# STATIC AND DYNAMIC MECHANICAL ANALYSIS OF CHEMICALLY MODIFIED RANDOMLY DISTRIBUTED SHORT BANANA FIBER REINFORCED HIGH-DENSITY POLYETHYLENE/POLY (Є-CAPROLACTONE) COMPOSITES

R.K.Misra[a*], Sandeep Kumar[b]

[a] *Department of Mechanical Engineering, B.I.T Mesra,*
*Ranchi, 835215, India.*
**E-mail: mishrark_kanpur@yahoo.com, Tel.No. +919431382611*
[b] *Scientist, Defence materials and stores research and development establishment, Kanpur, India.*

## ABSTRACT

Randomly distributed short banana fiber reinforced HDPE/PCL (high-density polyethylene / poly (Є-caprolactone)) composites have been fabricated to determine the mechanical behavior at static and dynamic loading. To enhance the mechanical properties of matrix, poly caprolactone has been blended with high-density polyethylene. Three samples of banana fibers were treated with sodium hydroxide, sebacoyl chloride, and toluene diisocyanate solutions separately. It has been observed that banana fibers treated with sodium hydroxide give better mechanical properties compared to the other two solutions. The role of fiber/matrix interactions in chemically treated banana fibers reinforced composites was investigated to predict the stiffness and damping properties and their different mechanical behavior is compared with untreated banana fibers. In order to study the static and dynamic response of HDPE, HDPE/PCL blend, treated and untreated banana fiber reinforced composite plate, a multiquadric radial basis function (MQRBF) method was developed. MQRBF is applied for spatial discretization and a Newmark implicit scheme is used for temporal discretization. The discretization of the differential equations generates a larger number of algebraic equations than the unknown coefficients. To

overcome this ill conditioning, the multiple linear regression analysis, which is based on the least square error norm, is employed to obtain the coefficients. Simple supported and clamped boundary conditions are considered. Numerical results are compared with those obtained by other analytical methods.

## Notations

| | |
|---|---|
| $a, b$ | Dimension of the plate |
| $h$ | Thickness of the plate |
| $Ŗ$ | Aspect ratio (*a/b*) |
| $\nu$ | Poisson's ratio |
| $\rho$ | Mass density of the plate |
| $m$ | Mass of the plate |
| $C_V^*, C_V$ | Viscous damping, dimensionless viscous damping |
| $D$ | Flexural rigidity |
| $E_R$ | Young's modulus |
| $G$ | Shear modulus |
| $q, Q$ | Transverse load, dimensionless transverse load |
| $t^*, t$ | Time, dimensionless time |
| $w^*$ | Displacement in $z^*$ direction |
| $w$ | Dimensionless displacement in z direction |
| $\omega$ | Natural frequency of vibration |
| $\rho_0$ | Surface density $(\rho_0 = \rho h)$ |
| $\lambda$ | Eigen value |

**Key words:** Short banana fiber, Polyethylene, multiquadric radial basis function, multiple linear regression analysis.

## 1. INTRODUCTION

Nowadays natural fiber-reinforced polymeric composite materials are becoming more popular day by day. Natural fiber-reinforced composites have many advantages compared to glass fiber and metallic fiber-reinforced composites. They have high strength to weight ratio, enhanced corrosion resistance, longer fatigue life, better mechanical properties and low cost. Vegetable fibers such as jute, coir, banana, sisal, etc are lignocellulosic materials comprising of lignin, cellulose, and hemicelluloses. Presently these fibers are being used only in limited applications such as manufacturing of yarns, mats, ropes, etc. The chemical composition and some properties of some of the natural fibers are given in Table 1. The traditional market areas of natural fibers have slowly been captured by synthetic fibers, particularly polypropylene. Considerable interest has been generated in the past few years for utilizing renewable raw materials like jute, sisal, banana, and pineapple in fibrous composites to supplement the present usage of high cost synthetic fibers like glass, carbon etc /1/. Comparative prices of some natural, synthetic fibers are given in Table 2 /2/. It is necessary to impart hydrophobicity to these fibers by reaction with suitable chemical reagents /3/. Natural fibers, with high impact strength and moderate tensile and flexural properties, are of particular interest /4/.

**Table 1**

Properties of different natural fibers

| **Property** | **Banana** | **Jute** | **Sisal** | **Pine apple** | **Coir** |
|---|---|---|---|---|---|
| Diameter (micrometer) | 80-250 | ---- | 50-200 | 20-80 | 100-450 |
| Density (g/cm$^3$) | 1.35 | 1.3 | 1.45 | 1.44 | 1.15 |
| Vol. resistivity at 100 V Ohm. cm x $10^5$ | 6.5-7 | ----- | 0.4-0.5 | 0.7-0.8 | 9-14 |
| Microfibrillar angle (degrees) | 11 | 8.1 | 10-22 | 14-18 | 30-49 |
| Cellulose / lignin contents % | 65/5 | 61/12 | 67/12 | 81/12 | 43/45 |
| Tenacity MPa | 529-754 | 440-533 | 568-640 | 413-1627 | 131-175 |
| Elongation % | 1-3.5 | 1-1.2 | 3-7 | 0.8-1.6 | 15-40 |

**Table 2**
Comparative prices of some natural and synthetic fibers

| Fibers | Cost in rupees per Kg |
|---|---|
| Carbon | 1800 |
| Stainless steel | 680 |
| Glass | 70 |
| Pineapple leaf | 70 |
| Banana | 22 |
| Palmyrah | 12 |
| Sisal | 8-10 |
| Coir | 6-8 |

In tropical countries, banana plants are available in abundance and it is an agricultural crop. Banana fibers are waste product of banana cultivation so hence, without any additional cost input, banana fibers can be obtained for industrial purposes. Banana fiber is known for its natural resilience, durability, and resistance to dampness, fungal and bacterial decomposition. Banana fibers are used for manufacturing of lightweight composites. Banana fibers reinforced composite is most suitable for agro industries. Fabrication of banana fibers reinforced low-density polyethylene / polycaprolacton composites has been reported by Kumar and Misra /5/. Bamboo and sisal fibers reinforced in thermoplastic resin have also been reported /6,7/.

Dynamic analysis of the composite is very important when it is used in structural applications. The stiffness and damping behavior of unidirectional banana fibers reinforced high-density polyethylene/ poly (Є-caprolactone) composites have been presented by Misra *et al.* /8/. When banana fibers are reinforced in brittle phenol formaldehyde resin then brittleness of the phenol formaldehyde resin decreases and composite becomes ductile due to strong adhesion between the lignocellulosic banana fibers and phenol formaldehyde resin.

Various numerical and analytical methods are used to predict the behavior of composite structures. In analytical methods, Fourier series and Rayleigh-Ritz methods are used. But analytical methods have been used only for simple plate geometries and boundary conditions. They cannot be applied when the problems involve complex geometries and boundary conditions.

Therefore, numerical methods are used, which are capable of solving such types of problems.

In numerical methods, finite element method is very popular. Although FEM is quite successful, the discretization of complex two- and three-dimensional geometry and re-meshing of the domain changing with time is very rigorous, time-consuming and a burdensome task in comparison of assembly and solution of the finite element equations. In spite of its numerous advantages and unparalleled success, it is not well suited for certain classes of problems, such as crack propagation and moving discontinuities, plate bending, solution of higher order partial differential equations, phase transformation, moving phase boundaries, modeling of multi-scale phenomena, dynamic impact problems such as turbine blades.

To avoid these problems, recently a class of new methods known as meshless methods has been developed. These classes of methods include diffuse element method /9/, element-free Galerkin method /10/, hp clouds /11/, reproducing kernel particle method /12/, modified smooth particle hydrodynamics /13/, boundary node method /14/, meshless local Petrov–Galerkin approach /15/, and local boundary integral equation method /16/.

Among various meshless methods, the importance of the radial basis function methods is increasing day by day for solving the engineering problems. In 1971 Hardy /17/ proposed the multi quadric radial basis function method for the interpolation of geographically scattered data. Later on, Franke /18/ studied the evaluation of radial basis function (RBF) for scattered data interpolation in terms of timing, accuracy and ease of implementation. In 1990, Kansa /19/ applied this function for the solution of partial differential equations. Chen et al. /20/ studied the free vibration analysis of circular and rectangular plates by employing the RBF. Misra et al /21,22/ applied the multiquadric radial basis function for the analysis of anisotropic plates and laminate. Ferreira presented the analysis of composite beams /23/ and plates /24/ by using RBF method.

This paper reports the fabrication, chemical treatment and evaluation of properties of randomly distributed short banana fiber-reinforced high-density polyethylene / polycaprolacton composites. After obtaining the properties experimentally, MQRBF method is employed for static and dynamic analysis of randomly distributed short banana fibers reinforced high-density polyethylene / polycaprolacton composite plate.

## 2. EXPERIMENTAL STUDIES

Modification and characterization of banana fibers are given below. The fabrication and testing of untreated/sodium hydroxide treated banana fibers with commercially available high-density polyethylene and polycaprolactone are described.

### • Materials

The banana fiber (BF) bundles were obtained from Regional Research Laboratory (CSIR) Jorhat, India. Sodium hydroxide, toluene, petroleum ether, tolylene diisocyanate and sebacoyl chloride were used as such.

High-density polyethylene (HDPE) grade HD50MA180 obtained from Indian Petrochemicals Corporation Limited, Vadodara. Its properties were as follows:

Melt flow index = 17.3 (@ $190^0$C/2.16 Kg)
Density = 0.98 g/cc
Melting temperature = $130^0$C

Tone polymer of Union Carbide Company, USA was used; as such, it is a low melting aliphatic polyester which is biodegradable, miscible with many polymers, pigments, fillers and has outstanding adhesion to a broad spectrum of substrates. Poly caprolactone (PCL) is non-toxic and acts as a die lubricant, extrusion aid, and mold release agent during processing. The properties of PCL are as follows:

Melt flow index = 2.2 (@$125^0$C/2.16 Kg)
Density = 1.145 g/cc
Melting temperature = $60^0$C

### • Separation of Fibers

In order to separate the individual fiber from the bundle, organic solvents such as acetone, toluene, and petroleum ether were used.

Better separation of fibers was achieved by using toluene followed by washing with petroleum ether. For this purpose a bundle of fibers were dipped in toluene for 4 hour and then placed in petroleum ether overnight. The solvent was removed and the fibers were dried in air atmosphere.

- **Chemical Treatment of Banana Fibers**

**1. *Modification of fibers with aqueous sodium hydroxide***

Fibers were dipped in 5% and 10 % aq-NaOH solution at room temperature for 4 hour then fibers were washed thoroughly with distilled water to ensure the complete removal of NaOH (no pink color with phenolphthalein indicator) and dried under vacuum at $60^0$C for 18-24 hours. The mass loss during the treatment was evaluated by weighing the bundles of fibers before and after alkali treatment.

**2. *Reaction with tolylene diisocyanate (TDI)***

The carbon-nitrogen double bond of the isocyanate group undergoes ionic addition reaction with a wide variety of functional groups, which contain an active hydrogen atom, such as water, alcohols, phenols, thiols, amines and carboxylic acid (Joseph et al., 1999). About 5 g of banana fibers, in oven dry condition, was added to DMSO (100 ml) containing 1.5 ml TDI. Reaction was carried out at $30^0$C for 4 hours. After that, fibers were taken out by decantation of the DMSO solution, washed three times with 50 ml portions each of dioxane, and finally dried over calcium chloride.

**3. *Reaction with sebacoyl chloride (SC)***

About 5 g banana fibers were added to dioxane (160 ml) containing 1.4 x $10^{-2}$ mole pyridine. This was followed by the addition of 0.5 g of sebacoyl chloride dissolved in 20 ml dioxane. The reaction was continued for 4 hours at 30°C, the fiber washed thrice with 50 ml portions each of acetone, and finally dried over calcium chloride.

- **Physical Appearance of Banana Fibers**

Banana fibers were pale yellowish with dull appearance. After modification of banana fibers with sodium hydroxide, sebacoyl chloride and tolylene diisocyanate, no significant change in the appearance of the fibers was observed on treatment with sebacoyl chloride and the fibers became somewhat brittle. On treatment with NaOH, the color became dark brown while treatment with TDI resulted in golden yellow color.

- **Weight Loss**

Before and after treatment, the bundle was weighed in each case and the results are summarized in Table 3.

**Table 3**
Weight Loss

| S. No. | Treated with | % Wt loss |
|---|---|---|
| 1. | 5% NaOH | 20.31 |
| 2. | 10%NaOH | 25.4 |
| 3. | Sebacoyl Chloride | 28.2 |
| 4. | Tolylene Diisocyanate | 23.1 |

- **Evaluation of Properties**

**1. *Denier calculation***

In order to determine the diameter of the fibers, the denier and the Tex of the fibers were evaluated. Tex is defined as the weight of 1000 m of fiber in gram and denier is the weight of 9000 m of fibers. The denier of the fibers was determined by weighing the fibers. The measurement was repeated four times with different fibers and the average of these values was calculated.

Denier of banana fibers decreased on NaOH treatment. There was a slight increase in the case of sebacoyl chloride, and tolylene diisocyanate treated fiber, which have been shown in Table 4. Fibers became finer after alkali treatment and since denier is a measure of linear density, it showed a decrease in denier and diameter.

**Table 4**
Tex and Denier of Modified Banana Fibers

| Fibers | Tex | Denier |
|---|---|---|
| Untreated | 14.2 | 127.8 |
| 5 % NaOH treated | 13.1 | 117.9 |
| 10% NaOH treated | 13.0 | 117 |
| Sebacoyl chloride | 14.6 | 131.9 |
| Tolylene Diisocyanate | 14.8 | 133.2 |

**2. *Density***

Density of the untreated and treated fibers was determined at room temperature using a density gradient column prepared from xylene ($\rho$ = 0.865 g/cm$^3$) and carbon tetrachloride ($\rho$ = 1.595 g/cm$^3$) which have low vapor pressure and low viscosity and are completely miscible with each other. The column was calibrated using standard floats. Small tightly packed bundles of fibers were dropped in the column and their height was noted, after that the density was computed from the calibration plot, and has been shown in Table 5. It is found to be lower than that of the other natural fibers, such as pineapple fiber (1.46 gm/cm$^3$) and ramie fibers (1.55 gm/cm$^3$). Lower density of the banana fibers supports the lower cellulose content (65%) in banana in comparison to pineapple (81%).

**Table 5**

Density of Modified Banana Fibers

| S. No. | Treated with | Density (g/cm$^3$) |
|---|---|---|
| 1. | ------ | 1.40 |
| 2. | 5% NaOH | 1.45 |
| 3. | 10% NaOH | 1.48 |
| 4. | Sebacoyl Chloride | 1.43 |
| 5. | Tolylene Diisocyanate | 1.43 |

**3. *Diameter calculation***

The diameter of the fibers was calculated from their density and denier according to the following expression:

$\rho = m / (\pi r^2 l)$

or $r = (m / \rho\pi l)^{1/2}$

where $\rho$ = density, m = mass of fibers, l =length of fibers

Since density and Tex are known for untreated and treated fibers, hence, r was calculated for l =1000 m and the results are presented in Table 6.

**Table 6**

Diameter of Modified Banana Fibers

| S. No. | Fibers types | Diameter (mm) |
|---|---|---|
| 1. | Untreated | 0.113 |
| 2. | 5% NaOH | 0.107 |
| 3. | 10% NaOH | 0.105 |
| 4. | Sebacoyl chloride | 0.1139 |
| 5. | Tolylene Diisocyanate | 0.114 |

- **Mechanical Properties**

Mechanical properties of fibers were evaluated using a Zwick tensile testing machine (model Z010). For each sample, 30 fibers were tested at the following specifications:

Gauge length= 2.5 cm,

Crosshead speed= 5 mm/min.

Constant lengths of the fibers were clamped in the jaws mounted on tensile testing machine and were stretched under constant rate of strain. The results of tensile strength, tensile modulus, and elongation at break of modified banana fibers are given in Table 7.

**Table 7**

Mechanical Properties of Fibers

| **Fiber tested** | **Maximum stress (G Pa)** | **Modulus (G Pa)** | **Strain at break %** |
|---|---|---|---|
| Untreated | $0.281 \pm 0.079$ | $11.2 \pm 4.4$ | $3.14 \pm 1.3$ |
| 5% NaOH treated | $0.248 \pm 0.089$ | $13.3 \pm 2.0$ | $5.68 \pm 0.9$ |
| 10 % NaOH treated | $0.256 \pm 0.092$ | $13.9 \pm 3.2$ | $5.88 \pm 1.1$ |
| SC treated | $0.238 \pm 0.08$ | $11.0 \pm 4.2$ | $2.88 \pm 1.6$ |
| TDI treated | $0.280 \pm 0.06$ | $12.1 \pm 4.0$ | $3.74 \pm 1.5$ |

- **Preparation of Blends of HDPE and PCL**

A Klockner Windsor single screw extruder having L/D ratio of 21:1 and screw diameter of 30 mm was used for blending HDPE and PCL. Five samples of different compositions as shown in Table 8 were prepared. The barrel temperature was varied from 130-175°C and the screw speed from 10-40 rpm. PCL having low melt viscosity at high temperature and high screw speed separated out from the high melt viscosity HDPE matrix. Thus, for proper mixing of these polymers, the optimum parameters shown in Table 9 were used. The extruded samples were cooled by quenching in a water bath and then chopped into granules and dried in oven at $70^0$C for 3 hours.

**Table 8**
Composition of Blend

| Sample No. | HDPE (g) | PCL (g) |
|---|---|---|
| 1. | 200 | 0 |
| 2. | 190 | 10 |
| 3. | 180 | 20 |
| 4 | 170 | 30 |
| 5. | 160 | 40 |

**Table 9**
Optimum Parameters for Mixing of Polymers

| Screw speed (rpm) | Barrel zone temperature ($^0$C) | | | Die zone Temperature | |
|---|---|---|---|---|---|
| 35 | 135 | 140 | 150 | 150 | 150 |

- **Fabrication of Composites**

The banana fibers were separated and then dried in oven for 3 hours at 80$^0$C for further processing. HDPE and polycaprolactone were dried in an oven at 55$^0$C.Weighing was done in the proportions given in Table 10.The total weight was taken to be 200 g in each experimental run. These Banana fibers were used for composite fabrication. The variation in the tensile properties with blend composition and banana fiber are shown in Table 11.

**Table 10**
Composition of Banana Fiber Reinforced Composites

| Sample No. | HDPE (g) | PCL (g) | Banana Fiber (g) |
|---|---|---|---|
| 1 | 160 | 40 | 10 (Untreated) |
| 2 | 160 | 40 | 10(treated) |
| 3 | 160 | 40 | 20 (untreated) |
| 4 | 160 | 40 | 20 (treated) |
| 5 | 160 | 40 | 30 (untreated) |
| 6 | 160 | 40 | 30 (treated) |

**Table 11**
Mechanical properties of HDPE/PCL blends and NaOH treated banana fiber

| **Sample** | | | | **Tensile** | **Tensile** | **Elongation** |
|---|---|---|---|---|---|---|
| **Code** | **HDPE (g)** | **PCL (g)** | **NaOH (g)** | **Modulus (MPa)** | **Strength (MPa)** | **at break (%)** |
| PCL000 | 200 | Nil | Nil | 190.2±14.2 | 11.21±4.3 | 7.49±1.4 |
| PCL005 | 190 | 10 | Nil | 204.58±7.3 | 13.45±3.5 | 7.01±1.8 |
| PCL010 | 180 | 20 | Nil | 235.45±8.4 | 13.83±2.8 | 6.98±2.1 |
| PCL015 | 170 | 30 | Nil | 255.94±7.8 | 14.08±5.1 | 6.75±2.5 |
| PCL020 | 160 | 40 | Nil | 266.76±13.4 | 14.48±4.4 | 4.5±1.9 |
| PCL5U | 160 | 40 | 10U | 270.86±21.3 | 14.5±2.9 | 3.44±1.5 |
| PCL5T | 160 | 40 | 10T | 280.9±19.4 | 15.3±6.1 | 3.40±1.2 |
| PCL10U | 160 | 40 | 20U | 273.9±18.4 | 14.8±4.3 | 3.0±1.5 |
| PCL10T | 160 | 40 | 20T | 289.3±23.6 | 16.8±5.6 | 2.9±0.8 |
| PCL15U | 160 | 40 | 30U | 275.5±21.6 | 14.9±5.5 | 2.9±1.4 |
| PCL15T | 160 | 40 | 30T | 296.3±18.4 | 17.1±6.8 | 2.8±0.9 |

Where U = untreated banana fiber and T = treated banana fiber

## 3. THEORETICAL ANALYSIS

After evaluating the mechanical properties of the chemically modified randomly distributed short banana fiber reinforced high-density polyethylene/ poly (Є-caprolactone) composites, Multiquadric radial basis function method is applied for static and dynamic mechanical analyses of the randomly distributed short banana fiber reinforced high density polyethylene/ poly (Є-caprolactone) composites plate to assess its response to external loading such as uniformly distributed load.

- ### The Multiquadric Radial Basis Function Method

Consider $P_1, P_2, P_3, P_{4,}, .........P_N$ being N collocation points in domain $\Omega$ of which $P_i\,(i = 1, 2, 3......N_I)$ are interior points and $P_b\,(b = N_{I+1}, .....N)$ are boundary points. Partial differential equation for boundary value problems has been given below:

$$Aw = f(x, y) \quad \text{in} \quad \Omega \tag{1}$$

$$Bw = g(x, y) \quad \text{on} \quad \partial\Omega \tag{2}$$

*A* is a linear differential operator, *B* is a linear boundary operator imposed on boundary conditions. The governing equation for a free flexural vibration of a uniform thin plate is written as follows /20/:

$$\nabla^4 w = \lambda^4 w(\mathrm{x}) \quad x \in \Omega \tag{3}$$

where $w$ is the lateral displacement, $\lambda^4 = \omega^2 \rho_0 h / D$, $\lambda$ is the frequency parameter, $\omega$ is the circular frequency, $\rho_0$ is the surface density, D is the flexural rigidity of the isotropic plate, and h is the thickness of the plate.

In the present research work, it is assumed that the randomly distributed short banana fiber reinforced composite plate behaves as an isotropic plate. Therefore, in Eigen value problems, the equations (1) and (2) reduce to

$$Aw = \lambda^4 w \tag{4}$$

$$Bw = 0 \tag{5}$$

Solution of equations (1) and (2) is expressed as

$$w(x, y) = \sum_{j=1}^{N} w_j \varphi_j(x, y) \tag{6}$$

$\left\{ w_j \right\}_{j=1}^{N}$ are the unknown coefficients to be determined, and $\varphi_j(x_j, y_j)$ is a basis function.

$$\varphi_{\mathrm{j}} = \sqrt{(x - x_j)^2 + (y - y_j)^2 + c^2} = \sqrt{r_j^{\,2} + c^2} \tag{7}$$

Other widely used radial basis functions are:

| | |
|---|---|
| $\varphi(r) = r^3$ | Cubic |
| $\varphi(r) = r^2 \log(r)$ | Thin plate splines |
| $\varphi(r) = (1-r)^m{}_+ p(r)$ | Wendland functions |
| $\varphi(r) = e^{-(cr)^2}$ | Gaussian |
| $\varphi(r) = (c^2 + r^2)^{-1/2}$ | Inverse multiquadrics |

Here $r = \|P - P_j\|$ is the Euclidean norm between points $P=(x,y)$ and $P_j = (x_j, y_j)$. The Euclidian distance $r$ is real and non-negative and $c$ is a shape parameter, a positive constant. Ling and Kansa /25/ discussed the shape parameter in detail.

- **Calculation of Eigen Values**

Applying the multiquadric radial basis in equation (4),

$$\sum_{j=1}^{N} w_j A\varphi \|P_i - P_j\| = \lambda^4 \sum_{j=1}^{N} w_j \varphi \|P_i - P_j\| \tag{8}$$

where $i = 1, 2, ............N_I$

## Define

$$\mathbf{L} = [A\phi(\|P_i - P_j\|)]_{N_I x N} \tag{9}$$

$$\mathbf{M} = [\phi\|P_i - P_j\|)]_{N_I x N} \tag{10}$$

Applying MQRBF in equation (5),

$$\sum_{j=1}^{N} w_j B\phi(\|P_b - P_j\|) = 0 \tag{11}$$

where

$b = N_{I+1}, N_{I+2}\ ......N$

$$\mathbf{K} = [B\phi(\|P_i - P_j\|)]_{N_b x N} \tag{12}$$

$$\mathbf{w} = [\mathrm{w}_1 \ \mathrm{w}_2 \ \mathrm{w}_3 \ldots \mathrm{w}_N]^T \tag{13}$$

Equations (8) and (12) can be written as

$$\mathbf{Lw} = \lambda^4 \mathbf{Mw} \tag{14}$$

$$\mathbf{Kw} = 0 \tag{15}$$

The general eigen value problem in the matrix form becomes

$$\begin{bmatrix} \mathbf{L} \\ \mathbf{K} \end{bmatrix} \mathbf{w} = \lambda^4 \begin{bmatrix} \mathbf{M} \\ 0 \end{bmatrix} \mathbf{w} \tag{16}$$

The following algorithm /26/ of the standard Eigen value problem has been used in the present analysis.

$$\mathbf{L}^1 \mathbf{w}^2 = \lambda^4 \mathbf{w}^2 \tag{17}$$

where $\mathbf{L}^1 = \mathbf{LD}^{-1} \begin{bmatrix} \mathbf{I}_{\mathrm{N_I} x \mathrm{N_I}} \\ \mathbf{0}_{\mathrm{N}bx\mathrm{N_I}} \end{bmatrix}$ (18)

$$\mathbf{w}^2 = [\mathrm{w}_1 \ \mathrm{w}_2 \ \mathrm{w}_3 \ldots \mathrm{w}_{\mathrm{N_I}}]^T \tag{19}$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{M} \\ \mathbf{K} \end{bmatrix} \tag{20}$$

## 4. RESULTS AND DISCUSSION

After separation and modification of banana fibers from chemicals, banana fibers-reinforced HDPE/PCL composite plates were prepared, and the evaluation of their mechanical properties, and theoretical dynamic analyses under self-weight of the composite plate have been presented in this work.

The hydrophilic nature of banana is primarily due to the hydroxyl groups of cellulosic component. An attempt was made to block this hydroxy group with several types of reagents using organic solvents. The reagents used were

- Tolylene diisocyanate (TDI) in dimethyl sulfoxide (DMSO) catalyst is triethylamine.

- Sebacoyl chloride (SC) in dioxane.
- Sodium hydroxide (NaOH)

The fibers acquired a dazzling, golden yellow color on treatment with TDI. No significant change in appearance of fibers was observed on treatment with SC, but the fibers showed some brittleness. Fibers became dark brown in color by treating with NaOH Reagents. Table 7 shows that lower concentration of alkali 5% and 10% resulted in a significant increase in modulus 18.75% and 24.1% respectively, and the strength values decreased by 11.74% and 8.1%. There is no change in strength values with TDI treatment but a significant drop of 15.3% in tensile strength on SC treatment was observed. This was perhaps due to liberation of HCL, which may bring about hydrolytic cleavage.

Table 8 shows the blending of high-density polyethylene (HDPE) with biodegradable polymer polycaprolactone (PCL) in the range of 0-20% PCL content. Effect of the mechanical properties due to blending of HDPE and PCL has been shown in Table 11. The mechanical properties such as Young's modulus and tensile strength of HDPE improve by the incorporation of PCL. The modulus increases by 41% and the tensile strength by 29% on introducing 20% (w/w) PCL into HDPE matrix. This may be attributed to the higher degree of crystallinity of PCL than HDPE and better room temperature mechanical properties. Elongation at break decreases significantly with the incorporation of PCL.

The properties of the HDPE/PCL/BF composites were studied to observe the effect of banana fiber by increasing its volume fraction and after treating it with sodium hydroxide, and have been shown in Table 11. An increase in the tensile parameter as in modulus and tensile strength of 11.07% and 18% respectively with 15% volume fraction of fibers in HDPE /PCL (80:20) blends and a significant decrease in strain at break value. With the untreated fibers, the improvement in properties is less which may be due to poor fiber/matrix interphase compared to treated fibers. Also, the composites are better than the neat blends because the fibers have a higher stiffness than polymer. There is not as much increase as expected; this may be due to clumping of the fibers and inadequate impregnation of fibers by the polymer during the molding process or treatments did not reach an optimum adhesion between fiber and matrix.

Using multi-quadric radial basis function (MQRBF), static and dynamic response and natural frequencies of short banana fibers reinforced high-density polyethylene / polycaprolacton composites are obtained.

## Case Study 1: Static and Dynamic Response of the Composite Plate

Figure 1 shows the geometry, coordinate system and loading in chemically modified randomly distributed short banana fiber reinforced high-density polyethylene/ poly (Є-caprolactone) composites plate. Neglecting the transverse shear and rotary inertia, governing equation of the plate is expressed in non-dimensional form as:

$$(w_{xxxx} + 2R^2 w_{xxyy} + R^4 w_{yyyy}) + w_{tt} + c_v w_t - Q(x, y, t) = 0 \quad (21)$$

where the subscript denotes the partial derivative with respect to the suffix following. The non-dimensional quantities are defined by

$$w = w^* / h, x = x^* / a, y = y^* / b, R = a / b, t = t^* \sqrt{D / (\rho a^4 h)}$$
$$Q = qa^4 / (Dh), C_v = (C_v^* / M)\sqrt{(\rho a^4 h) / D}, m = \rho abh$$

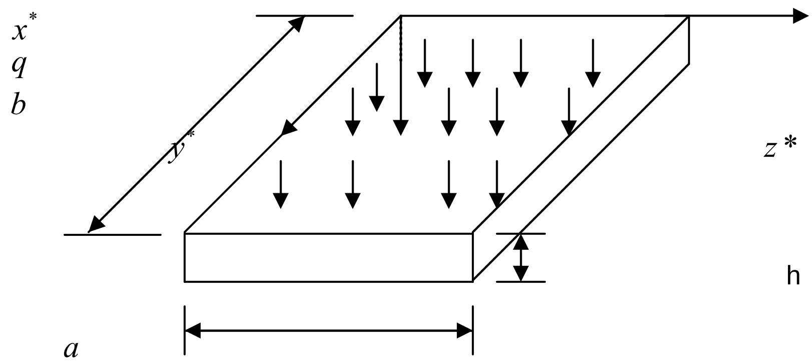


**Fig. 1:** Geometry of the banana fiber reinforced HDPE/ PCL blend composite plate

**Boundary conditions**

***(a) Simply supported boundary conditions are***

$$x = 0,1 \quad w = 0 \tag{22a}$$

$$x = 0,1 \quad w_{xx} = 0 \tag{22b}$$

$$y = 0,1 \quad w = 0 \tag{22c}$$

$$y = 0,1 \quad w_{xx} = 0 \tag{22d}$$

***(b) Clamped edge boundary conditions are***

$$x = 0,1 \quad w = 0 \tag{23a}$$

$$x = 0,1 \quad w_x = 0 \tag{23b}$$

$$y = 0,1 \quad w = 0 \tag{23c}$$

$$y = 0,1 \quad w_y = 0 \tag{23d}$$

***(c) For initial conditions at*** $t = 0$

$$w = 0 \quad \text{and} \quad \frac{\partial w}{\partial t} = 0, \quad \frac{\partial^2 w}{\partial t^2} = 0 \tag{24}$$

The governing equation (21) is solved using multiquadric radial basis function and boundary conditions, equations (22) and (23), for simply supported and clamped edge plates respectively, and has been presented in Appendix A. In this collocation method, the number of equations is more than the number of unknown coefficients; hence, this method creates ill conditioning. To overcome this ill-conditioning, multiple linear regressions analysis (Appendix B) based on least-square error norms is employed.

Deflection of the pure HDPE, HDPE/PCL blend and banana fiber reinforced composite plate at various aspect ratios of the present method for simply supported boundary condition has been compared with those obtained by Timoshenko and Woinowsky-Krieger /27/ as shown in Tables 12-18. There is good agreement between the present results and reference solution. The following features are observed after analyzing Tables 12-18 for simple supported boundary conditions:

- As aspect ratio increases deflection decreases due to decrease in load.
- HDPE plate gives maximum deflection and 15% treated banana fiber reinforced composite plate gives least deflection. It means strength of the HDPE plate is least and treated banana fiber reinforced composite plate is maximum.
- As percentage of the PCL and banana fiber increases, deflection decreases

and strength increases.

- 5 % treated banana fiber reinforced composite plate gives more strength than 15% untreated banana fiber reinforced composite plate. Due to chemical treatment, quantity of the banana fiber decreased.

**Table 12**

Deflection of simple supported pure HDPE (PCL000) plate

| q ($N/m^2$) | b (mm) | a (mm) | b/a | h (mm) | Analytical Method $w_{max}$ /27/ (mm) | MQRBF Method $w_{max}$ (mm) |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.018888 | 0.018985 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0029443 | 0.0029477 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00070216 | 0.00069933 |
| 10 | 300 | 50 | 6.0 | 10 | 0.00004672 | 0.000046630 |

**Table 13**

Deflection of simple supported HDPE/PCL blend (PCL005) plate

| q ($N/m^2$) | b (mm) | a (mm) | b/a | h (mm) | Analytical Method $w_{max}$ /27/ (mm) | MQRBF Method $w_{max}$ (mm) |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.017554 | 0.017651 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0027374 | 0.00 27405 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00065281 | 0.00065017 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000043436 | 0.00043352 |

**Table 14**

Deflection of simple supported HDPE/PCL blend (PCL020) plate

| **q ($N/m^2$)** | **b (mm)** | **a (mm)** | **b/a** | **h (mm)** | **Analytical Method $w_{max}$ /27/ (mm)** | **MQRBF Method $w_{max}$ (mm)** |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.013462 | 0.013536 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0020993 | 0.0021017 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00050064 | 0.00049862 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000033311 | 0.000033247 |

**Table 15**

Deflection of simple supported untreated banana fiber reinforced HDPE/PCL blend (PCL5U) composite plate

| **q ($N/m^2$)** | **b (mm)** | **a (mm)** | **b/a** | **h (mm)** | **Analytical Method $w_{max}$ /27/ (mm)** | **MQRBF Method $w_{max}$ (mm)** |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.013260 | 0.013334 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0020679 | 0.0020703 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00049315 | 0.00049115 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000032813 | 0.000032749 |

**Table 16**

Deflection of simple supported treated banana fiber reinforced HDPE/PCL blend (PCL5T) composite plate

| **q ($N/m^2$)** | **b (mm)** | **a (mm)** | **b/a** | **h (mm)** | **Analytical Method $w_{max}$ /27/ (mm)** | **MQRBF Method $w_{max}$ (mm)** |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.012784 | 0.012855 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0019936 | 0.0019959 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00047544 | 0.00047352 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000031635 | 0.000031574 |

**Table 17**
Deflection of simple supported untreated banana fiber reinforced HDPE/PCL blend (PCL 15U) composite plate

| **q ($N/m^2$)** | **b (mm)** | **a (mm)** | **b/a** | **h (mm)** | **Analytical Method $w_{max}$ /27/ (mm)** | **MQRBF Method $w_{max}$ (mm)** |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.013035 | 0.013107 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0020327 | 0.0020351 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00048476 | 0.00048280 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000032255 | 0.000032193 |

**Table 18**
Deflection of simple supported treated banana fiber reinforced HDPE/PCL blend (PCL15T) composite plate

| **q ($N/m^2$)** | **b (mm)** | **a (mm)** | **b/a** | **h (mm)** | **Analytical Method $w_{max}$ /27/ (mm)** | **MQRBF Method $w_{max}$ (mm)** |
|---|---|---|---|---|---|---|
| 10 | 300 | 300 | 1.0 | 10 | 0.012120 | 0.012187 |
| 10 | 300 | 150 | 2.0 | 10 | 0.0018900 | 0.0018922 |
| 10 | 300 | 100 | 3.0 | 10 | 0.00045073 | 0.00044891 |
| 10 | 300 | 50 | 6.0 | 10 | 0.000029990 | 0.000029933 |

Similar behavior is observed at clamped edge boundary conditions as shown in Table 19. A computer program based on the finite difference method (FDM) proposed by Chadrashekara /28/ is also developed. The damped response of simple supported composite plates obtained by the present method and finite difference method has been compared and shown in Figure 2 at uniformly distributed load q = 10 ($N/m^2$), sides a = b = 300 (mm) and thickness h = 10 (mm) at non-dimensional viscous damping factor $C_v$ = 1.25. There is good agreement in the results. The following features are observed after analyzing Figures 3-9 and Table 20 for the simple supported boundary condition:

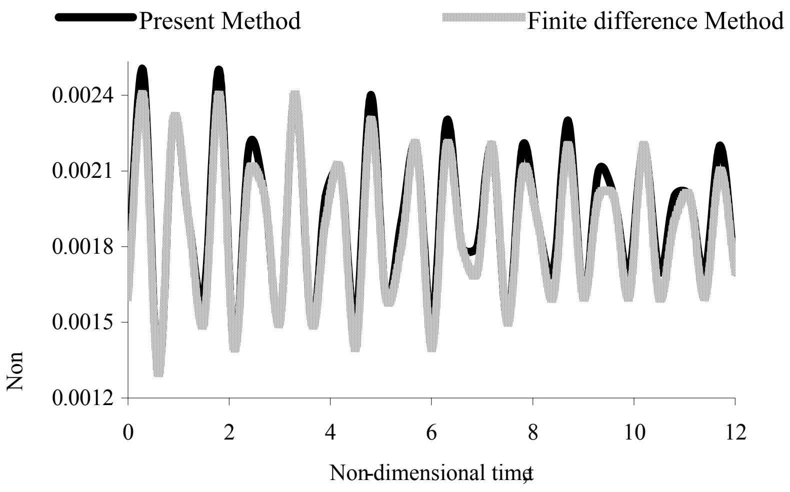


**Fig. 2:** Damped response of a simple supported pure HDPE (PCL000) square isotropic plate at damping coefficient factor $C_v$ =1.25

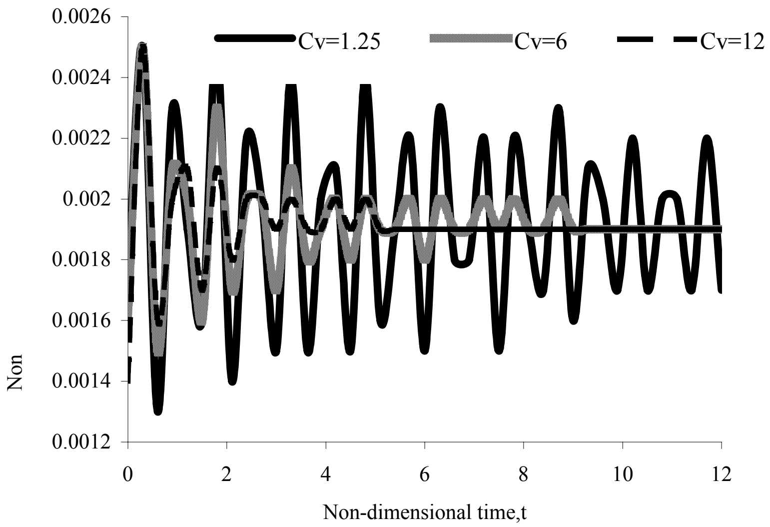


**Fig. 3:** Damped response of a simple supported pure HDPE (PCL000) square isotropic plate

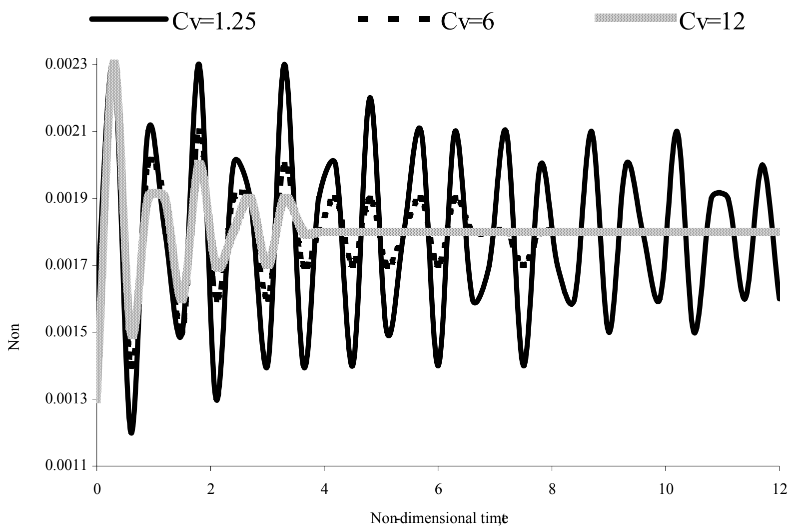


**Fig. 4:** Damped response of a simple supported HDPE/PCL blend (PCL005) square isotropic plate

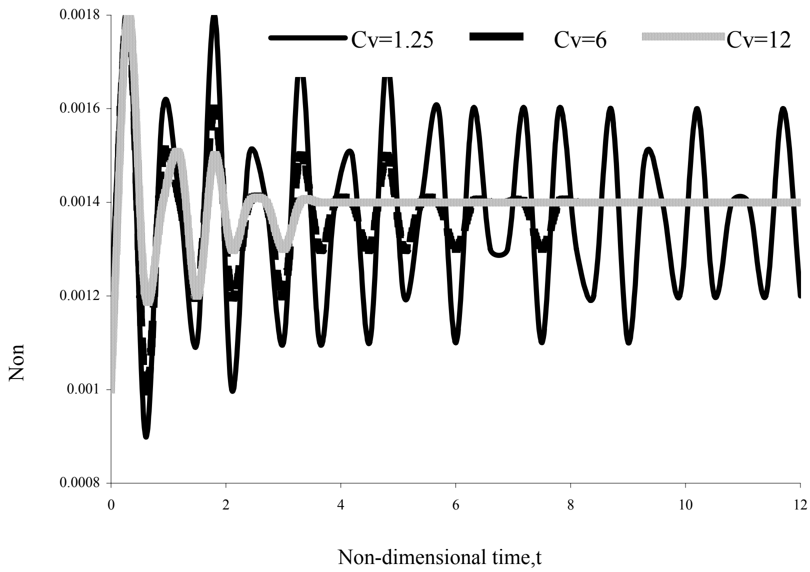


**Fig. 5:** Damped response of a simple supported HDPE/PCL blend (PCL020) square isotropic plate

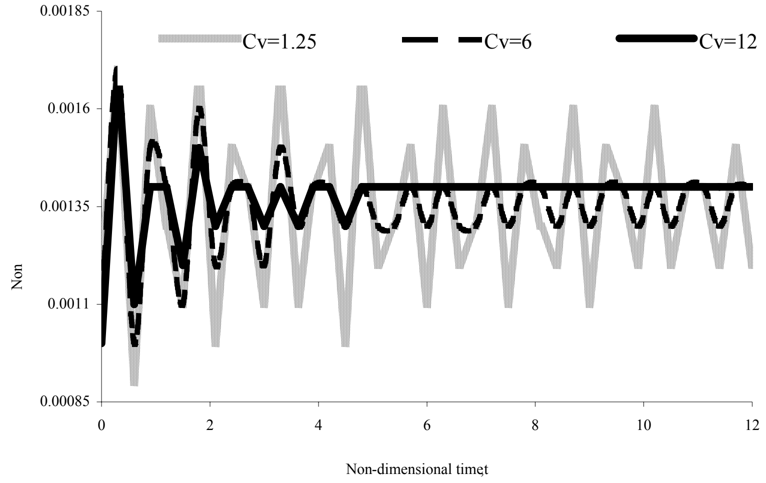


**Fig. 6:** Damped response of a simple supported untreated banana fiber reinforced (PCL5U) HDPE/PCL blend square composite plate

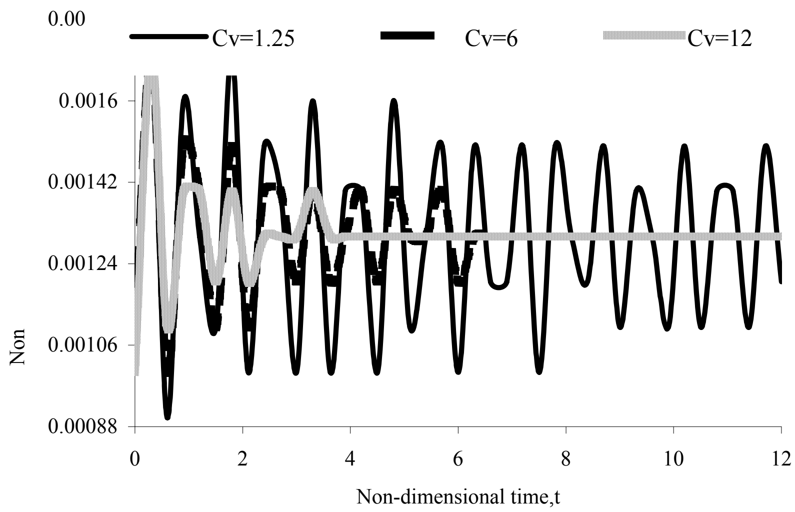


**Fig. 7:** Damped response of a simple supported treated banana fiber reinforced (PCL5T) HDPE/PCL blend square composite plate

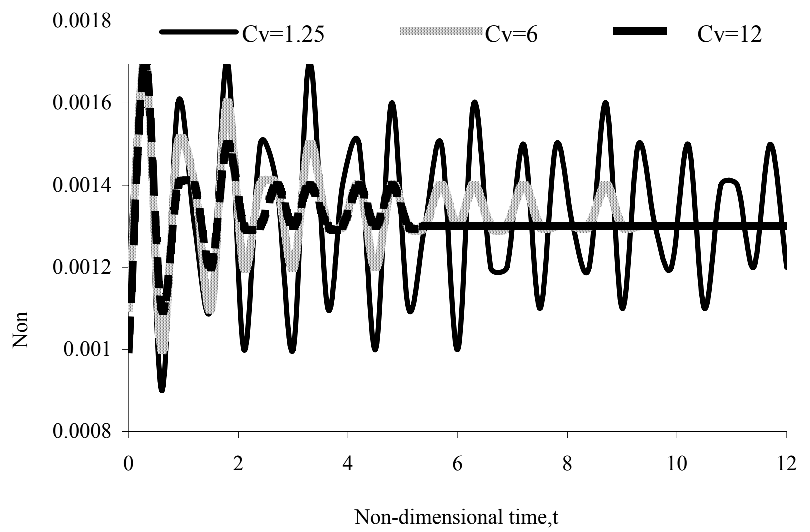


**Fig. 8:** Damped response of a simple supported untreated banana fiber reinforced (PCL15U) HDPE/PCL blend square composite plate

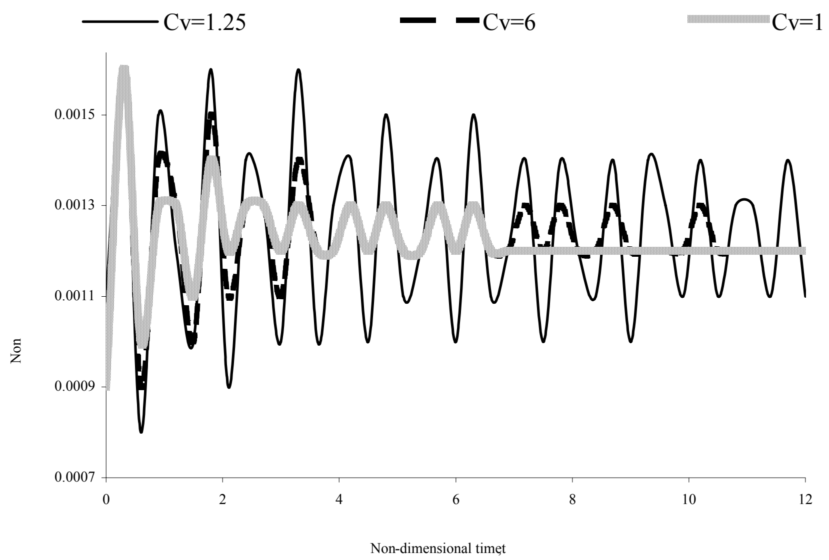


**Fig. 9:** Damped response of a simple supported treated banana fiber reinforced (PCL15T) HDPE/PCL blend square composite plate

**Table 19**

Deflection of clamped edge HDPE, HDPE/PCL blend and banana fiber reinforced composite plate

| | Sample Code | Analytical Method $w_{max}$ /27/ (mm) | MQRBF Method $w_{max}$ (mm) |
|---|---|---|---|
| q = 10 (N/m$^2$)<br>a = b = 300 (mm)<br>h = 10 (mm) | PCL000 | 0.0058596 | 0.0055879 |
| | PCL005 | 0.0054477 | 0.0051951 |
| | PCL020 | 0.0041779 | 0.0039842 |
| | PCL5U | 0.0041153 | 0.0039245 |
| | PCL5T | 0.0039676 | 0.0037836 |
| | PCL15U | 0.00404541 | 0.0038578 |
| | PCL15T | 0.0037614 | 0.0035870 |

**Table 20**

Damped response of a simple supported HDPE, HDPE/PCL blend and banana fiber reinforced composite plate

| Material | Cv=6.0 | Cv=12.0 |
|---|---|---|
| | **Stability Time** | **Stability Time** |
| PCL000 | Stabilize after 9 | Stabilize after 5.4 |
| PCL005 | Stabilize after 8.1 | Stabilize after 3.63 |
| PCL020 | Stabilize after 8.1 | Stabilize after 3.63 |
| PCL5U | Did not stabilize after 8.1 | Stabilize after 4.8 |
| PCL15U | Stabilize after 9.3 | Stabilize after 5.4 |
| PCL5T | Stabilize after 6.6 | Stabilize after 3.9 |
| PCL15T | Stabilize after 10.8 | Stabilize after 6.9 |

- HDPE/PCL blend orthotropic plate takes less time to stabilize compared to HDPE and banana fiber reinforced composite plate.
- When damping coefficient increases from 1.25-12, Non-dimensional maximum deflection decreases in banana fiber reinforced composite plate compared to HDPE and HDPE/PCL blend orthotropic plate.
- 5 % treated banana fiber reinforced composite plate stabilizes early as compared to 5 % untreated banana fiber reinforced composite plate. But a different phenomenon is observed in case of 15 % treated and untreated banana fiber reinforced composite plate. 15 % untreated banana fiber reinforced composite plate stabilizes fast as compared to 15% treated banana fiber reinforced composite plate.
- After incorporation of fibers in composite materials, tensile properties and stiffness improves and damping properties decreases. Damping means capacity of the materials to absorb the impact. When stiffness increases and damping decreases in materials, its capacity to dissipate energy decreases. Therefore, banana fiber reinforced composite plate takes more time to stabilize as compared to HDPE and HDPE/PCL blend orthotropic plate. After chemical treatment of banana fibers, interfacial bonding improves between fibers and matrix. Due to improvement of the interfacial bonding movement of the molecules at the interface decreases and damping properties decreases. Composites, which have poor interfacial bonding between fiber and matrix dissipates more energy compared to better interfacial bonding composites. This chemical treatment of banana fibers removes the foreign materials and decreases the diameter of banana fibers. This result increases the matrix materials. Matrix dissipates more energy compared to banana fibers. When quantity of the banana fibers is less, matrix plays important role. Therefore 5 % treated banana fiber reinforced composite plate stabilizes early as compared to 5 % untreated banana fiber reinforced composite plate. But 15 % untreated banana fiber reinforced composite plate stabilizes fast as compared to 15% treated banana fiber reinforced composite plate.

Similar behavior is observed at clamped edge boundary conditions as shown in Figure 10, but clamped edge plate takes more time to stabilize compared to simple supported plate because clamped edge plate dissipate less energy.

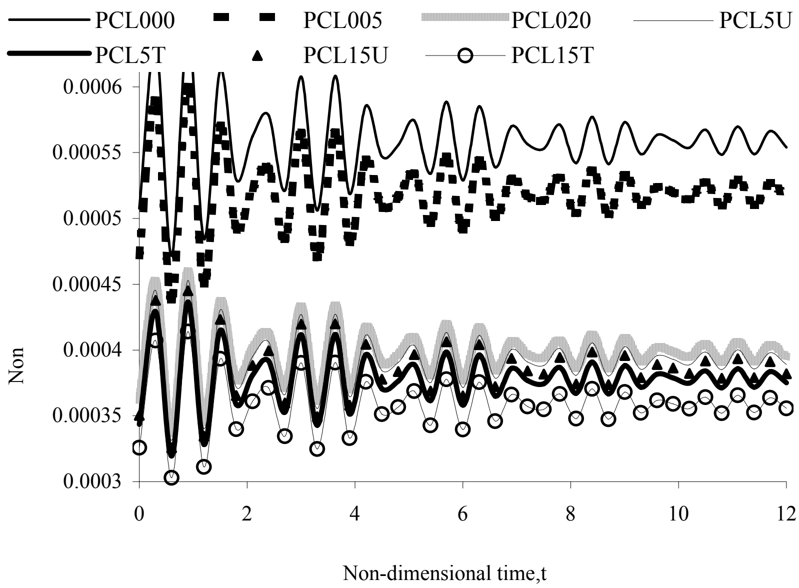


**Fig. 10:** Damped responses of a clamped edge HDPE, HDPE/PCL blends, treated and untreated banana fiber reinforced composite plate at damping coefficient factor $C_v$ =1.25

**Case Study 2: Natural frequency of chemically modified randomly distributed short banana fiber reinforced high-density polyethylene/ polycaprolactone composites plate.**

Randomly distributed short banana fiber reinforced high-density polyethylene/ polycaprolactone composites plate behaves similarly to isotropic plate. Elasticity is the same in all the direction in this composite plate; therefore governing differential equation is same. The governing differential equation for a free flexural vibration of a uniform thin isotropic plate in non-dimensional form is written as follows:

$$\frac{1}{a^4}\left(\frac{\partial^4}{\partial x^4}+2R^2\frac{\partial^4}{\partial x^2\partial y^2}+R^4\frac{\partial^4}{\partial y^4}\right)w = \lambda^4 w(x, y) \quad (25)$$

$$\lambda^4 = \omega^2 \rho_0 h / D \quad (26)$$

where w is displacement, $\rho_0$ is surface density, $\lambda$ and $\omega$ are eigen values and natural frequency respectively. The flexural rigidity $D = Eh^3 / 12(1-\nu^2)$, where *E* is Young's modulus of the coir/epoxy micro composite plate, $\nu$ is Poisson ratio and *h* is the plate thickness. The natural frequency of simply supported isotropic plate is obtained by Ashton and Whitney /29/ as

$$\omega_{mn} = \frac{\pi^2}{R^2 b^2} \sqrt{\frac{D}{\rho_0}} \sqrt{m^4 + 2m^2 n^2 R^2 + n^4 R^4} \tag{27}$$

where *m* and *n* are integers. Different natural frequencies are obtained at different combinations of *m* and *n*. Substituting the property values of pure HDPE, HDPE/PCL blend and short banana fiber reinforced high-density polyethylene/ polycaprolactone composites plate in equation 27. The natural frequencies of free vibration at simple supported boundary conditions have been evaluated by Ashton and Whitney /29/. After that, MQRBF method is applied in equation 25 to find out natural frequencies at simple supported boundary conditions. After analyzing Tables 21-27, it is observed that as soon as quantity of the PCL increases, the value of the natural frequencies increases and the value of the natural frequency is higher in treated banana fiber reinforced composite plate compared to untreated banana fiber reinforced composite plate.

**Table 21**

Lowest four natural frequencies of simple supported pure HDPE (PCL000) plate

| **q = 10 (N/m$^2$)**<br>**a = b = 300 (mm)**<br>**h = 10 (mm)** | **Mode** | **m** | **n** | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **Density = 0.98 gm/cm$^3$** | 1st | 1 | 1 | **26.270** | **26.322** |
| | 2nd | 1 | 2 | **65.675** | **66.331** |
| | 3rd | 2 | 1 | **65.675** | **66.331** |
| | 4th | 2 | 2 | **105.079** | **105.592** |

**Table 22**

Lowest four natural frequencies of simple supported HDPE/PCL blend (PCL005) plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)** **a = b = 300 (mm)** **h = 10 (mm)** **Density = 0.9871 gm/cm$^3$** | 1st | 1 | 1 | **27.1466** | **27.2009** |
| | 2nd | 1 | 2 | **67.8665** | **68.5452** |
| | 3rd | 2 | 1 | **67.8665** | **68.5452** |
| | 4th | 2 | 2 | **108.5865** | **109.1158** |

**Table 23**

Lowest four natural frequencies of simple supported HDPE/PCL blend (PCL020) plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)** **a = b = 300 (mm)** **h = 10 (mm)** **Density =1.0091 gm/cm$^3$** | 1st | 1 | 1 | **30.6590** | **30.7204** |
| | 2nd | 1 | 2 | **76.6476** | **77.4141** |
| | 3rd | 2 | 1 | **76.6476** | **77.4141** |
| | 4th | 2 | 2 | **122.6362** | **123.2340** |

**Table 24**

Lowest four natural frequencies of simple supported untreated banana fiber reinforced HDPE/PCL blend (PCL5U) composite plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)** **a = b = 300 (mm)** **h = 10 (mm)** **Density = 1.0227 gm/cm$^3$** | 1st | 1 | 1 | **30.6851** | **30.7465** |
| | 2nd | 1 | 2 | **76.7128** | **77.4800** |
| | 3rd | 2 | 1 | **76.7128** | **77.4800** |
| | 4th | 2 | 2 | **122.7406** | **123.3389** |

**Table 25**

Lowest four natural frequencies of simple supported treated banana fiber reinforced HDPE/PCL blend (PCL5T) composite plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)<br>a = b = 300 (mm)<br>h = 10 (mm)<br>Density = 1.0246 gm/cm$^3$** | 1st | 1 | 1 | **31.2222** | **31.2846** |
| | 2nd | 1 | 2 | **78.0555** | **78.8360** |
| | 3rd | 2 | 1 | **78.0555** | **78.8360** |
| | 4th | 2 | 2 | **124.8888** | **125.4976** |

**Table 26**

Lowest four natural frequencies of simple supported untreated banana fiber reinforced HDPE/PCL blend (PCL 15U) composite plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)<br>a = b = 300 (mm)<br>h = 10 (mm)<br>Density = 1.0472 gm/cm$^3$** | 1st | 1 | 1 | **30.5852** | **30.6463** |
| | 2nd | 1 | 2 | **76.4629** | **77.2275** |
| | 3rd | 2 | 1 | **76.4629** | **77.2275** |
| | 4th | 2 | 2 | **122.3406** | **122.9370** |

**Table 27**

Lowest four natural frequencies of simple supported treated banana fiber reinforced HDPE/PCL blend (PCL15T) composite plate

| | Mode | m | n | Analytical Method /29/ | MQRBF Method |
|---|---|---|---|---|---|
| **q = 10 (N/m$^2$)<br>a = b = 300 (mm)<br>h = 10 (mm)<br>Density = 1.0528 gm/cm$^3$** | 1st | 1 | 1 | **31.6343** | **31.6975** |
| | 2nd | 1 | 2 | **79.0857** | **79.8766** |
| | 3rd | 2 | 1 | **79.0857** | **79.8766** |
| | 4th | 2 | 2 | **126.5371** | **127.1540** |

## 5. CONCLUSIONS

Chemically modified randomly distributed short banana fiber reinforced HDPE/PCL composites plate has been prepared to evaluate the mechanical properties. Tensile strength, stiffness and damping properties greatly improved after blending PCL with HDPE. Tremendous changes occur in mechanical properties when banana fibers are treated with chemical. HDPE, HDPE/PCL blend, treated and untreated banana fibers reinforced composite plate show different mechanical behavior at static and dynamic loading. When the quantity of the treated banana fibers is less as compared to matrix in composite plate, damping properties are more than untreated banana fibers reinforced composite plate; but, as soon as the quantity of the treated banana fibers increases in composite plate, damping properties decrease. Due to incorporation of fibers in matrix, tensile properties and stiffness increase but damping properties decrease. The natural frequencies are determined for the self-weight of the composite plate. It was found that MQRBF method is more effective in static and dynamic response studies of the composite plates.

## APPENDIX

### A. Multiquadric radial basis function method for governing differential equation

Substitution of multiquadric radial basis function in equation (21) gives

$$\left(\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^4}\varphi_j + 2R^2\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^2 y^2}\varphi_j + \sum_{j=1}^{n} R^4 w_j \frac{\partial^4}{\partial y^4}\varphi_j\right) + \frac{\partial^2 w}{\partial t^2} - C_v\frac{\partial w}{\partial t} - Q = 0 \tag{28}$$

Substitution of multiquadric radial basis function in equation (25) gives

$$\frac{1}{a^4}\left(\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^4}\varphi_j + 2R^2\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^2 y^2}\varphi_j + R^4\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial y^4}\varphi_j\right) = \lambda^4\sum_{j=1}^{N} w_j\varphi_j \tag{29}$$

$$\frac{1}{a^4}\sum_{j=1}^{N} w_j \left(\frac{\partial^4}{\partial x^4}\varphi_j + 2R^2\frac{\partial^4}{\partial x^2 y^2}\varphi_j + R^4\frac{\partial^4}{\partial y^4}\varphi_j\right) = \lambda^4 \sum_{j=1}^{N} w_j\ \varphi_j \quad (30)$$

***For simple supported edge***

$$x = 0, a \qquad \sum_{j=1}^{N} w_j\varphi_j = 0 \quad (31a)$$

$$y = 0, b \qquad \sum_{j=1}^{N} w_j\varphi_j = 0 \quad (31b)$$

$$x = 0, a \qquad \sum_{j=1}^{N} w_j \frac{\partial^2}{\partial x^2}\varphi_j = 0 \quad (31c)$$

$$y = 0, b \qquad \sum_{j=1}^{N} w_j \frac{\partial^2}{\partial y^2}\varphi_j = 0 \quad (31d)$$

*For clamped edge*

$$x = 0, a \qquad \sum_{j=1}^{N} w_j\varphi_j = 0 \quad (32a)$$

$$y = 0, b \qquad \sum_{j=1}^{N} w_j\varphi_j = 0 \quad (32b)$$

$$x = 0, a \qquad \sum_{j=1}^{N} w_j \frac{\partial}{\partial x}\varphi_j = 0 \quad (32c)$$

$$y = 0, b \qquad \sum_{j=1}^{N} w_j \frac{\partial}{\partial y}\varphi_j = 0 \quad (32d)$$

**B. Multiple regression analysis**

$$A.a = p$$

where $A$ is $(l*k)$ coefficient matrix, $a$ is (k*1) vector, $p$ is $(l*1)$ load vector. Approximating the solution by introducing the error vector $e$, we get

$$p = Aa + e$$

where $e$ is $(l*1)$ vector. To minimize the error norm, let us define a function S as $S(a) = e^T e = (p - Aa)^T (p - Aa)$

The least-square norm must satisfy

$$(\partial S / \partial a)_a = -2A^T p + 2A^T Aa = 0$$

This can be expressed as

$$a = (A^T A)^{-1} A^T P$$

or

$$a = B.P$$

The matrix *B* is evaluated once and stored for subsequent usage.

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