Newton's Laws of Motion Based Substantial Aether Theory for Electro-Magnetic Wave

T.-W. Lin and H. Lin

Journal of Mechanics / Volume 30 / Issue 04 / August 2014, pp 435 - 442
DOI: 10.1017/jmech.2014.18, Published online: 13 March 2014

NEWTON'S LAWS OF MOTION BASED SUBSTANTIAL AETHER THEORY FOR ELECTRO-MAGNETIC WAVE

T.-W. Lin *

National Taiwan University
Taipei, Taiwan 10617, R.O.C.

H. Lin

General Motors Company
Pontiac, Michigan 48340, USA

ABSTRACT

Even though electro-magnetic wave can be calculated from Maxwell's equations, the cause of electro-magnetic waves has not been fully understood. This paper proposes a Newton's laws of motion based aether theory to derive identical results as those from Maxwell's equations for free field. The authors suggest that every aether particle has a mass and occupies a volume in space. Every aether particle has translational movement and particle spin movement. The translational movement is similar to the gas particle moving in the air and it does not produce an electro-magnetic wave. The particle spin movement generates shear and a spin wave that will be shown to have the same results as Maxwell's equations. Detailed derivation of electro-magnetic wave solutions from the proposed aether theory and Maxwell's equations is presented in this paper to show the validation of this model.

Keywords: Maxwell's equation, Substantial aether theory, Newton's laws of motion, Kinetic theory of gas, Electro-magnetic wave, Navier equation, Constitutive law for solid, Constitutive law for particle, Equation of motion.

1. INTRODUCTION

Since Isaac Newton [1], many physicists [2] considered the aether as a static massless medium. Therefore, Michelson and Morley [3], and many others tried to look for the absolute stilled aether from its relative motion to the Earth, but only got a small fraction of the predicted relative speed.

Miller then considered the aether as tangible with sufficient substance to be entrained at the Earth's surface. At Mount Wilson (6000' from sea level), from long term observations Miller [4] acquired 200,000 data points to prove the existence of the aether.

In this paper, we view the aether as a small particle with mass which was the same as Miller's experiments proved. The energy to keep the motion of the planet is provided by the Sun as an energy source. It is the aether with mass to transmit the energy to the planets. And the energy field provided by the Sun is stored in the aether inside the volume. The aether is the medium not only to transfer the energy but also to transmit the electro-magnetic wave and light.

Identical results of electro-magnetic wave from Maxwell's equations for free field can be obtained from constitution laws of aether particles considering two rotational motions. The constitution equation is extended from the well-known equation of motion for elastic solid discovered by Navier in 1827 [5]. Two terms are added to the constitution law to include self-spin and group rotation. The mechanical interpretation of Maxwell's equations had been studied by Jaswon in 1969 [6]. Jaswon followed the gyrostatic aether concept proposed by MacCullagh in 1837 [7]. A Derivation of Maxwell's equations from aether had also been proposed by Larson in 1998 [8]. But they only considered the effect of the group rotation, that is the curl of the translational displacement. As shown in the paper by Lin in 1996 [9], self-spin of particle about its center is important for relating magnetic force to electric force.

The wave equation for aether particle will be derived from its constitution equation in Cartesian coordinates similar to solving wave equation for solids. Newton's second law of motion is used with extended constitution equations of aether particles to get it's equations of motion. Then, the equations of motion are decoupled into deformation and rotation terms by taking divergence and curl of equations of motions. Similar to solve a vibration problem, an eigenvalue problem is solved for the sinusoid wave of electric force and magnetic force. Finally, the wave solutions are compared to Maxwell's equations to verify the aether theory.

2. CONSTITUTIVE LAW OF AETHER PARTICLES

The constitution equation for homogeneous isotropic solid medium relates stress strain as

* Corresponding author (twlin@ntu.edu.tw)

DOI : 10.1017/jmech.2014.18

$$\sigma_{ij} = K\delta_{ij}\,\epsilon_{kk} + 2\mu(\epsilon_{ij} - \frac{1}{3}\delta_{ij}\,\epsilon_{kk}) \tag{1}$$

where

σ_{ij} is stress tensor,

ϵ_{ij} is strain tensor,

δ_{ij} is Kronecker delta,

K is bulk modulus,

μ is shear modulus.

Bulk modulus K is related to pure volume change and hydro-static pressure. Shear modulus μ (also called bulk viscosity, volume viscosity or second viscosity) is related to shape distortion and shear stresses.

Rearrange Eq. (1) by combing terms of $\delta_{ij}\,\epsilon_{kk}$ and get

$$\sigma_{ij} = \lambda\delta_{ij}\,\epsilon_{kk} + 2\mu\epsilon_{ij} \tag{2}$$

where λ is Lame's constant and is defined as

$$\lambda = K - \frac{2}{3}\mu \tag{3}$$

For small deformation, the strain tensor can be related to displacement vector, u_i as

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \tag{4}$$

Define rotation strain tensor ω_{ij} and corresponding rotation vector ω_k as

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) \tag{5}$$

and

$$\omega_k = \frac{1}{2}e_{kij}\,\omega_{ij} \tag{6}$$

where

x_i is coordinate in i direction,

u_i is displacement in i direction,

e_{ijk} is permutation symbol and defined as: $e_{123} = e_{231} = e_{312} = 1$, $e_{321} = e_{132} = e_{213} = -1$, otherwise is 0.

The rotation strain tensor can be written in matrix form to show some inter-changeable relationship as

$$\omega_{ij} = \begin{bmatrix} 0 & \omega_{12} & \omega_{13} \\ \omega_{21} & 0 & \omega_{23} \\ \omega_{31} & \omega_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_{21} & -\omega_{31} \\ -\omega_{12} & 0 & -\omega_{32} \\ -\omega_{13} & -\omega_{23} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{7}$$

The rotation vector is related to the curl of the displacement vector. This relationship is shown in the following expansion of rotation vector in matrix form as

$$\vec{\omega} = \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \frac{1}{2}\begin{Bmatrix} \omega_{23} - \omega_{32} \\ \omega_{31} - \omega_{13} \\ \omega_{12} - \omega_{21} \end{Bmatrix} = \begin{Bmatrix} \omega_{23} \\ \omega_{31} \\ \omega_{12} \end{Bmatrix} = \begin{Bmatrix} -\omega_{32} \\ -\omega_{13} \\ -\omega_{21} \end{Bmatrix}$$
$$= \frac{1}{2}\begin{Bmatrix} \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} & 0 & -\frac{\partial}{\partial x_1} \\ -\frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$
$$= \frac{1}{2}\nabla \times \vec{u} \tag{8}$$

Stress-strain relationship for rotational medium is based on friction forces (with-out energy loss) from the rotational movement. The constitution equation for aether particle are shown in Fig. 2 and written as

$$\sigma_{ij} = -2\xi e_{ijk}(\dot{\omega}_k - \dot{\theta}_k) \tag{9}$$

Spin vector, $\dot{\vec{\theta}}$, represents the mean particle spin angular velocity, rotation vector, $\dot{\vec{\omega}}$, represents the volume rotation angular velocity. .

Equation (9) can be written in matrix form as

$$\sigma_{ij} = \begin{bmatrix} 0 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & 0 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 0 \end{bmatrix} = 2\xi\begin{bmatrix} 0 & -\dot{\omega}_3 & \dot{\omega}_2 \\ \dot{\omega}_3 & 0 & -\dot{\omega}_1 \\ -\dot{\omega}_2 & \dot{\omega}_1 & 0 \end{bmatrix} - 2\xi\begin{bmatrix} 0 & -\dot{\theta}_3 & \dot{\theta}_2 \\ \dot{\theta}_3 & 0 & -\dot{\theta}_1 \\ -\dot{\theta}_2 & \dot{\theta}_1 & 0 \end{bmatrix} \tag{10}$$

The constitution equation for aether particles considering stresses due to volume deformation, volume rotation and particle spin can be completed by combing Eqs. (2) and (9) as

$$\sigma_{ij} = \lambda\delta_{ij}\,\epsilon_{kk} + 2\mu\epsilon_{ij} - 2\xi e_{ijk}(\dot{\omega}_k - \dot{\theta}_k) \tag{11}$$

3. EQUATIONS OF MOTION

The equation of translational motion can be develop from Newton's second law of motion as shown in Fig. 3 and written as

$$\rho\Delta x_1\Delta x_2\Delta x_3\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}\Delta x_1\Delta x_2\Delta x_3 + b_i\,\Delta x_1\Delta x_2\Delta x_3 \tag{12}$$

where

b_i is body force vector,

ρ is mass density,

x_1, x_2, x_3 are coordinates.

Eliminate the volume, $(\Delta x_1\Delta x_2\Delta x_3)$, and simplify Eq. (12) as

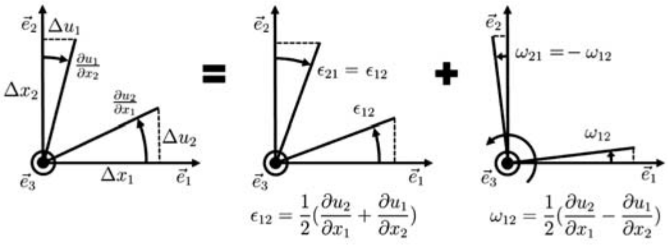

Fig. 1 Deformation strain and rotation strain from displacement

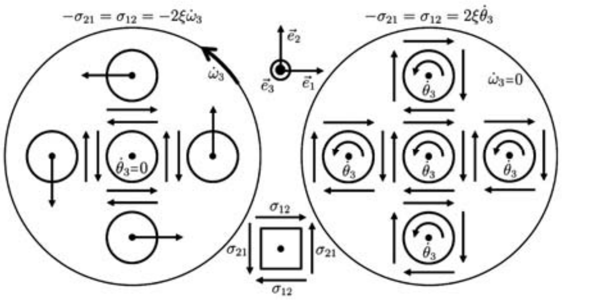

Fig. 2 Rotation stress-strain relationship

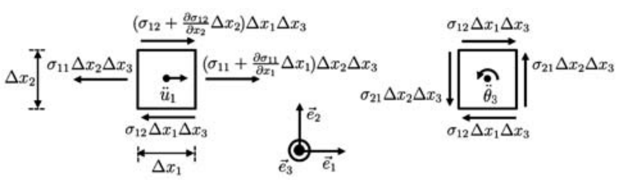

Fig. 3 Force and torque balance diagram

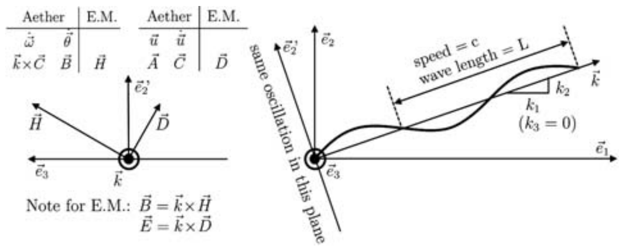

Fig. 4 Orthogonal oscillation vectors of plane wave

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{\partial\sigma_{ij}}{\partial x_j}+b_i \tag{13}$$

The equation of rotational motion can also be derived from Newton's second law of motion as shown in Fig. 3 and written as

$$\rho R^2\Delta x_1\Delta x_2\Delta x_3\,\frac{\partial^2\theta_k}{\partial t^2}=-e_{ijk}\sigma_{ij}\Delta x_1\Delta x_2\Delta x_3+q_k\Delta x_1\Delta x_2\Delta x_3 \tag{14}$$

where

q_k is body torque vector,

R is radius of gyration of a particle.

Eliminate volume ($\Delta x_1\Delta x_2\Delta x_3$) and simplify Eq. (14) as

$$\rho R^2\,\frac{\partial^2\theta_k}{\partial t^2}=-e_{ijk}\sigma_{ij}+q_k \tag{15}$$

For equations of motion in free field, the body force vector $b_i = 0$ and body torque vector $q_k = 0$ in Eqs. (13) and (15), and the equations of motion for translational motion and rotational motion are

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{\partial\sigma_{ij}}{\partial x_j} \tag{16a}$$

$$\rho R^2\,\frac{\partial^2\theta_k}{\partial t^2}=-e_{ijk}\,\sigma_{ij} \tag{16b}$$

Substitution of Eq. (11) into Eq. (16) yields

$$\begin{aligned}\rho\frac{\partial^2 u_i}{\partial t^2}&=\frac{\partial}{\partial x_j}\left[\lambda\delta_{ij}\,\epsilon_{kk}+2\mu\epsilon_{ij}-2\xi e_{ijk}(\dot\omega_k-\dot\theta_k)\right]\\
&=\lambda\delta_{ij}\frac{\partial}{\partial x_j}\left(\frac{\partial u_k}{\partial x_k}\right)+\mu\frac{\partial}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)\\
&\quad-2\xi e_{ijk}\frac{\partial\dot\omega_k}{\partial x_j}+2\xi e_{ijk}\frac{\partial\dot\theta_k}{\partial x_j}\\
&=\lambda\frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right)+\mu\frac{\partial}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i}\right)+\mu\frac{\partial^2 u_i}{\partial x_j\,\partial x_j}\\
&\quad-2\xi e_{ijk}\frac{\partial\dot\omega_k}{\partial x_j}+2\xi e_{ijk}\frac{\partial\dot\theta_k}{\partial x_j}\\
&=(\lambda+\mu)\frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right)+\mu\frac{\partial^2 u_i}{\partial x_k\partial x_k}-\xi e_{ijk}\left(\frac{\partial\dot\omega_k}{\partial x_j}-\frac{\partial\dot\omega_j}{\partial x_k}\right)\\
&\quad+\xi e_{ijk}\left(\frac{\partial\dot\theta_k}{\partial x_j}-\frac{\partial\dot\theta_j}{\partial x_k}\right)\end{aligned} \tag{17a}$$

$$\begin{aligned}\rho R^2\,\frac{\partial^2\theta_k}{\partial t^2}&=-e_{ijk}\sigma_{ij}\\
&=2\xi e_{ijk}\,e_{ijn}\,\dot\omega_n-2\xi e_{ijk}\,e_{ijn}\,\dot\theta_n\\
&=4\xi\dot\omega_k-4\xi\dot\theta_k\\
&=\xi e_{ijk}\left(\frac{\partial\dot u_j}{\partial x_i}-\frac{\partial\dot u_i}{\partial x_j}\right)-4\xi\dot\theta_k\end{aligned} \tag{17b}$$

Noted that the stress related to ϵ_{ij} and ϵ_{kk} are symmetric, so they do not include in Eq. (17b). Equation (17) can be written in vector form as

$$\rho\ddot{\vec u}=\mu\nabla^2\vec u+(\lambda+\mu)\nabla(\nabla\cdot\vec u)+2\xi\nabla\times\dot{\vec\theta}+\xi\nabla^2\dot{\vec u}-\xi\nabla(\nabla\cdot\dot{\vec u}) \tag{18a}$$

$$=(\lambda+2\mu)\nabla^2\vec u+(\lambda+\mu)\nabla\times(\nabla\times\vec u)+2\xi\nabla\times\dot{\vec\theta}-\xi\nabla\times(\nabla\times\dot{\vec u}) \tag{18b}$$

$$\rho R^2\ddot{\vec\theta}=-4\xi\dot{\vec\theta}+2\xi\nabla\times\dot{\vec u} \tag{18c}$$

Equations (18a) and (18b) are identical by the identity: $\nabla^2\vec u=\nabla(\nabla\cdot\vec u)-\nabla\times(\nabla\times\vec u)$. Figure A1 is a flowchart for deriving the equations of motion.

4. DECOUPLING EQUATION OF MOTION

Displacement $\vec u$ and particle spin $\vec\theta$ can be decoupled by taking divergence of Eq. (18).

$$\rho \frac{\partial^2 (\nabla \cdot \vec{u})}{\partial t^2} = (\lambda + 2\mu)\nabla^2 (\nabla \cdot \vec{u}) \tag{19a}$$

$$\rho R^2 \frac{\partial^2 (\nabla \cdot \vec{\theta})}{\partial t^2} = -4\xi(\nabla \cdot \vec{\theta}) \tag{19b}$$

Divergence of displacement and spin can be solved from Eq. (19). However, the solutions from Eq. (19) do not produce the electro-magnetic wave. Therefore, solutions from Eq. (19) are not shown and discussed. That is, electro-magnetic waves are only related to the curl of displacement and spin as shown in Eq. (20). And solutions from Eq. (19) are independent of solutions from Eq. (20).

Rotational properties of aether field can be obtained by taking curl of Eq. (18).

$$\rho \frac{\partial^2 (\nabla \times \vec{u})}{\partial t^2} = \mu \nabla^2 (\nabla \times \vec{u}) + 2\xi \nabla \times (\nabla \times \vec{\theta}) + \xi \nabla^2 (\nabla \times \dot{\vec{u}}) \tag{20a}$$

$$\rho R^2 \frac{\partial^2 (\nabla \times \vec{\theta})}{\partial t^2} = -4\xi(\nabla \times \dot{\vec{\theta}}) + 2\xi \nabla \times (\nabla \times \dot{\vec{u}}) \tag{20b}$$

Because Eq. (19) do not contain the terms $\nabla \times \vec{u}$ and $\nabla \times \vec{\theta}$, $\nabla \times \vec{u}$ and $\nabla \times \vec{\theta}$ can only be solved from Eq. (20). Since, both $\nabla \times \vec{u}$ and $\nabla \times \vec{\theta}$ appear in both Eqs. (20a) and (20b), the solutions of $\nabla \times \vec{u}$ and $\nabla \times \vec{\theta}$ are coupled and must be solved together. Equation (20) can be organized and simplified as

$$-\left(\rho \partial_t^2 - (\mu + \xi \partial_t)\nabla^2\right)(\nabla \times \vec{u}) + (2\xi \partial_t)\nabla \times (\nabla \times \vec{\theta}) = 0 \tag{21a}$$

$$-(\rho R^2 \partial_t^2 + 4\xi \partial_t)(\nabla \times \vec{\theta}) + (2\xi \partial_t)\nabla \times (\nabla \times \vec{u}) = 0 \tag{21b}$$

And organize the coupled Eq. (21) into matrix forms as

$$\begin{bmatrix} -\left(\rho \partial_t^2 - (\mu + \xi \partial_t)\nabla^2\right)[I] & 2\xi \partial_t [\nabla] \\ 2\xi \partial_t [\nabla] & -(\rho R^2 \partial_t^2 + 4\xi \partial_t)[I] \end{bmatrix} \begin{bmatrix} [\nabla] & [0] \\ [0] & [\nabla] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{\theta\} \end{Bmatrix} = 0 \tag{22}$$

where

$$\partial_t = \frac{\partial}{\partial t} \tag{23a}$$

$$\partial_i = \frac{\partial}{\partial x_i} \tag{23b}$$

$$[\nabla] = \begin{bmatrix} 0 & -\partial_3 & \partial_2 \\ \partial_3 & 0 & -\partial_1 \\ -\partial_2 & \partial_1 & 0 \end{bmatrix} \tag{23c}$$

$$< \nabla > = < \partial_1 \ \ \partial_2 \ \ \partial_3 > \tag{23d}$$

$$\{\nabla\} = \begin{Bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{Bmatrix} \tag{23e}$$

$$\nabla^2 = < \nabla > \{\nabla\} = \partial_1^2 + \partial_2^2 + \partial_3^2 \tag{23f}$$

5. SOLVING EIGENVALUE PROBLEM FOR SPEED OF LIGHT

To solve Eq. (22) for $\vec{u}$ and $\vec{\theta}$, we assume harmonic solutions for them in Cartesian coordinate as

$$\vec{u} = \vec{A} \exp\left[j(\vec{k} \cdot \vec{x} - \omega t) \right] \tag{24a}$$

$$\vec{\theta} = \vec{B} \exp\left[j(\vec{k} \cdot \vec{x} - \omega t) \right] \tag{24b}$$

where

- $\vec{A}$ is wave oscillation vector of $\vec{u}$,
- $\vec{B}$ is wave oscillation vector of $\vec{\theta}$,
- $\vec{k}$ is wave traveling vector,
- $\vec{x}$ is position vector,
- ω is oscillation angular velocity,

and

$$\vec{k} = k_1 \vec{e}_1 + k_2 \vec{e}_2 + k_3 \vec{e}_3 \tag{25}$$

$$k = |\vec{k}| = \frac{2\pi}{L(= \text{wave length})} \tag{26}$$

$$\omega = \frac{2\pi}{T(= \text{vibration period})} \tag{27}$$

The amplitude of $\vec{k}$ is k and is called wave number. The wave traveling speed for a three dimensional wave is

$$c = \frac{\omega}{k} = \frac{L}{T} \tag{28}$$

where c is wave traveling speed or called speed of light. Based on the assumed harmonic solution for $\vec{u}$ and $\vec{\theta}$ as shown in Eq. (24), the following operator when acting on $\vec{u}$ or $\vec{\theta}$ can be replaced by

$$\partial_t = -j\omega \tag{29a}$$

$$< \nabla > \{\nabla\} = \nabla^2 = -k^2 \tag{29b}$$

$$[\nabla] = j\,|k| \tag{29c}$$

$$[\nabla][\nabla] = -[k][k] = -\{k\} < k > + k^2 [I] \tag{29d}$$

$$[\nabla][\nabla][\nabla] = jk^2 [k] \tag{29e}$$

$$< \nabla > [\nabla] = - < k > [k] = 0 \quad (29f)$$

where

$$\{k\} = \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} \quad (29g)$$

$$< k > = < k_1 \;\; k_2 \;\; k_3 > \quad (29h)$$

$$k^2 = < k > \{k\} \quad (29i)$$

$$[k] = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix} \quad (29j)$$

$j\,[k]$ in Eq. (29c) and $[\nabla]$ in Eq. (23c) is interchangeable when these operators are acting on assumed harmonic solution of $\vec{u}$ and $\vec{\theta}$.

Substitute Eq. (24) into Eq. (22) and use relations in Eq. (29) to replace u and θ with A and B as

$$j\begin{bmatrix} \left(\rho\omega^2 - (\mu - j\xi\omega)k^2\right)[I] & 2\xi\omega[k] \\ 2\xi\omega[k] & (\rho R^2\omega^2 + j4\xi\omega)[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (30)$$

For $[k]\{A\}$ and $[k]\{B\}$ to be nontrivial, the determinant in Eq. (30) must be equal to zero as

$$\begin{Vmatrix} \left(\rho\omega^2 - (\mu - j\xi\omega)k^2\right)[I] & 2\xi\omega[k] \\ 2\xi\omega[k] & (\rho R^2\omega^2 + j4\xi\omega)[I] \end{Vmatrix} = 0 \quad (31)$$

Equation (31) is equal to

$$\left\| \left(\rho\omega^2 - (\mu - j\xi\omega)k^2\right)(\rho R^2\omega^2 + j4\xi\omega)[I] - 4\xi^2\omega^2[k][k] \right\| = 0 \quad (32)$$

Substitution of Eq. (29d) into Eq. (32) yields

$$\left(\rho\omega^2 - (\mu - j\xi\omega)k^2\right)(\rho R^2\omega^2 + j4\xi\omega) + 4\xi^2\omega^2 k^2 = 0 \quad (33)$$

Expand Eq. (33) and separate real and imaginary part in the first term to get

$$\left((\rho\omega^2 - \mu k^2)\,\rho R^2\omega^2 - 4\xi^2 k^2\omega^2\right) + j\xi\omega(4\rho\omega^2 - 4\mu k^2 + \rho R^2 k^2\omega^2) + 4\xi^2 k^2\omega^2 = 0 \quad (34)$$

Simplify Eq. (34) and separate real and imaginary part as

$$(\rho\omega^2 - \mu k^2)\,\rho R^2\omega^2 + j\xi\omega(4\rho\omega^2 - 4\mu k^2 + \rho R^2 k^2\omega^2) = 0 \quad (35)$$

Equation (35) is a complex number equation and can be separated into two equations

$$(\rho\omega^2 - \mu k^2)\,\rho R^2\omega^2 = 0 \quad (36a)$$

$$\xi\omega(4\rho\omega^2 - 4\mu k^2 + \rho R^2 k^2\omega^2) = 0 \quad (36b)$$

In Eq. (36a), we have Eq. (38) since

$$\rho R^2\omega^2 \neq 0 \quad (37)$$

$$\frac{\omega}{k} = \sqrt{\frac{\mu}{\rho}} \quad (38)$$

As a result, the speed of light (based on Eq. (28)) can be expressed by shear module μ and mass density ρ as

$$c = \frac{\omega}{k} = \sqrt{\frac{\mu}{\rho}} \quad (39)$$

The same speed of light can be and should be concluded from Eq. (36b) based on the argument follows.

Since the radius of gyration R of the undetectable aether particle is far smaller than measurable electromagnetic wave length L, therefore

$$R \ll L = 2\pi / k \quad (40)$$

or

$$Rk \ll 1 \quad (41)$$

Equation (36b) can be rewritten as

$$\xi\omega\left((4 + R^2k^2)\,\rho\omega^2 - 4\mu k^2\right) = 0 \quad (42)$$

Substitution of Eq. (41) into Eq. (42) yields

$$\xi\omega(4\rho\omega^2 - 4\mu k^2) = 0 \quad (43)$$

In Eq. (43), we have Eq. (45) since

$$\xi\omega \neq 0 \quad (44)$$

$$4\rho\omega^2 - 4\mu k^2 = 0 \quad (45)$$

Since Eq. (45) is the same as Eq. (38), the same speed of light c is concluded from either real part Eq. (36a) or imaginary part Eq. (36b) which guarantee a nontrivial solution for $\vec{u}$ and $\vec{\theta}$.

6. SOLVING EIGENVECTOR FOR ELECTRO-MAGNETIC WAVE EQUATION

The eigenvalue as shown in Eq. (38) or Eq. (45) will be used to solve the eigenvector for the electromagnetic wave equation from the curled equation of motion (EOM) in Eq. (30). Substitute eigenvalue in Eq. (45) into Eq. (30)) to eliminate $\rho\omega^2$ and μk^2 terms to get

$$\begin{bmatrix} (j\xi k^2\omega)[I] & (2\xi\omega)[k] \\ (2\xi\omega)[k] & (j4\xi\omega + \rho R^2\omega^2)[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (46)$$

The first term in Eq. (46) can be simplified by eliminating two common terms ξ and ω, and change Eq. (46) into

$$\begin{bmatrix} (jk^2)[I] & (2)[k] \\ (2\xi\omega)[k] & (j4\xi\omega+\rho R^2\omega^2)[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (47)$$

The second term in Eq. (46) can be rewritten by the relation of $\rho\omega^2 = \mu k^2$ and $j4\xi\omega + \rho\omega^2 R^2 = j4\xi\omega + \mu k^2 R^2 \cong j4\xi\omega$, to get

$$\begin{bmatrix} (jk^2)[I] & (2)[k] \\ (2\xi\omega)[k] & \mu(j4\omega\xi/\mu+k^2R^2)[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (48)$$

In the second equation of Eq. (46),the following conclusion can be reasoned as: (1) Oscillation angular velocity ω are measurable should not be too small, (2) The ratio of dynamic viscosity ξ and shear module μ should not be too small, (3) k^2R^2 is much smaller than 1 according to Eq. (41), Therefore, k^2R^2 is negligible when comparing with $\omega\xi/\mu$. Equation (48) can be simplified as

$$\begin{bmatrix} (jk^2)[I] & (2)[k] \\ (2\xi\omega)[k] & (j4\omega\xi)[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (49)$$

The second equation in Eq. (46) can be simplified by eliminating common factor 2ξω.

$$\begin{bmatrix} jk^2[I] & 2[k] \\ [k] & j2[I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{B\} \end{Bmatrix} = 0 \quad (50)$$

Expand Matrix to get

$$[k]\{A\} = j\frac{2}{k^2}[k][k]\{B\} \quad (51a)$$

$$[k]\{B\} = j\frac{1}{2}[k][k]\{A\} \quad (51b)$$

Equations (51a) and (51b) can be shown to be equivalent by multiplying both side of Eq. (51a) or Eq. (51b) by $[k]$ and replacing $[k][k][k]$ with $-k^2[k]$ as shown in Eq. (29e).

Equation (51) are derived in matrix form. The matrix form can be translated into vector form as

$$\vec{k}\times\vec{A} = j\frac{2}{k^2}\vec{k}\times\vec{k}\times\vec{B} \quad (52a)$$

$$\vec{k}\times\vec{B} = j\frac{1}{2}\vec{k}\times\vec{k}\times\vec{A} \quad (52b)$$

In Eq. (52), $\vec{A}$ and $\vec{B}$ are wave oscillation vectors of translational and rotational displacements $\vec{u}$ and $\vec{\theta}$ of electro-magnetic wave as defined in Eq. (24). The translational velocity can be written as

$$\dot{\vec{u}} = -2\vec{C}\exp\left(j(\vec{k}\cdot\vec{x}-\omega t)\right) \quad (53)$$

where

$$\vec{C} = \frac{1}{2}j\omega\vec{A} \quad (54)$$

Substitution of Eq. (54) into Eq. (52) yields

$$\vec{k}\times\vec{C} = \frac{-\omega}{k^2}\vec{k}\times\vec{k}\times\vec{B} \quad (55a)$$

$$\vec{k}\times\vec{B} = \frac{1}{\omega}\vec{k}\times\vec{k}\times\vec{C} \quad (55b)$$

Equation (55) are the final electro-magnetic wave solution derived from Newton's second law of motion and constitution equations of aether particles considering (1) hydro-static pressure, (2) shear distortion, (3) particle spin and (4) volume rotation. Equation (55) will be proved to be the same as Maxwell's equations for free field (no source terms) in the next section.

7. COMPARING TO THE SOLUTIONS OF MAXWELL'S EQUATIONS

The Maxwell's equations with source are

$$\nabla\cdot\vec{E} = 4\pi\rho \quad (56a)$$

$$\nabla\cdot\vec{B} = 0 \quad (56b)$$

$$\nabla\times\vec{B} - \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \frac{4\pi}{c^2}\vec{J} \quad (56c)$$

$$\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \quad (56d)$$

where

ρ is electric source,
$\vec{j}$ is magnetic source,
$\vec{E}$ is electric force vector,
$\vec{B}$ magnetic force vector.

The Maxwell's equations for free field ($\rho = 0$, $\vec{j} = 0$) are

$$\nabla\cdot\vec{E} = 0 \quad (57a)$$

$$\nabla\cdot\vec{B} = 0 \quad (57b)$$

$$\nabla\times\vec{B} - \frac{\partial\vec{E}}{c^2\partial t} = 0 \quad (57c)$$

$$\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \quad (57d)$$

For Eqs. (57a) and (57b) to be satisfied, vector $\vec{E}$ and $\vec{B}$ can be redefined by $\vec{F}$ and $\vec{G}$ as

$$\vec{E} = \nabla \times \vec{F} \tag{58a}$$

$$\vec{B} = \nabla \times \vec{G} \tag{58b}$$

Substitution of Eq. (58) into Eq. (57cd) yields

$$\nabla \times \nabla \times \vec{G} - \frac{\partial}{c^2 \partial t}(\nabla \times \vec{F}) = 0 \tag{59a}$$

$$\nabla \times \nabla \times \vec{F} + \frac{\partial}{\partial t}(\nabla \times \vec{G}) = 0 \tag{59b}$$

Organize Eq. (59) into matrix form as

$$\begin{bmatrix} -c^{-2}\partial_t [I] & [\nabla] \\ [\nabla] & \partial_t [I] \end{bmatrix} \begin{bmatrix} [\nabla] & [0] \\ [0] & [\nabla] \end{bmatrix} \begin{Bmatrix} \{F\} \\ \{G\} \end{Bmatrix} = 0 \tag{60}$$

Similar to solving coupled equations in Eq. (22), Eq. (60) can be solved by assuming harmonic solution for { $\vec{F}$ } and { $\vec{G}$ } as

$$\vec{F} = \vec{D} \exp\left(j(\vec{k} \cdot \vec{x} - \omega t) \right) \tag{61a}$$

$$\vec{G} = \vec{H} \exp\left(j(\vec{k} \cdot \vec{x} - \omega t) \right) \tag{61b}$$

where

$\vec{D}$ is wave oscillation vector of $\vec{F}$,

$\vec{H}$ is wave oscillation vector of $\vec{G}$.

Substitute Eq. (61) into Eq. (60) and use Eq. (29) to modify Eq. (60).

$$-\begin{bmatrix} c^{-2}\omega [I] & [k] \\ [k] & -\omega [I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{D\} \\ \{H\} \end{Bmatrix} = 0 \tag{62}$$

For $[k]\{D\}$ and $[k]\{H\}$ to be nontrivial, the determinant in Eq. (62) must equal to zero as

$$\begin{Vmatrix} c^{-2}\omega [I] & [k] \\ [k] & -\omega [I] \end{Vmatrix} = 0 \tag{63}$$

Equation (63) is equal to

$$\left\| -c^{-2}\omega^2 [I] - [k][k] \right\| = 0 \tag{64}$$

Substitution of Eq. (29d) into Eq. (64), and note that $\|\{k\}<k>\| = 0$, yields

$$-c^{-2}\omega^2 + k^2 = 0 \tag{65}$$

Therefore, the speed of light is

$$c = \pm \frac{\omega}{k} \tag{66}$$

Use the eigenvalue $c = \pm \frac{\omega}{k}$ from Eq. (66) to eliminate c in Eq. (62) and rewrite Eq. (62) as

$$-\begin{bmatrix} \omega^{-1} k^2 [I] & [k] \\ [k] & -\omega [I] \end{bmatrix} \begin{bmatrix} [k] & [0] \\ [0] & [k] \end{bmatrix} \begin{Bmatrix} \{D\} \\ \{H\} \end{Bmatrix} = 0 \tag{67}$$

Expand Eq. (67) to get

$$[k]\{D\} = \frac{-\omega}{k^2}[k][k]\{H\} \tag{68a}$$

$$[k]\{H\} = \frac{1}{\omega}[k][k]\{D\} \tag{68b}$$

The Matrix form of Eq. (68) can be translated to vector form as

$$\vec{k} \times \vec{D} = \frac{-\omega}{k^2} \vec{k} \times \vec{k} \times \vec{H} \tag{69a}$$

$$\vec{k} \times \vec{H} = \frac{1}{\omega} \vec{k} \times \vec{k} \times \vec{D} \tag{69b}$$

Equation (69) are electro-magnetic wave solutions for solving Maxwell's equations for free field. Equation (55) are electro-magnetic wave solutions derived from constitution law of aether particles. It shows that Eqs. (69) and (55) are exactly the same. These conclusions suggested that substantial aether model can be used to obtain the wave of Maxwell's equations from its constitution equations. Figure A2 is a flowchart for solving electro-magnetic waves.

8. CONCLUSIONS

Aether particles with mass that move and spin is proven in this paper to generate an electro-magnetic wave. Electro-magnetic force is, therefore, a contact force transmitted through aether particles.

ACKNOWLEDGMENTS

We are deeply indebted to Professor Emeritus Chao-Chung Yu, former President of the National Taiwan University, and Professor Zissimos P. Mourelatos, Chair of the Mechanical Engineering Department of Oakland University for their supports and valuable discussions and suggestions in the research of this subject. We are grateful to Dr. Steven K. Lang, Applications Engineering Director of Intel Microelectronics Asia Ltd. Taiwan Branch, for his valuable suggestions. We are also thankful to the reviewers for their constructive comments.

APPENDIX A
FLOWCHARTS

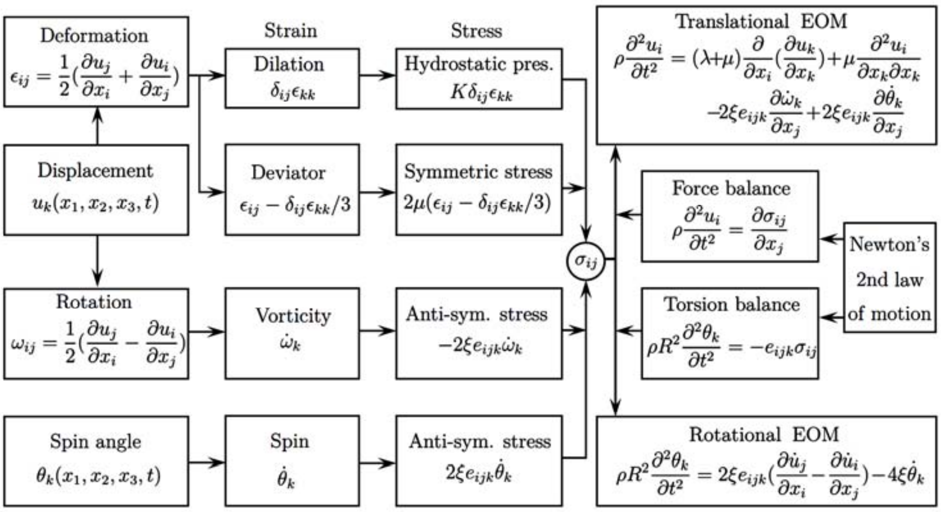

Fig. A1 Flowchart to derive the equations of motion

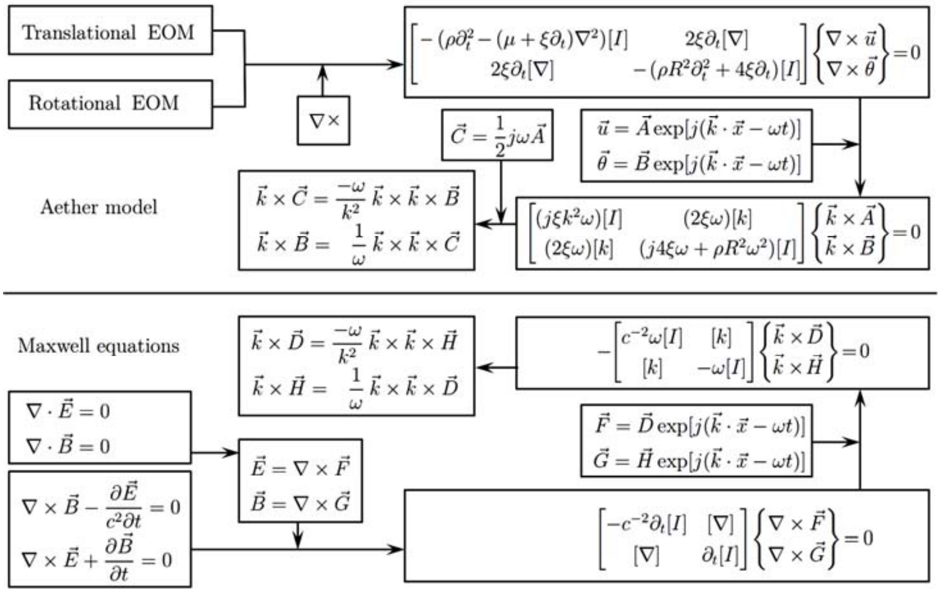

Fig. A2 Flowchart to solve electro-magnetic waves

REFERENCES

1. Newton, I., *Opticks: Or a Treatise of the Reflections, Refractions, Inflections and Colours of Light*, Royal Society, London (1704).
2. DeMeo, J., "A Dynamic and Substantive Cosmological Ether," *Proceedings of the Natural Philosophy Alliance*, Arlington, MA, **1**, **1** (2004).
3. Michelson, A. A. and Morley, E. W., "On the Relative Motion of the Earth and the Luminiferous Ether," *The American Journal of Science*, **34**, **203**, pp. 333–345 (1887).
4. Miller, D. C., "The Ether-Drift Experiment and the Determination of the Absolute Motion of the Earth," *Reviews of Modern Physics*, **5**, pp. 203–242 (1933).
5. Navier, C. L. M. H., *Memoirs of the Academy*, **7**, p. 375 (1827).
6. Jaswon, M. A., "The Mechanical Interpretation of Maxwell's Equations," *Nature*, **224** (1969).
7. MacCullagh, J., "An Essay Towards a Dynamical Theory of Crystalline Reflection and Refraction," *The Royal Irish Academy*, xxi (1839).
8. Larson, D. J., "A Derivation of Maxwell's Equation from a Simple Two-Component Solid-Mechanical Aether," *Physics Essays*, **11**, **4** (1998).
9. Lin, T. W., "The Mechanism Connecting Magnetic and Electric Forces," *The Chinese Journal of Mechanics*, **12**, **1**, pp. 109–115 (1996).

(Manuscript received April 6, 2013, accepted for publication November 12, 2013.)