國立台灣大學應用力學研究所碩士論文

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On Hadamard Principal Value and Boundary Integral Formulation of Fracture Mechanics

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The general solution is thus

$$\phi(s) = C_1\phi_1(s) + C_2\phi_2(s) + \phi^*(s)$$

where C_1, C_2 are undetermined constants and $\phi^*(s)$ is a particular solution. In order to determine the two constants, the corresponding physical problem has two auxiliary (subsidiary) conditions $\phi(\pm\ell) = 0$ to secure a unique solution. The subsidiary condition in our problem just means no crack extension at the crack tip.

Now we proceed to answer the question of how to evaluate the Hadamard principal value. First, let us look into the one-dimensional Hadamard principal value (58). When the two-dimensional, explicit expression of M_{kp} given in Sec.VI is used, the crucial difficulty of the evaluation of Eq.(34) lies in how to evaluate

$$\text{H.P.V.} \int_a^c \frac{\phi(s)}{(x-s)^2}\, ds$$

$$= -\frac{d}{dx}\, \text{C.P.V.} \int_a^c \frac{\phi(s)}{(x-s)}\, ds$$

$$= \lim_{y \to 0} \int_a^c k(x,y,s)\phi(s)\, ds$$

$$= \text{H.P.V.} \int_a^c \lim_{y \to 0} k(x,y,s)\phi(s)\, ds$$

where $\phi(s)$ is an unknown density function and $k(x,y,s)$ is a kernel function with the asymptotic property

$$\lim_{y \to 0} k(x,y,s) = \frac{1}{(x-s)^2}$$

For later usage we also define $\bar{k}(x,y,s)$ by

$$\frac{\partial \bar{k}(x,y,s)}{\partial s} = k(x,y,s)$$

such that

$$\lim_{y\to 0} \bar{k}(x,y,s) = \frac{1}{(x-s)}$$

With these we are ready to provide three methods for evaluation of the Hadamard principal value (58) as $a < x < c$. They are often called the regularization of divergent integral.