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Inverse identification of boundary conditions for potential problems of thin body using BEM

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Abstract: The analytical integral formulas and singular value decomposition were employed to treat the 2D potential inverse boundary condition determination problems of thin body. The Cauchy potential inverse problems were considered. The nearly singular integrals in the boundary element method (BEM) of thin body problems were dealt with by the analytical integral formulas. The system equation was solved by singular value decomposition technique. Thin body numerical examples with the thickness-to-length ratio down to 1E−8 or 1E−7 demonstrate the effectiveness and accuracy of the present algorithm.

Key words: inverse potential problem; boundary element method (BEM); nearly singular integral; analytical integral; singular value decomposition

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薄体位势边界条件识别反问题正则化边界元方法

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摘要:针对二维薄体位势柯西边界条件识别反问题,提出了解析积分和奇异值分解联合正则化算法.解析积分用于薄体位势问题边界元法中几乎奇异积分的正则化.奇异值分解技术用来求解系统方程.数值算例研究了狭长比为1E−8和1E−7的薄体问题,计算结果表明该算法的有效性和精确性.

关键词:反位势问题;边界元法;几乎奇异积分;解析积分;奇异值分解

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0 Introduction

The boundary element method (BEM), also named boundary integral equation method (BIE), is an important numerical method, which has continually attracted the attention of researchers and engineers and has been increasingly applied to different types of engineering problems[1].

On the one hand, it is well known that there are singular integrals and nearly singular integrals arising in the BEM. Earlier researches on the regularization of singular integrals were reviewed by Tanaka et al.[2]. The principal strategies and the related references developed to deal with the various kinds of singular integrals were introduced by Sladek, Gray, Guiggiani et al.[3]. Unlike singular integrals, nearly singular integrals are not singular in the sense of mathematics. Conventional Gauss numerical quadrature is invalid for evaluating these integrals. Nearly singular integrals usually exist in two situations. One is known as boundary layer effect problem when the physical quantities at the interior points very close to the boundary are calculated; the other is named thin body effect problem when the thickness of the considered domain is small. If the physical quantities at interior points of a thin domain are calculated, boundary layer effect problems and thin body effect problems occur together.

The boundary element analysis of problems with thin bodies has consequentially been carried out recently. Liu et al.[4~6] has undertaken research in this field. The size of elements in the thickness direction can be too smaller than that of elements in the length direction. So the total number of boundary elements is not very large. The nearly singular surface integrals are transformed into a sum of weakly singular integrals and non-singular line integrals, and two shell-like thin structures were investigated using the 3D elastic BEM model[4]. The nearly singular line integrals are transformed into function evaluations at the two end points of the element of integration for 2D elastic thin structures, and a new nonlinear coordinate transformation was developed for nearly weakly-singular integrals. Then very promising results were obtained for the examples with the thickness-to-length ratio in the orders of 1E−6 to 1E−9[5]. The theory was also applied to interfacial stress analysis for multi-coating systems[6].

Semi-analytical or analytical integral algorithm proposed by Niu et al.[7,8] is used to solve elasticity problems with thin structures. The nearly singular surface integrals are transformed into a series of line integrals along the contour of the 3-point triangular element, and the singular factor is separated from the remaining regular integrals. Consequently standard Gaussian quadrature can provide very accurate evaluation of the resulting line integrals[7]. A set of general analytical integral formulas were derived in the BEM for 2D elasticity. The formulas were derived with integration by parts for linear boundary elements[8]. New direct analytical integral formulas for orthotropic potential problems were derived by Zhou et al.[9], which are also applied to solve problems of thin body with thickness-to-length ratios down to 1E−8[10]. Problems with boundary layer effect or thin body effect in the BEM were systematically studied[11].

There are other analytical integral expressions derived for 2D isotropic potential problems[12,13], for 2D anisotropic potential problems[14], 2D elastic problems[15], and 3D elastic problems[16], respectively. These analytical integrations have potentials to effectively treat nearly singular integrals.

On the other hand, inverse engineering problems are becoming more and more important in BEM[17]. It often occurs that all the potentials and fluxes are known on a part of the boundary and no boundary data can be directly measured on the rest of the boundary in engineering for potential problems. This is Cauchy inverse problem. Inverse problems are in general unstable in the sense that small noise in the input data may amplify significantly the errors in the solution. So the regularization techniques are very important to the inverse numerical analysis[18].

The mathematical mechanism of the ill-posed Cauchy problem for the Laplace equation was analyzed by Chen[19]. An iterative method was applied to solve Cauchy inverse problems for the Laplace equation by Lesnic et al.[20]. The approximate solutions to the Cauchy problem in linear elasticity were determined by Marin et al., by using an alternating iterative BEM which reduces the problem to a sequence of well-posed boundary value problems[21]. Truncated singular value decomposition (TSVD) was also proposed to solve the same problem[22].

In this paper, the boundary potentials and fluxes on a part of boundary need be inversely identificatified by using the over-specified boundary conditions on the remaining boundary in the BEM for 2D potential problems of thin body. The corresponding direct potential problem of thin body is analyzed in Ref. [11]. The completely analytical integral algorithm is applied to evaluate the nearly singular integrals. The analytical integral formulas are directly derived with integration by parts. The system equation is solved by singular value decomposition technique. The unknown potential and flux boundary conditions for thin structures with very small thickness-to-length ratios are accurately calculated.

1 The boundary integral equations and their discretized integrals

Consider two-dimensional potential problems. For an interior point $\boldsymbol{y}$, the boundary integral equation can be given as

$$u(\boldsymbol{y}) = \int_\Gamma u^*(\boldsymbol{x},\boldsymbol{y})q(\boldsymbol{x})\mathrm{d}\Gamma - \int_\Gamma q^*(\boldsymbol{x},\boldsymbol{y})u(\boldsymbol{x})\mathrm{d}\Gamma \tag{1}$$

in which $u^*(\boldsymbol{x},\boldsymbol{y}) = -\frac{1}{2\pi}\ln r$ is the fundamental solution of 2D potential problems, $q^*(\boldsymbol{x},\boldsymbol{y}) = -\frac{1}{2\pi r}r_{,i}n_i$ is the potential gradient with respect to an outward normal to the boundary, where r is the distance between the source point $\boldsymbol{y}$ and arbitrary field point $\boldsymbol{x}$ on the boundary Γ. When the source point $\boldsymbol{y}$ is on the boundary Γ, the boundary integral equation for a boundary node can be written as

$$C(\boldsymbol{y})u(\boldsymbol{y}) = \int_\Gamma u^*(\boldsymbol{x},\boldsymbol{y})q(\boldsymbol{x}) - \int_\Gamma q^*(\boldsymbol{x},\boldsymbol{y})u(\boldsymbol{x})\mathrm{d}\Gamma \tag{2}$$

where $C(\boldsymbol{y})$ is the boundary singular coefficient, which is determined by the boundary geometry characterization.

For a direct potential problem, if the boundary conditions are sufficiently given, the unknown boundary conditions, such as potential and potential gradient with respect to boundary outward normal, can be calculated by discretizing Eq. (2) based on the forward method. Gaussian elimination can give satisfactory results for the system equation.

The potential gradients q_k at the interior point $\boldsymbol{y}$ with respect to directions x_k can be obtained by differentiating Eq. (1)

$$q_k(\boldsymbol{y}) = \int_\Gamma \frac{\partial u^*(\boldsymbol{x},\boldsymbol{y})}{\partial y_k}q(\boldsymbol{x})\mathrm{d}\Gamma - \int_\Gamma \frac{\partial q^*(\boldsymbol{x},\boldsymbol{y})}{\partial y_k}u(\boldsymbol{x})\mathrm{d}\Gamma \quad (k=1,2) \tag{3}$$

The boundary is assumed to be divided into N linear elements. Taking the linear interpolation functions $N_1=(1-\xi)/2$ and $N_2=(1+\xi)/2$, the influence coefficients of the discretized integrals of Eqs. (1)~(2) are, respectively

$$\left.\begin{aligned}\int_{\Gamma_j} u^*(\boldsymbol{x},\boldsymbol{y})N_k\mathrm{d}\Gamma = -\frac{1}{2\pi}\int_{\Gamma_j}\ln rN_k\mathrm{d}\Gamma \\ (k=1,2)\end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned}\int_{\Gamma_j} q^*(\boldsymbol{x},\boldsymbol{y})N_k\mathrm{d}\Gamma = -\frac{1}{2\pi}\int_{\Gamma_j}(r_{,i}n_iN_k/r)\mathrm{d}\Gamma \\ (i,k=1,2)\end{aligned}\right\} \tag{5}$$

Similar to Eqs. (4) and (5), the integral influence coefficients for the discretized form of Eq. (3) can be written as

$$\left.\begin{aligned}\int_{\Gamma_j} \frac{\partial u^*(\boldsymbol{x},\boldsymbol{y})}{\partial y_k}N_l\mathrm{d}\Gamma = \frac{1}{2\pi}\int_{\Gamma_j}\frac{1}{r}r_{,k}N_l\mathrm{d}\Gamma \\ (k,l=1,2)\end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned}\int_{\Gamma_j}\frac{\partial q^*(\boldsymbol{x},\boldsymbol{y})}{\partial y_k}N_l\mathrm{d}\Gamma = \frac{1}{2\pi}\int_{\Gamma_j}\left(\frac{1}{r}r_{,m}n_m\right)_{,k}N_l\mathrm{d}\Gamma\\ (k,m,l=1,2)\end{aligned}\right\} \tag{7}$$

where $r_{,k}=\partial r/\partial x_k=-\partial r/\partial y_k$.

When the considered domain is a thin structure, some boundary nodes are very close to some boundary elements. The boundary integrals in Eqs. (4)～(5) have near singularity. The conventional Gaussian quadrature is invalid to evaluate these integrals. If these nearly singular integrals can not be treated effectively, the boundary unknowns can not be accurately calculated. This problem also exists in inverse problems. Thereinafter, the nearly singular integrals for Eqs. (4)～(7) are treated with the analytical integral algorithm.

2　The evaluation of nearly singular integrals

The linear interpolation is employed to define geometry characteristic and physics quantities on a linear element. Let the coordinates of the first node I and the last node F of the element be (x_{1I}, x_{2I}) and (x_{1F}, x_{2F}), respectively. The coordinate system Ox_1x_2 of the element is transformed into the intrinsic coordinate system. The transformation equation is given by

$$\left.\begin{aligned}x_i = (s_i/2)\xi + (x_{iF}+x_{iI})/2\\ (\xi\in[-1,1], i=1,2)\end{aligned}\right\} \tag{8}$$

where $s_i=x_{iF}-x_{iI}$. Denoting $\varepsilon_i=(x_{iF}+x_{iI})/2-y_i$ and $r_i=x_i-y_i$, we have

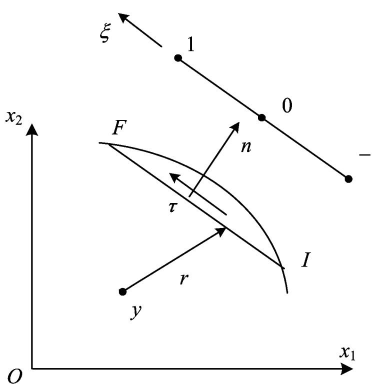

Fig. 1　Linear element

$$r_{,i}=r_i/r=[(s_i/2)\xi+\varepsilon_i]/r \tag{9}$$

$$R=r^2=r_ir_i=a\xi^2+b\xi+c=a[(\xi-d)^2+e^2] \tag{10a}$$

For a boundary node, when it is located on the extension of a linear element, e equals zero, Eq. (10a) can be written as

$$R=r^2=r_ir_i=a\xi^2+b\xi+c=a(\xi-d)^2 \tag{10b}$$

In the above two equations

$$\left.\begin{aligned}&a=s^2/4,\ b=s_i\varepsilon_i,\ c=\varepsilon_i\varepsilon_i,\\ &\delta=\sqrt{4ac-b^2},\ d=-b/(2a),\ e=2\delta/s^2\end{aligned}\right\} \tag{11}$$

The parameters a,b,c,d and e are defined by the coordinates of the nodes of the element and the source point $\boldsymbol{y}$.

The integral kernels of Eqs. (4)～(7) can be written in the following forms, respectively

$$\ln rN_k=\ln\sqrt{R}N_k\quad(k=1,2) \tag{12}$$

$$r_{,i}n_iN_k/r=r_in_iN_k/r^2=r_in_iN_k/R\quad(i,k=1,2) \tag{13}$$

$$r_{,k}N_l/r=r_kN_l/r^2=r_kN_l/R\quad(k,l=1,2) \tag{14}$$

$$\left(\frac{1}{r}r_{,m}n_m\right)_{,k}=\frac{n_kN_l}{r^2}-\frac{2r_mn_mr_kN_l}{r^4}=\frac{n_kN_l}{R}-\frac{2r_mn_mr_kN_l}{R^2}\quad(k,m,l=1,2) \tag{15}$$

Note that $\mathrm{d}\Gamma=\frac{s}{2}\mathrm{d}\xi$ for linear elements. So the integrals in Eq. (4) contain the following form

$$I_0=\int_{-1}^{1}\ln\sqrt{R}N_k\mathrm{d}\xi \tag{16}$$

The integrals in Eqs. (5)～(7) contain the following two forms

$$I_1=\int_{-1}^{1}\frac{P_1}{R}\mathrm{d}\xi,\ I_2=\int_{-1}^{1}\frac{P_2}{R^2}\mathrm{d}\xi \tag{17}$$

The corresponding polynomials P_1 and P_2 in Eqs. (5)～(7) can be expressed as

$$P_{1a}=n_ir_iN_k,\ P_{1b}=r_kN_l,\ P_{1c}=n_kN_l \tag{18}$$

$$P_2=2r_mn_mr_kN_l \tag{19}$$

Taking derivatives of P_1 and P_2 with respect to ξ, it is obvious that

$$P'''_{1a}=P'''_{1b}=P'''_{1c}=0,\ P_2^{(4)}=0 \tag{20}$$

When a source point approaches boundary,

$r\to 0$ and $R\to 0$, standard Gaussian quadrature can not exactly calculate these integrals. The analytical expressions of these singular integrals are derived as follows[11]

Case 1 $e\neq 0$ ($\delta\neq 0$)

$$I_0=\int_{-1}^{1}(\ln\sqrt{R})N_1\mathrm{d}\xi=\int_{-1}^{1}\ln\sqrt{a\xi^2+b\xi+c}\,\frac{1-\xi}{2}\mathrm{d}\xi=\left\{\frac{-(4a+b)\xi}{8a}+\frac{\xi^2}{8}+\frac{(-2ab^2-b^3+8a^2c+4abc)g}{8a^2\delta}-\frac{(-2+\xi)\xi\ln\sqrt{R}}{4}+\frac{(2ab+b^2-2ac)\ln R}{16a^2}\right\}\Bigg|_{\xi=-1}^{1} \tag{21a}$$

$$I_0=\int_{-1}^{1}(\ln\sqrt{R})N_2\mathrm{d}\xi=\int_{-1}^{1}\ln\sqrt{a\xi^2+b\xi+c}\,\frac{1+\xi}{2}\mathrm{d}\xi=\left\{\frac{(-4a+b)\xi}{8a}-\frac{\xi^2}{8}+\frac{(-2ab^2+b^3+8a^2c-4abc)g}{8a^2\delta}+\frac{(2+\xi)\xi\ln\sqrt{R}}{4}+\frac{(2ab-b^2+2ac)\ln R}{16a^2}\right\}\Bigg|_{\xi=-1}^{1} \tag{21b}$$

$$I_1=\int_{-1}^{1}\frac{p_1(\xi)}{R(\xi)}\mathrm{d}\xi=\int_{-1}^{1}\frac{P_1(\xi)}{a\xi^2+b\xi+c}\mathrm{d}\xi=\left\{\frac{2g}{\delta}P_1(\xi)-\left(\frac{R'}{a\delta}g-\frac{\ln R}{2a}\right)P'_1(\xi)+\frac{2g(bR'-2ac+2a^2\xi^2)+\delta(2a\xi-R'\ln R)}{4a^2\delta}\cdot P''_1(\xi)\right\}\Bigg|_{\xi=-1}^{1} \tag{22}$$

$$I_2=\int_{-1}^{1}\frac{p_2(\xi)}{[R(\xi)]^2}\mathrm{d}\xi=\int_{-1}^{1}\frac{P_2(\xi)}{(a\xi^2+b\xi+c)^2}\mathrm{d}\xi=\left\{\left(\frac{R'}{\delta^2R}+\frac{4ag}{\delta^3}\right)P_2(\xi)-\left(\frac{2bg}{\delta^3}+\frac{4a\xi g}{\delta^3}\right)P'_2(\xi)+\left[-\frac{\xi}{\delta^2}+\frac{2cg}{\delta^3}+\frac{2(b\xi+a\xi^2)g}{\delta^3}\right]P''_2(\xi)-\left[-\frac{b\xi}{6a\delta^2}-\frac{2\xi^2}{3\delta^2}-\frac{(b^3-6abc)g}{6a^2\delta^3}+\frac{\xi g(6c+3b\xi+2a\xi^2)}{3\delta^3}-\frac{\ln R}{12a^2}\right]P'''_2(\xi)\right\}\Bigg|_{\xi=-1}^{1} \tag{23}$$

in which $g=\arctan(R'/\delta)$, where $R'=2a\xi+b$.

Case 2 $e=0$ ($\delta=0$)

For this case, the complete analytical expressions of integral I_0 and I_1 can also be deduced as

$$I_0=\int_{-1}^{1}(\ln\sqrt{R})N_1\mathrm{d}\xi=\int_{-1}^{1}\ln\sqrt{a(\xi-d)^2}\,\frac{1-\xi}{2}\mathrm{d}\xi=\left\{\frac{(d-1)(\xi-d)}{2}+\frac{(\xi-d)^2}{8}-\frac{(\xi-d)(\xi+d-2)\ln\sqrt{R}}{4}\right\}\Bigg|_{\xi=-1}^{1} \tag{24a}$$

$$I_0=\int_{-1}^{1}(\ln\sqrt{R})N_2\mathrm{d}\xi=\int_{-1}^{1}\ln\sqrt{a(\xi-d)^2}\,\frac{1+\xi}{2}\mathrm{d}\xi=\left\{\frac{(-d-1)(\xi-d)}{2}-\frac{(\xi-d)^2}{8}+\frac{(\xi-d)(\xi+d+2)\ln\sqrt{R}}{4}\right\}\Bigg|_{\xi=-1}^{1} \tag{24b}$$

$$I_1=\int_{-1}^{1}\frac{P_1(\xi)}{R(\xi)}\mathrm{d}\xi=\int_{-1}^{1}\frac{P_1(\xi)}{a(\xi-d)^2}\mathrm{d}\xi=\frac{1}{a}\left\{\frac{1}{d-\xi}P_1(\xi)+P'_1(\xi)\ln|\xi-d|-P''_1(\xi)[(\xi-d)\ln|\xi-d|-\xi]\right\}\Bigg|_{\xi=-1}^{1} \tag{25}$$

3 Singular value decomposition

There exists a very powerful set of techniques for dealing with sets of equations or matrices that are either singular or numerically very close to being singular. In many cases where Gaussian elimination and LU decomposition fail to give satisfactory results, this set of techniques, known as singular value decomposition[23,24], will diagnose precisely what the problem is. In some cases, SVD will not only diagnose the problem, but also give a useful numerical answer.

SVD is based on the following theorem of linear algebra: Any $M\times N$ matrix $\boldsymbol{A}$ whose number of rows M is greater than or equal to its number of columns N, can be written as the product of an

$M \times N$ column-orthogonal matrix $\boldsymbol{U}$, an $N \times N$ diagonal matrix $\boldsymbol{W}$ with positive or zero elements (the singular values), and the transpose of an $N \times N$ orthogonal matrix $\boldsymbol{V}$.

$$\boldsymbol{A} = \boldsymbol{UWV}^{\mathrm{T}} \tag{26}$$

where $\boldsymbol{U} = (\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_N) \in \boldsymbol{R}^{M \times N}$ and $\boldsymbol{V} = (\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_N) \in \boldsymbol{R}^{N \times N}$ are each orthogonal in the sense that their columns are orthonormal. $\boldsymbol{W} = \mathrm{diag}(w_1, w_2, \cdots, w_n)$, the singular values w_i are nonnegative and are typically written in a non-increasing order

$$w_1 \geqslant w_2 \geqslant \cdots \geqslant w_n \geqslant 0 \tag{27}$$

For Cauchy inverse problems, the system equation of discretized Eq. (2) is

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{28}$$

where matrix $\boldsymbol{A}$ has M rows and N columns. M equals the number of the boundary nodes or the linear elements. N equals the number of the unknown boundary potential and normal flux conditions.

In the ideal setting, without perturbations and rounding errors, the treatment of ill-conditioned system Eq. (28) is easy, i. e. the SVD components associated with zero singular values can be ignored. Therefore, by applying SVD techniques, the minimum norm least square solution to Eq. (28) can be expressed as using the Moore-Penrose generalized inverse $\boldsymbol{A}^{+}$.

$$\boldsymbol{x} = \boldsymbol{A}^{+}\boldsymbol{b} = \sum_{i=1}^{\mathrm{rank}(\boldsymbol{A})} \frac{\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{b}}{w_i}\boldsymbol{v}_i \tag{29}$$

In practice, $\boldsymbol{A}$ is never exactly mathematically rank deficient, but instead numerically rank deficient, i. e. it has one or more small nonzero singular values w_i for some i greater than k, $1 < k < N$. The very small singular values inevitably give rise to the error and undulation of the solution $\boldsymbol{x}$.

Fortunately, the condition number of matrix $\boldsymbol{A}$ of the system equation is not large for the thin body numerical examples considered in the next section, which does not have very small singular values. Therefore no very small singular values need be truncated. The ordinary SVD technique can provide accurate results.

4 Numerical examples and conclusion

Example 1 Heat conduction in a thin rectangular domain is shown in Fig. 2. The length of Side AB and CD of the rectangle is $a = 10$ m. The width of the rectangle is $b = 1.0\mathrm{E}-7$ m. Both temperature and flux conditions on Side AB, BC and CD are specified. Neither temperature nor the flux conditions on Side DA are known. 10 uniform linear elements are divided on each side of the rectangle. Each element on Side AB and CD is 1 m long, and each element on Side BC and DA is 1.0E−8 m long.

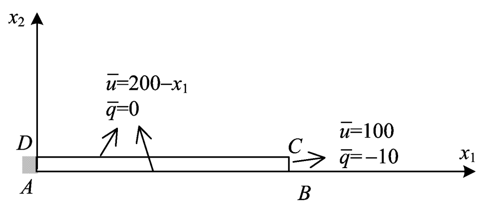

Fig. 2 Heat conduction in a thin rectangular domain

For this thin body problem, the thickness-to-length ratio is defined as b/a, thus the ratio equals 1E−8.

For the nodes on Side AD, Fig. 3 and Fig. 4 present the numerical temperature and flux results and their exact solutions, respectively. It can be seen that the accurate results of the temperature and flux are obtained by the present BEM, in which the nearly singular integrals are treated by the analytical integral formulas. The conventional BEM can not obtain accurate results due to the invalid standard Gauss quadrature.

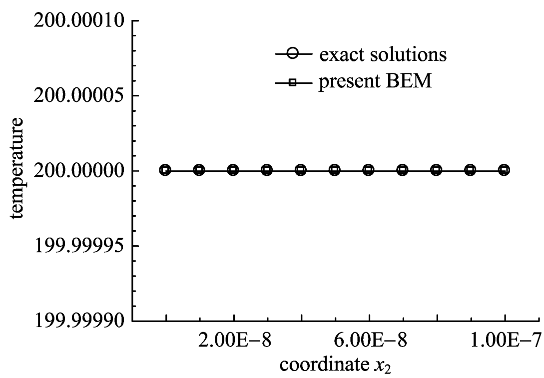

Fig. 3 Temperature solutions on Side AD

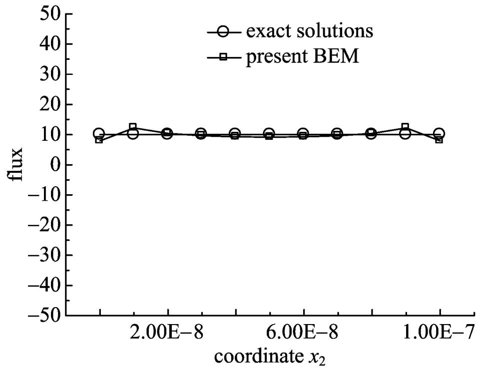

Fig. 4 Flux solutions on Side *AD*

Example 2 Heat conduction in a thin ring is shown in Fig 5. The inner and outer radii of the ring are a and b, respectively. $a=10$ m, $b=10.000\,001$ m. The temperature and flux on the outer surface are 10 and $-9\,999\,999.5$. The temperature and flux on the inner surface are unknown. 120 linear elements are divided on the inner and outer boundaries.

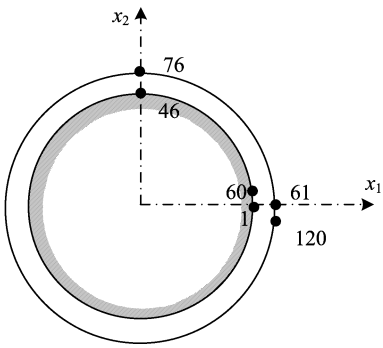

Fig. 5 Heat conduction in a thin ring

For this thin body problem, the thickness-to-length ratio is defined as $(b-a)/a$, thus the ratio equals 1E−7.

Fig. 6 shows the numerical temperature results and their exact results. Fig. 7 shows the numerical flux results and their exact solutions. It can be seen that the very accurate results of the temperature and flux are obtained by the present method. Both the temperature and flux errors are less than 0.1%.

The analytical integral algorithm is applied to evaluate the nearly singular integrals in the BEM of 2D potential problems of thin bodies. The system equation is solved by singular value

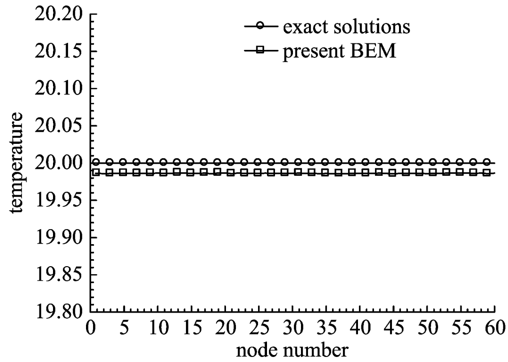

Fig. 6 Temperature solutions on inner surface

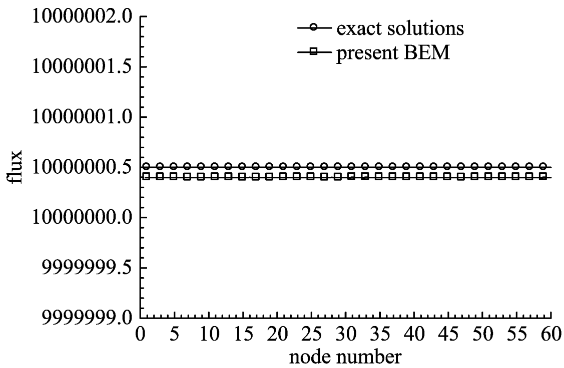

Fig. 7 Flux solutions on inner surface

decomposition technique. The condition number of the coefficient matrix of the system equation for the thin body problem is not large. The singular values have no numerical rank-deficient phenomenon[23]. Therefore the ordinary singular value decomposition technique can provide accurate results for this kind of inverse problem. The unknown potentials and fluxes on boundary for thin structures with very small thickness-to-length ratios are accurately calculated. The algorithm is being applied to 2D anisotropic and 3D potential Cauchy inverse problems. Multidimensional elasticity inverse problems are also being studied.

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