots, respectively. The basis function (1) is actually the zero-order non-singular general solution [10] of the governing equation of the thin vibration plate. Thus, the NDIF method is based on the radial basis function [11], truly meshless [12]. And only the boundary discretization [13,14] is required.

The governing equation for a free flexural vibration of a thin vibration plate can be expressed as

$$\nabla^4 W(r) - \lambda^4 W(r) = 0, \tag{2}$$

where $W(r)$ represents the vibration mode in term of the Euclidian distance r of the domain knots, and λ denotes the frequency parameter, which can be stated as

$$\lambda = \sqrt[4]{\frac{\bar{m}\omega^2}{D}}, \tag{3}$$

where $\bar{m}$ is the surface density, ω represents the nature frequency, D is the flexural rigidity expressed as $D = Eh^3/12(1-\nu^2)$ in

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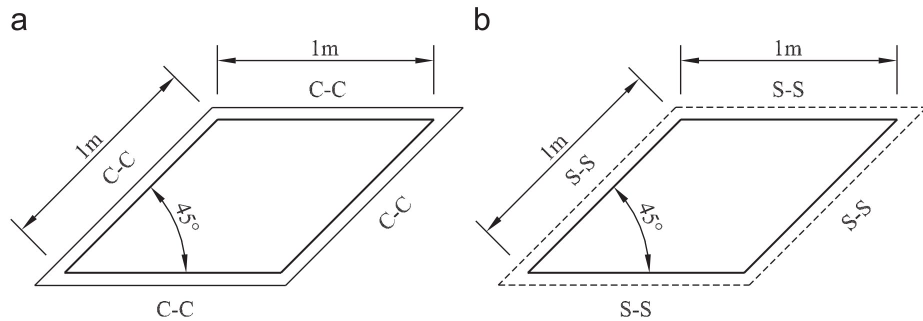

Fig. 1. (a) Geometric configuration of a parallelogram clamped plate; (b) geometric configuration of a parallelogram simply-supported plate.

terms of Young's modulus E, the Poisson ratio v, and the plate thickness h.

Without the loss of generalization, consider the clamped (CC) plate and discretize the boundary into N installments. The NDIF method forces its basis function (1) to satisfy the displacement and slope conditions on the boundary, and then we get the following discretization equation:

$$W(r_i)=\sum_{j=1}^{N}\alpha_j J_0(\lambda r_{ij})+\beta_j I_0(\lambda r_{ij})=0$$
$$\frac{\partial W(r_i)}{\partial n_i}=\sum_{j=1}^{N}\alpha_j\frac{\partial J_0(\lambda r_{ij})}{\partial n_i}+\beta_j\frac{\partial I_0(\lambda r_{ij})}{\partial n_i}=0. \quad (4)$$

Similarly, the basis function (1) is also forced to satisfy the displacement and moment conditions on the boundary, the discretization equation for a simply-supported (SS) plate is stated as

$$W(r_i)=\sum_{j=1}^{N}\alpha_j J_0(\lambda r_{ij})+\beta_j I_0(\lambda r_{ij})=0$$
$$\left(\frac{\partial^2}{\partial n_i^2}+v\frac{\partial^2}{\partial \tau_i^2}\right)W(r_i)=\sum_{j=1}^{N}\alpha_j\left(\frac{\partial^2}{\partial n_i^2}+v\frac{\partial^2}{\partial \tau_i^2}\right)J_0(\lambda r_{ij})$$
$$+\beta_j\left(\frac{\partial^2}{\partial n_i^2}+v\frac{\partial^2}{\partial \tau_i^2}\right)I_0(\lambda r_{ij})=0, \quad (5)$$

where n_i and τ_i represent the outer normal and tangential vector of the ith boundary knot, respectively, α_j and β_j are unknown coefficients, and r_{ij} the distance between the ith and jth boundary knots.

For clarity, Eq. (4) can be written in the form of a matrix

$$\begin{bmatrix} J_0(\lambda r_{11}) & J_0(\lambda r_{12}) & \cdots & J_0(\lambda r_{1N}) & I_0(\lambda r_{11}) & I_0(\lambda r_{12}) & \cdots & I_0(\lambda r_{1N}) \\ J_0(\lambda r_{21}) & J_0(\lambda r_{22}) & \cdots & J_0(\lambda r_{2N}) & I_0(\lambda r_{21}) & I_0(\lambda r_{22}) & \cdots & I_0(\lambda r_{2N}) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ J_0(\lambda r_{N1}) & J_0(\lambda r_{N2}) & \cdots & J_0(\lambda r_{NN}) & I_0(\lambda r_{N1}) & I_0(\lambda r_{N2}) & \cdots & I_0(\lambda r_{NN}) \\ \frac{\partial J_0(\lambda r_{11})}{\partial n_1} & \frac{\partial J_0(\lambda r_{12})}{\partial n_1} & \cdots & \frac{\partial J_0(\lambda r_{1N})}{\partial n_1} & \frac{\partial I_0(\lambda r_{11})}{\partial n_1} & \frac{\partial I_0(\lambda r_{12})}{\partial n_1} & \cdots & \frac{\partial I_0(\lambda r_{1N})}{\partial n_1} \\ \frac{\partial J_0(\lambda r_{21})}{\partial n_2} & \frac{\partial J_0(\lambda r_{22})}{\partial n_2} & \cdots & \frac{\partial J_0(\lambda r_{2N})}{\partial n_2} & \frac{\partial I_0(\lambda r_{21})}{\partial n_2} & \frac{\partial I_0(\lambda r_{22})}{\partial n_2} & \cdots & \frac{\partial I_0(\lambda r_{2N})}{\partial n_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial J_0(\lambda r_{N1})}{\partial n_N} & \frac{\partial J_0(\lambda r_{N2})}{\partial n_N} & \cdots & \frac{\partial J_0(\lambda r_{NN})}{\partial n_N} & \frac{\partial I_0(\lambda r_{N1})}{\partial n_N} & \frac{\partial I_0(\lambda r_{N2})}{\partial n_N} & \cdots & \frac{\partial I_0(\lambda r_{NN})}{\partial n_N} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix} = 0, \quad (6)$$

and Eq. (5) can be rewritten in a similar matrix fashion.

For the existence of non-trivial solution of $[\boldsymbol{\alpha}, \boldsymbol{\beta}]^T$, the determinants of the influence matrix of Eq. (6) must become zero, namely,

$$\begin{vmatrix} J_0(\lambda r_{11}) & J_0(\lambda r_{12}) & \cdots & J_0(\lambda r_{1N}) & I_0(\lambda r_{11}) & I_0(\lambda r_{12}) & \cdots & I_0(\lambda r_{1N}) \\ J_0(\lambda r_{21}) & J_0(\lambda r_{22}) & \cdots & J_0(\lambda r_{2N}) & I_0(\lambda r_{21}) & I_0(\lambda r_{22}) & \cdots & I_0(\lambda r_{2N}) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ J_0(\lambda r_{N1}) & J_0(\lambda r_{N2}) & \cdots & J_0(\lambda r_{NN}) & I_0(\lambda r_{N1}) & I_0(\lambda r_{N2}) & \cdots & I_0(\lambda r_{NN}) \\ \frac{\partial J_0(\lambda r_{11})}{\partial n_1} & \frac{\partial J_0(\lambda r_{12})}{\partial n_1} & \cdots & \frac{\partial J_0(\lambda r_{1N})}{\partial n_1} & \frac{\partial I_0(\lambda r_{11})}{\partial n_1} & \frac{\partial I_0(\lambda r_{12})}{\partial n_1} & \cdots & \frac{\partial I_0(\lambda r_{1N})}{\partial n_1} \\ \frac{\partial J_0(\lambda r_{21})}{\partial n_2} & \frac{\partial J_0(\lambda r_{22})}{\partial n_2} & \cdots & \frac{\partial J_0(\lambda r_{2N})}{\partial n_2} & \frac{\partial I_0(\lambda r_{21})}{\partial n_2} & \frac{\partial I_0(\lambda r_{22})}{\partial n_2} & \cdots & \frac{\partial I_0(\lambda r_{2N})}{\partial n_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial J_0(\lambda r_{N1})}{\partial n_N} & \frac{\partial J_0(\lambda r_{N2})}{\partial n_N} & \cdots & \frac{\partial J_0(\lambda r_{NN})}{\partial n_N} & \frac{\partial I_0(\lambda r_{N1})}{\partial n_N} & \frac{\partial I_0(\lambda r_{N2})}{\partial n_N} & \cdots & \frac{\partial I_0(\lambda r_{NN})}{\partial n_N} \end{vmatrix} = 0. \quad (7)$$

Then, we can obtain the eigenvalues by searching eigenvalue λ in frequency domain. However, the spurious eigenvalues will be encountered in solving certain eigenvalue problems. Kang et al. [4] reduce the size of Eq. (6) matrix from $2N\times 2N$ to $N\times N$, and then multiply it by an inverse matrix, which contains the spurious eigenvalues. Consequently, the spurious eigenvalues can be removed. The problem in this approach is that we have to know prior the types of spurious eigenvalues. In addition, this technique dramatically increases computing cost and worsens condition number of the NDIF influence matrix. Thus, we need to recognize the spurious eigenvalues by numerical experiments, and then consider the application of Kang's approach to remove these spurious eigenvalues.

3. Numerical results and discussion

In this section, we will discuss the the NDIF spurious eigenvalues of plates vibration of different shapes and under various boundary conditions. Figs. 1(a) and (b) are respectively the geometric configurations of a parallelogram clamped plate and a parallelogram simply-supported plate, as illustrated in Fig. 1. Fig. 2

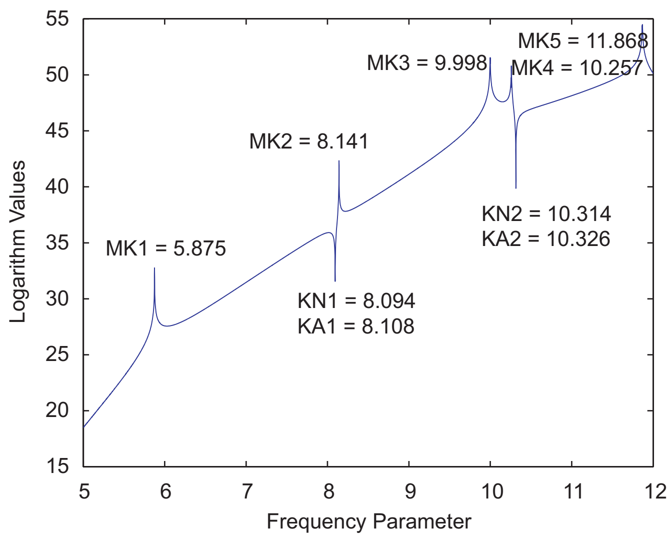

Fig. 2. The eigenvalue curve of a parallelogram clamped plate as shown in Fig. 1(a) ($N = 24$), KK and KA represent the numerical solutions of the NDIF and ANSYS, respectively; MK denotes the spurious eigenvalues, namely, the eigenvalues of corresponding membrane.

is the eigenvalue curve of the frequency parameter of plate shown in Fig. 1(a), where KN represents the eigenvalues of the parallelogram clamped plate calculated by the NDIF method, and KA denotes the results by ANSYS, a commercial finite element method code, and MK is the eigenvalues of the corresponding shaped membrane, namely, the spurious eigenvalues. In parallelogram clamped plate case, the NDIF method yields spurious eigenvalues. Compared with numerical results in Ref. [4], we can conclude that the NDIF method will produce spurious eigenvalues in calculating the eigensolutions of arbitrarily shaped clamped plate, and these spurious eigenvalues are the frequency parameters of the corresponding shaped membrane.

Further numerical experiments show that the spurious eigenvalues calculated by NDIF method will be more perplexing for plates with simply-supported or mixed boundary. The curve shown in Fig. 3 is the NDIF results of a unit simply-supported circular plate, where KN represents the numerical results simulated by NDIF, KE denotes the analytical solutions, and MK presents the spurious eigenvalues. We can also find that the spurious eigenvalues, which are the frequency parameters of the corresponding membrane, are contained in the NDIF numerical results of the circular simply-supported plate in Fig. 3.

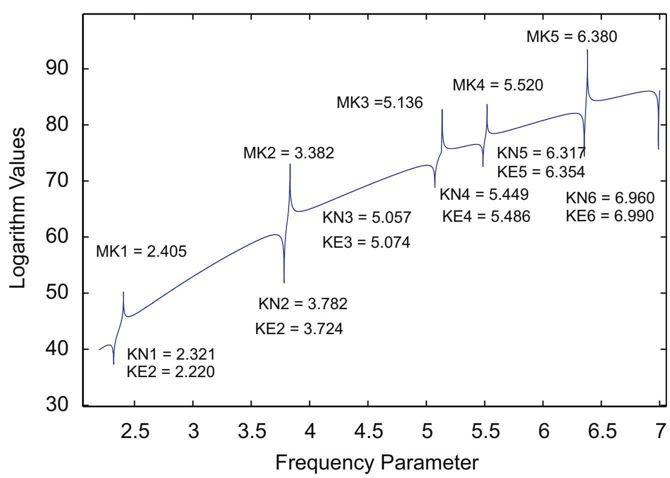

Fig. 3. The eigenvalue curve of a simply-supported circular plate ($N = 20$); KN and KE represent the NDIF numerical solutions and the analytical solution, respectively; MK denotes the spurious eigenvalues, namely, the eigenvalues of corresponding membrane.

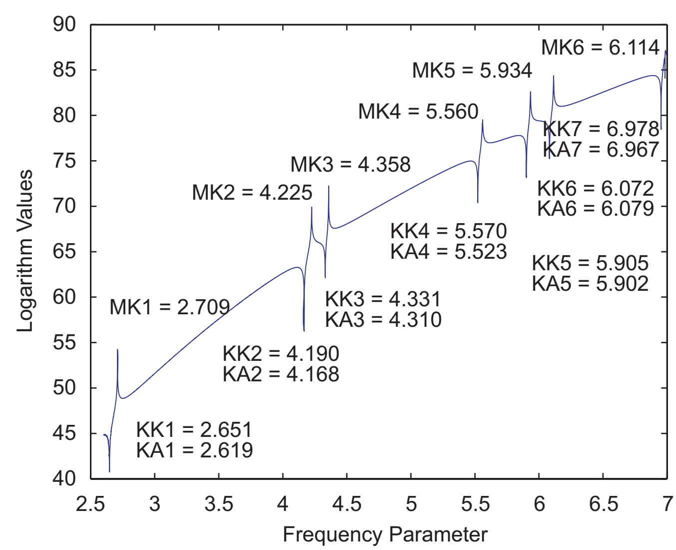

Fig. 5. The eigenvalue curve of a simply-supported plate as shown in Fig. 4(a) ($N = 20$); KK and KA represent the numerical solutions of the NDIF and ANSYS, respectively; MK denotes the spurious eigenvalues, namely, the eigenvalues of corresponding membrane.

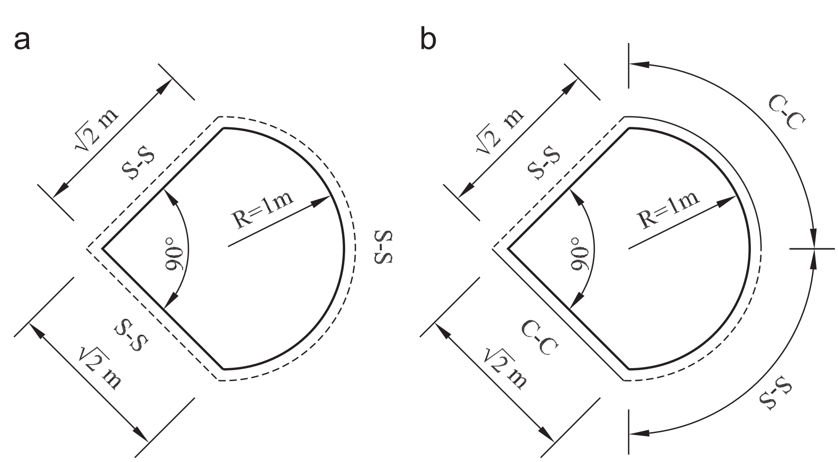

Fig. 4. (a) Geometric configuration of a simply-supported plate; (b) geometric configuration of a SS–CC–SS–CC plate.

We apply the NDIF method to the arbitrarily shaped simply-supported and SS–CC–SS–CC plates as shown in Figs. 4(a) and (b). The NDIF numerical solutions are displayed in Figs. 5 and 6, where MK represents numerical spurious eigenvalues by the NDIF method; meanwhile, KK and KA denote the numerical eigenvalues calculated by the NDIF method and ANSYS, respectively. It is observed that the eigenvalue solutions of the NDIF and ANSYS agree very well. However, the NDIF method encounters the spurious eigenvalues, indicated by MK in Figs. 5 and 6. We note that these spurious solutions are actually eigenvalues of the corresponding fixed membrane. Interestingly, it is also observed that the NDIF solutions of the arbitrarily shaped clamped plate [4] also contain the same spurious eigenvalues. Therefore, we can further improve our above conclusion from our numerical experiments that all spurious eigenvalues of plates including circular-arc-shaped boundary with different boundary conditions are the eigenvalues of the corresponding fixed membrane.

When the NDIF method is applied to simulate the eigenvalues of the simply-supported rectangular or parallelogram plate, the spurious eigenvalues, however, behave differently from the above-mentioned numerical experiments. For instance, Fig. 7 is the eigenvalue curve of a unit simply-supported square plate, and Fig. 8 the curve of the simply-supported plate as shown in Fig. 1(b), where KN presents the numerical solutions by NDIF, KE stands for the analytical solution, and KA the ANSNS solution. It is obvious that the NDIF solutions, compared with the analytical or ANSYS solutions, are highly precise in Figs. 7 and 8. However, we find that these two examples do not encounter the spurious eigenvalues. It seems that there is no spurious eigenvalue for the polygonal plates with simply-supported boundaries.

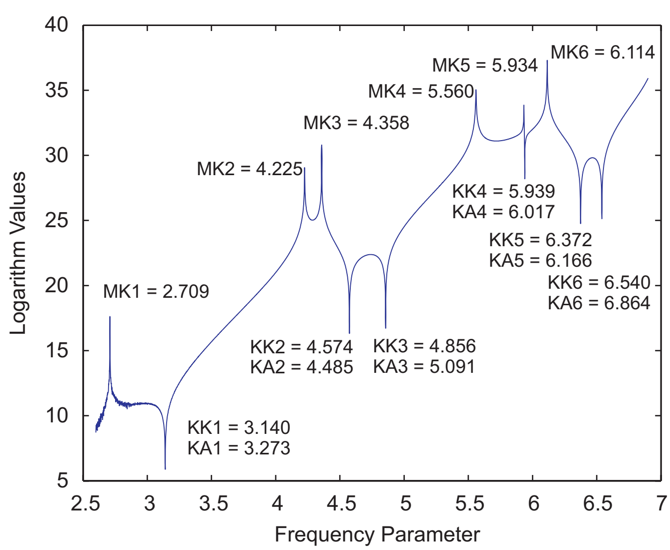

Fig. 6. The eigenvalue curve of a SS–CC–SS–CC plate as shown in Fig. 4(b) (***N*** = 20); KK and KA represent the numerical solutions of the NDIF and ANSYS, respectively; MK denotes the spurious eigenvalues, namely, the eigenvalues of corresponding membrane.

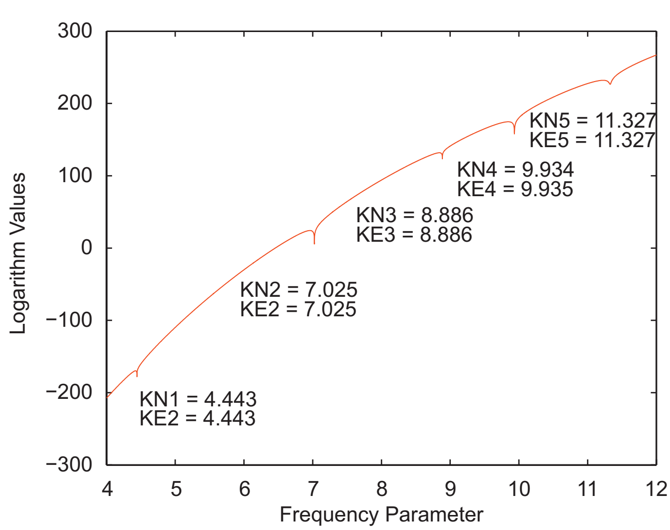

Fig. 7. The eigenvalue curve of a unit SS–SS–SS–SS square plate (total boundary nodes 20); KN and KE represent its numerical and analytical eigenvalues, respectively.

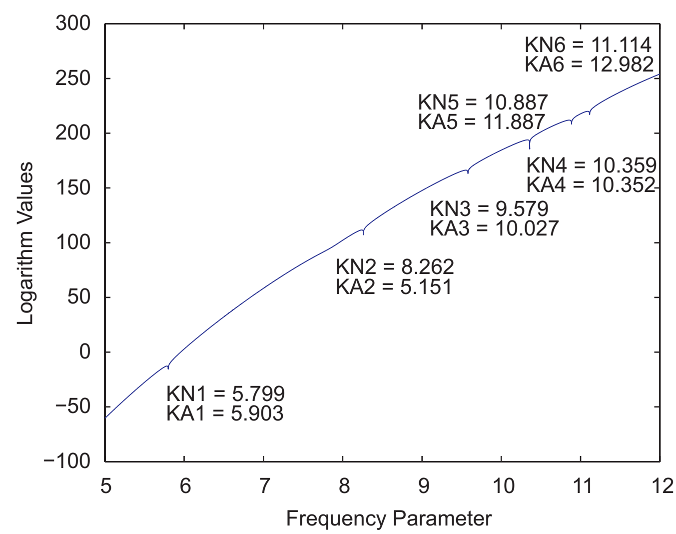

Fig. 8. The eigenvalue curve of a parallelogram clamped plate as shown in Fig. 1(b) (***N*** = 20), KK and KA represent the NDIF and ANSYS numerical solutions, respectively.

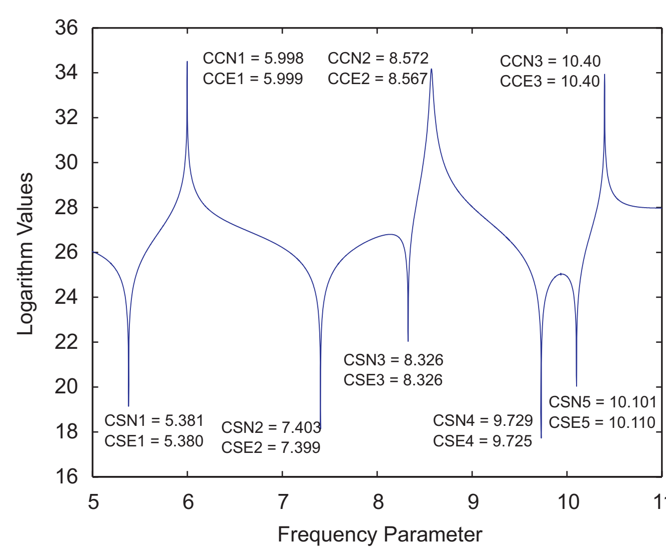

Fig. 9. The eigenvalue curve of a unit CC–SS–CC–SS square plate (total boundary nodes 20), CCN and CCE represent the numerical and analytical eigenvalues of a CC–CC–CC–CC square plate, respectively; CSN and CSE denote the numerical and analytical eigenvalues of a CC–SS–CC–SS square plate, respectively.

Furthermore, the NDIF solution of a square plate subjected to a variety of mixed boundary conditions is also found to encounter spurious eigenvalues. For example, the NDIF solution of a unit SS–CC–SS–CC square plate contains the spurious eigenvalues of the corresponding shaped fully clamped square plate, indicated by CCN as shown in Fig. 9.

Thus, we can get another conclusion that the NDIF spurious eigenvalues of the plate with mixed boundary conditions are the eigenvalues of plate with stricter boundary conditions. For instance, the NDIF spurious eigenvalues of a CC–FF–CC–SS square plate are also the eigenvalues of the corresponding CC–CC–CC–CC square plate.

4. Concluding remarks

The reason for the NDIF spurious eigenvalues is still open to study. As mentioned before, the NDIF kernel function consists of the two functions $J_0(\lambda r)$ and $I_0(\lambda r)$. It is important to note that $J_0(\lambda r)$ is also the non-singular general solution of the governing equation of the membrane vibration problems. Thus, the NDIF interpolation matrix of a plate has something to do with that of the corresponding membrane. This partly interprets why the spurious eigenvalues of a plate are often found to be the eigenvalues of the corresponding membrane. However, we cannot explain why the spurious eigenvalues of square or parallelogram plates with simply-supported or mixed boundary conditions behave in such a different way.

It is worthy of stressing that the spurious eigenvalues of the NDIF method vary with boundary conditions in the above-mentioned various cases. It seems that we cannot simply identify the spurious eigenvalues of a real-world problem. But, fortunately, we can observe from our numerical experiments that there are a few general behaviors of the NDIF solution of the plate vibration eigenvalue, which can be helpful in real-world applications.

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