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Fast Multipole Boundary Element Method for Sound Scattering from Aerodynamic Bodies

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A multi-stage adaptive Fast Multipole Method (FMM) is used to accelerate the matrix-vector products arising from the boundary integral equations formed in a Boundary Element Method (BEM). The present study considers the scattering of acoustic waves, generated by localized sources, from bodies with rigid surfaces. Details on the implementation of the multi-stage adaptive FMM are described for 2-D and 3-D formulations. The code is verified through the solution of well documented test cases. The FMM is tested for acoustic scattering problems of single and multiple bodies and a discussion is provided on the performance of the method. Results for engineering problems with complex geometries, such as a multi-element wing, are presented in order to assess the implemented capability.

I. Introduction

The development of physics-based noise prediction tools for analysis of jet and airframe noise sources is of paramount importance since noise regulations have become more stringent. The design of 3-D complex configurations requires the use of time consuming numerical simulations for the study and mitigation of noise sources. In the context of aeroacoutics, several acoustic scattering codes such as the Fast Scattering Code (FSC) from NASA Langley Research Center,[1,2,3] and the ACTI3S and the ACTIPOLE codes from Airbus-F,[4] are under development. These scattering codes solve the Helmholtz equation through the discretization of boundary integral equations by the Boundary Element Method (BEM) or by the Equivalent Source Method (ESM). In the present work, we develop a scattering code for 2-D and 3-D simulations using an accelerated BEM. In the future, the code will be used for the analysis of airframe noise sources and for exploring strategies for their mitigation.

The use of BEM for solving scattering and radiation problems in acoustics provides several advantages over Finite Element Methods (FEM) and Finite Difference Methods (FDM). Among these, one can cite the advantage of boundary discretization only, the simplified pre-processing and the accurate modelling of infinite domains. However, for large scale problems, the solution of the non-symmetric dense matrices appearing in the BEM have computational complexity proportional to $O(n^2)$ when an iterative solver is used, which makes the method prohibitive to use. Here, n is the number of boundary elements used in the discretization. In order to reduce the computational complexity of the BEM one can use Fast Multipole Methods (FMM) to accelerate the solution of the linear systems and reduce the computational complexity to an order proportional to $O(n \log n)$.

The Fast Multipole Method was introduced by Rokhlin[5] for the solution of integral equations of classical potential theory. However, the method was further developed and became famous for the solution of $N-$body problems in the paper of Greengard and Rokhlin.[6] The application of the FMM for acoustic scattering was introduced by Rokhlin[7] for the solution of integral equations of scattering theory in 2D. Since then,

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