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Special Issue on Computational Mathematics

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Preface

This special issue is dedicated to Professor Shi Zhong-Ci on the occasion of his 80th birthday.

Professor Shi was born on December 5, 1933, in Ningbo, Zhejiang Province. He completed the undergraduate study in Fudan University in 1955 and had studied computational mathematics in Steklov Institute of Mathematics in Moscow in 1956–1960. Professor Shi has made profound and significant contributions in many areas of scientific computing including finite element methods, domain decomposition methods, and multigrid methods. His achievements in the development of nonconforming finite element methods have been particularly recognized by the community.

Professor Shi is an outstanding representative of the Chinese community of computational mathematics. He served as the Director of the Computing Center of Chinese Academy of Sciences in 1987–1991. He was the chief scientist of the State Key Project "Large Scale Scientific and Engineering Computing Research" in 1993–1999. He has been the President of the Chinese Computational Mathematics Society and the chief editor of *Journal of Computational Mathematics*, *Mathematica Numerica Sinica*, and *Journal on Numerical Methods and Computer Applications* since 1994. He has served as the chairman of the selection committee for the well-known Feng Kang Prize of Scientific Computing since 1995.

This special issue includes 23 papers, of which 4 are survey papers and 19 are research articles. All the submissions have been rigorously refereed. The authors of the papers are internationally well recognized experts covering wide spectrum of scientific computing. The content of the papers reflects the current international research trend. We are grateful to all the authors for their outstanding contributions.

Finally, we wish Professor Shi Zhong-Ci happy birthday and good health!

Editors:
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SCIENCE CHINA
Mathematics

• REVIEWS •

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doi: 10.1007/s11425-013-4660-7

Some progress in spectral methods

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract In this paper, we review some results on the spectral methods. We first consider the Jacobi spectral method and the generalized Jacobi spectral method for various problems, including degenerated and singular differential equations. Then we present the generalized Jacobi quasi-orthogonal approximation and its applications to the spectral element methods for high order problems with mixed inhomogeneous boundary conditions. We also discuss the related spectral methods for non-rectangular domains and the irrational spectral methods for unbounded domains. Next, we consider the Hermite spectral method and the generalized Hermite spectral method with their applications. Finally, we consider the Laguerre spectral method and the generalized Laguerre spectral method for many problems defined on unbounded domains. We also present the generalized Laguerre quasi-orthogonal approximation and its applications to certain problems of non-standard type and exterior problems.

Keywords Jacobi, Hermite and Laguerre spectral approximations, Jacobi and Laguerre quasi-orthogonal approximations, spectral and spectral element methods, degenerated and singular problems, problems on non-rectangular and unbounded domains, problems of non-standard type, exterior problems

MSC(2010) 65M70, 65N35, 41A10, 41A20, 41A30, 35J25, 35J35, 35K20, 35M13

Citation: Guo B Y. Some progress in spectral methods. Sci China Math, 2013, 56: 2411–2438, doi: 10.1007/s11425-013-4660-7

1 Introduction

The spectral methods have been successfully used in scientific computations, see the books of Bernardi and Maday [8,9], Bernardi et al. [10], Boyd [13], Canuto et al. [14–16], Funaro [25], Gottlieb and Orszag [27], Guo [30], Hesthaven et al. [82], Shen and Tang [104], Shen et al. [105], Karniadakis and Sherwin [87], the review papers of Guo [38], Guo et al. [76], and Shen and Wang [111], and the references therein.

The traditional spectral methods are available for periodic problems and many problems defined on rectangular domains. Their mathematical foundations are the Fourier, Legendre and Chebyshev approximations. Guo [34,35], and Guo and Wang [54,56] developed the Jacobi approximation, and proposed the Jacobi spectral method for degenerated problems. We also refer to the work of Babuška and Guo [3], Funaro [25], and Ma and Sun [95]. Later, Guo et al. [47,48] provided the generalized Jacobi approximation, which leads to a class of new spectral methods for high order problems. It is also appreciated for singular problems. Recently, Guo et al. [53], and Guo and Wang [60] proposed the Jacobi quasi-orthogonal approximation, which plays an important role in the spectral element methods for mixed inhomogeneous boundary value problems of various differential equations.

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A survey on artificial boundary method

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract The artificial boundary method is one of the most popular and effective numerical methods for solving partial differential equations on unbounded domains, with more than thirty years development. The artificial boundary method has reached maturity in recent years. It has been applied to various problems in scientific and engineering computations, and the theoretical issues such as the convergence and error estimates of the artificial boundary method have been solved gradually. This paper reviews the development and discusses different forms of the artificial boundary method.

Keywords artificial boundary method, global artificial boundary condition, local artificial boundary condition, discrete artificial boundary condition, implicit artificial boundary condition, nonlinear artificial boundary condition

MSC(2010) 35Q30, 35Q72, 65M06, 65M12, 65M99, 65N06, 65N12, 65N99

Citation: Han H D, Wu X N. A survey on artificial boundary method. Sci China Math, 2013, 56: 2439–2488, doi: 10.1007/s11425-013-4694-x

1 Introduction

Many problems in science and engineering are described by partial differential equations on unbounded domains, and they must be solved numerically. Flow around an airfoil (see Figure 1(a)), stress analysis of a dam with an infinite foundation (see Figure 1(b)), and wave propagation in the space (sound wave, elastic wave, electric magnetic wave, etc.) are typical examples. For these problems, one of the main difficulties is the unboundedness of the domain. The normal numerical methods, such as the finite difference and finite element method, cannot be applied directly to solve the above problems. One way to solve the problems is to introduce an artificial boundary, and divide the physical domain into two parts: the bounded computational domain and the remaining unbounded domain. The artificial boundary becomes the boundary (or a part of the boundary) of the computational domain. If we can find the boundary condition on the artificial boundary satisfied by the solution of the original problem, then we can reduce the original problem to a boundary value problem on the bounded computational domain, and solve it numerically. In early literatures, the boundary condition at infinity is usually applied directly on the artificial boundary. The Dirichlet boundary condition (or Neumann boundary condition) is the commonly used boundary condition. In general, this boundary condition is not the exact boundary condition satisfied by the solution of the original problem. Suppose that $\Omega_0 \subset \mathbb{R}^n$ is a bounded domain,

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Sparse time-frequency representation of nonlinear and nonstationary data

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract Adaptive data analysis provides an important tool in extracting hidden physical information from multiscale data that arise from various applications. In this paper, we review two data-driven time-frequency analysis methods that we introduced recently to study trend and instantaneous frequency of nonlinear and nonstationary data. These methods are inspired by the empirical mode decomposition method (EMD) and the recently developed compressed (compressive) sensing theory. The main idea is to look for the sparsest representation of multiscale data within the largest possible dictionary consisting of intrinsic mode functions of the form $\{a(t)\cos(\theta(t))\}$, where a is assumed to be less oscillatory than $\cos(\theta(t))$ and $\theta' \geqslant 0$. This problem can be formulated as a nonlinear l^0 optimization problem. We have proposed two methods to solve this nonlinear optimization problem. The first one is based on nonlinear basis pursuit and the second one is based on nonlinear matching pursuit. Convergence analysis has been carried out for the nonlinear matching pursuit method. Some numerical experiments are given to demonstrate the effectiveness of the proposed methods.

Keywords sparse representation, time-frequency analysis, data-driven

MSC(2010) 65N21, 94A12

Citation: Hou T Y, Shi Z Q. Sparse time-frequency representation of nonlinear and nonstationary data. Sci China Math, 2013, 56: 2489–2506, doi: 10.1007/s11425-013-4733-7

1 Introduction

Nowadays we must process a massive amount of data in our daily life and in our scientific research. How to extract hidden pattern or physical information from these data has become essential in our scientific discovery. This calls for the need to develop a truly adaptive data analysis method. Traditional data analysis methods based on pre-determined basis provide an effective tool to process linear and stationary data. When applying these methods to analyze nonlinear and nonstationary data, they tend to generate some artificial harmonics. These limitations have been alleviated to some extent with the introduction of time-frequency analysis which represents a signal with a joint function of both time and frequency. Wavelet analysis provides an excellent tool for time-frequency analysis by introducing multi-scales to characterize signals, see [10, 23, 26, 31].

The concept of instantaneous frequency is an important development in time-frequency analysis. It can be traced back to Van der Pol [42], who introduced the so-called analytic signal method (AS) that

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Some recent advances on vertex centered finite volume element methods for elliptic equations

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract In this paper, we report our recent advances on vertex centered finite volume element methods (FVEMs) for second order partial differential equations (PDEs). We begin with a brief review on linear and quadratic finite volume schemes. Then we present our recent advances on finite volume schemes of arbitrary order. For each scheme, we first explain its construction and then perform its error analysis under both H^1 and L^2 norms along with study of superconvergence properties.

Keywords high order, finite volume method, inf-sup condition, superconvergence

MSC(2010) Primary 65N30; Secondary 45N08

Citation: Zhang Z M, Zou Q S. Some recent advances on vertex centered finite volume element methods for elliptic equations. Sci China Math, 2013, 56: 2507–2522, doi: 10.1007/s11425-013-4740-8

1 Introduction

The finite volume method (FVM) is one of the most commonly used numerical methods for solving PDEs in practice. Due to its local conservation of numerical fluxes (a property not shared by the finite element method (FEM)), the capability of handling domains with complex geometries (a property shared by FEM but not by the finite difference method (FDM)), and other advantages, the FVM enjoyed a great population in computational fluid mechanics, heat transfer and hyperbolic equations (cf. [9, 31, 36, 44, 48, 53, 69–71, 73, 83, 87] and the references cited therein).

Earlier work on FVMs may be traced back to MacNeal's difference schemes on irregular networks [59], Samarskii and Tichonov's discretization schemes for convection-diffusion equations [76, 77, 89], and the work of McDonald [62] and MacCormack and Paullay [58]. Since then, the FVM has attracted much attention. Roughly speaking, there are two categories of FVM schemes: *cell-centered FVM* with unknown variables associated with centers of computational cells [1–3, 8, 29, 37, 42, 46, 47, 60, 66, 91, 97], and *vertex-centered FVM* with unknown variables associated with vertices of computational cells. There is also hybrid finite volume method, which uses a simple control volume partitions for higher-order schemes and more flexible algorithm construction [14]. In this article we concentrate only on vertex centered finite

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Rotated block triangular preconditioning based on PMHSS

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract Based on the PMHSS preconditioning matrix, we construct a class of rotated block triangular preconditioners for block two-by-two matrices of real square blocks, and analyze the eigen-properties of the corresponding preconditioned matrices. Numerical experiments show that these rotated block triangular preconditioners can be competitive to and even more efficient than the PMHSS preconditioner when they are used to accelerate Krylov subspace iteration methods for solving block two-by-two linear systems with coefficient matrices possibly of nonsymmetric sub-blocks.

Keywords block two-by-two matrix, PMHSS preconditioner, block triangular preconditioning, product-type preconditioning, eigen-properties

MSC(2010) 65F10, 65F50

Citation: Bai Z-Z. Rotated block triangular preconditioning based on PMHSS. Sci China Math, 2013, 56: 2523–2538, doi: 10.1007/s11425-013-4695-9

1 Introduction

Given two real and square matrices W and T, we consider preconditioning block two-by-two matrices of the form

$$A = \begin{pmatrix} W & -T \\ T & W \end{pmatrix}. \tag{1.1}$$

Note that A is nonsingular if and only if

$$\mathrm{null}(W) \cap \mathrm{null}(T) = \{0\}$$

and $\pm\mathrm{i}$ is not a generalized eigenvalue of the matrix pair (W, T) (i.e., $Tx \neq \mp\mathrm{i}Wx$ for some $x \neq 0$), where $\mathrm{null}(\cdot)$ denotes the null space of the corresponding matrix and $\mathrm{i} = \sqrt{-1}$ indicates the imaginary unit. For some application backgrounds of this class of matrices, we refer the readers to [1, 3, 7, 8, 13, 19, 21] for details.

By modifying and preconditioning the *Hermitian and skew-Hermitian splitting* (HSS) iteration method [10], Bai et al. [7] established a class of *preconditioned and modified HSS* (PMHSS) iteration

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Reconstruction of a defect in an open waveguide

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract The paper is concerned with the reconstruction of a defect in the core of a two-dimensional open waveguide from the scattering data. Since only a finite numbers of modes can propagate without attenuation inside the core, the problem is similar to the one-dimensional inverse medium problem. In particular, the inverse problem suffers from a lack of uniqueness and is known to be severely ill-posed. To overcome these difficulties, we consider multi-frequency scattering data. The uniqueness of solution to the inverse problem is established from the far field scattering information over an interval of low frequencies.

Keywords inverse problem, Helmholtz equation, open waveguide, Green function, multi-frequency data

MSC(2010) 35R30

Citation: Bao G, Triki F. Reconstruction of a defect in an open waveguide. Sci China Math, 2013, 56: 2539–2548, doi: 10.1007/s11425-013-4696-8

1 The optical waveguide scattering problem

In this paper, we seek to locate and reconstruct the refractive index of a defect in the core of a homogeneous $2d$ open waveguide from information contained in the scattering data. This problem arises from many physical applications, such as geophysics, integrated optics and ultrasonic nondestructive testing. An optical waveguide is said to be homogeneous if its index of refraction does not depend on the direction of propagation of the guided waves (x-axis) (see [16]). As shown in Figure 1, the waveguide is assumed to occupy the upper half space ($y > 0$). A thin conductor is deposited on the line $y = 0$. The region $0 < y < h$ is considered to be the core of the fiber, while the rest is considered the cladding.

Assume that the index of refraction is a non-increasing function with respect to the vertical variable y. The homogeneous and normalized index of refraction of the waveguide in absence of the defect is given by

$$n(y) = \begin{cases} n_c(y), & \text{in } (0,h), \\ 1, & \text{in } (h,+\infty). \end{cases}$$

The positive function $n_c(y)$ is C^2, decreasing on $(0,h)$ and satisfies $n_c(h) = \boldsymbol{n}_c > 1$. The perturbed waveguide has an index of refraction that depends on both variables x and y, and can be formulated as

$$\boldsymbol{n}^2(x,y) = n^2(y) + p(x,y). \tag{1.1}$$

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A tensor model for liquid crystals on a spherical surface

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract Rod-like molecules confined on a spherical surface can organize themselves into nematic liquid crystal phases. This can give rise to novel textures displayed on the surface, which has been observed in experiments. An important theoretical question is how to find and predict these textures. Mathematically, a stable configuration of the nematic fluid corresponds to a local minimum in the free energy landscape. By applying Taylor expansion and Bingham approximation to a general molecular model, we obtain a closed-form tensor model, which gives a free energy form that is different from the classic Landau-de Gennes model. Based on the tensor model, we implement an efficient numerical algorithm to locate the local minimum of the free energy. Our model successfully predicts the splay, tennis-ball and rectangle textures. Among them, the tennis-ball configuration has the lowest free energy.

Keywords liquid crystal, nematic texture, Bingham closure approximation, tennis-ball configuration

MSC(2010) 33C55, 76A15, 81T80, 82D30

Citation: Cheng H, Zhang P W. A tensor model for liquid crystals on a spherical surface. Sci China Math, 2013, 56: 2549–2559, doi: 10.1007/s11425-013-4746-2

1 Introduction

Liquid crystal phase formation confined on a spherical surface has attracted a lot of attention recently. A curved surface coated with anisotropic rod-like molecules, such as nanorods, polymers, or gemini lipids, may display various kinds of textures, which contain defects as a result of discontinuity in the orientation of molecules. The textures and defects formed by the nematic liquid are strongly affected by the geometric and topological properties of the surface that they embedded in, a feature that we can manipulate in designing materials with novel properties. For example, as proposed by Nelson [10], a tennis-ball texture with four defects sitting at the vertices of a regular tetrahedron can be used to create a four-fold tetravalent colloid particle. Thus, it is of great importance in predicting the configuration of rod-like molecules confined on a curved surface.

Consider some nematic liquid crystals confined on a spherical surface. According to the hairy ball theorem [3], the tangent vector field corresponding to the orientation of molecules must contain defects, or discontinuities in the orientation field. In addition, the Poincaré-Hopf theorem [5] states that the sum of the topological indexes of all defects must equal to two—the Euler characteristic of the spherical surface. Recent studies have suggested four possible textures presented in Figure 1. The splay texture

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Fast integral equation solver for Maxwell's equations in layered media with FMM for Bessel functions

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract The paper presents a new fast integral equation solver for Maxwell's equations in 3-D layered media. First, the spectral domain dyadic Green's function is derived, and the 0-th and the 1-st order Hankel transforms or Sommerfeld-type integrals are used to recover all components of the dyadic Green's function in real space. The Hankel transforms are performed with the adaptive generalized Gaussian quadrature points and window functions to minimize the computational cost. Subsequently, a fast integral equation solver with $O(N_z^2 N_x N_y \log(N_x N_y))$ in layered media is developed by rewriting the layered media integral operator in terms of Hankel transforms and using the new fast multipole method for the n-th order Bessel function in 2-D. Computational cost and parallel efficiency of the new algorithm are presented.

Keywords Maxwell's equations, Helmholtz equation, layered media, Green's function, fast multipole method

MSC(2010) 65F10, 65R20, 65N22

Citation: Cho M H, Cai W. Fast integral equation solver for Maxwell's equations in layered media with FMM for Bessel functions. Sci China Math, 2013, 56: 2561–2570, doi: 10.1007/s11425-013-4719-5

1 Introduction

Wave propagation in inhomogeneous media has widespread applications ranging from geosciences to quantum mechanics. Planar-layered media is the most fundamental inhomogeneous structure in science and engineering applications such as microstrip antennae [9, 14, 32], thin-film solar cells [2, 15, 31], and negative refractive index metamaterials [16, 17]. Numerical simulation of Maxwell's equations in layered media plays an important role for these applications as they can be employed for design optimization and reduce the cost of experiments or fabrication. Therefore, the development of fast and robust solvers on high performance parallel computers is critically important. Furthermore, in order to take advantage of modern high performance computers, one requires a well-designed algorithm that can utilize all the potential computing resources whether in personal computers or clusters. In this paper, several numerical algorithms and techniques will be presented regarding parallel fast integral equation solvers for Maxwell's equations on different computing platforms based on an efficient representation of the layered media Green's function.

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The effect of ghost forces for a quasicontinuum method in three dimension

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We study the effect of "ghost forces" for a quasicontinuum method in three dimension with a planar interface. "Ghost forces" are the inconsistency of the quasicontinuum method across the interface between the atomistic region and the continuum region. Numerical results suggest that "ghost forces" may lead to a negilible error on the solution, while lead to a finite size error on the gradient of the solution. The error has a layer-like profile, and the interfacial layer width is of $O(\varepsilon)$. The error in certain component of the displacement gradient decays algebraically from $O(1)$ to $O(\varepsilon)$ away from the interface. A surrogate model is proposed and analyzed, which suggests the same scenario for the effect of "ghost forces". Our analysis is based on the explicit solution of the surrogate model.

Keywords quasicontinuum method, atomistic-to-continuum, ghost force

MSC(2010) 65N30, 65N12, 65N06, 74G20, 74G15

Citation: Cui L, Ming P B. The effect of ghost forces for a quasicontinuum method in three dimension. Sci China Math, 2013, 56: 2571–2589, doi: 10.1007/s11425-013-4726-6

1 Introduction

Multiscale methods that couple the atomistic model and the continuum model have been extensively studied over the past few decades, and these methods have been employed to simulate various properties of materials, in particular for solids [1, 5, 14]. One critical issue of these methods is the so-called *ghost force*, which is the none-zero force that atoms experience at the undeformed state [19]. From the numerical analysis aspect of view, ghost force is nothing but the inconsistency of the method. Some efforts have been devoted to studying the effect of the ghost force. The ghost force issue for the one-dimensional problem is well-understood now, see [4, 15, 16]. The main findings are nicely summarized in [5, p. 278]: (1) the ghost force induces a negligible error on the solution, which is usually as small as the lattice spacing, while it may lead to an $\mathcal{O}(1)$ error on the gradient of the solution; (2) the influence of the ghost force decays exponentially fast away from the interface; (3) away from the interfacial layer of width $\mathcal{O}(\varepsilon|\ln\varepsilon|)$, the error in the displacement gradient is of $\mathcal{O}(\varepsilon)$ with ε being the lattice spacing. The effect of the ghost force for a two-dimensional problem with a planar atomistic to continuum (a/c) interface has recently been carried out in [3]. The story is almost the same except that the decay of the error in the displacement gradient is much slower than that of the one-dimensional problem. The decay rate is algebraical, and

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Mean field limit of a dynamical model of polymer systems

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80*th Birthday*

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Abstract This paper provides a mathematically rigorous foundation for self-consistent mean field theory of the polymeric physics. We study a new model for dynamics of mono-polymer systems. Every polymer is regarded as a string of points which are moving randomly as Brownian motions and under elastic forces. Every two points on the same string or on two different strings also interact under a pairwise potential V. The dynamics of the system is described by a system of N coupled stochastic partial differential equations (SPDEs). We show that the mean field limit as $N \to \infty$ of the system is a self-consistent McKean-Vlasov type equation, under suitable assumptions on the initial and boundary conditions and regularity of V. We also prove that both the SPDE system of the polymers and the mean field limit equation are well-posed.

Keywords polymers, mean field limit, stochastic PDEs, McKean-Vlasov equation

MSC(2010) 60H15, 65Z05, 82D60

Citation: E W, Shen H. Mean field limit of a dynamical model of polymer systems. Sci China Math, 2013, 56: 2591–2598, doi: 10.1007/s11425-013-4713-y

1 Introduction

In this paper, we study a new model for the dynamics of mono-polymer systems. The system consists of N polymers. Each polymer is modeled as an elastic string of fixed length $L = 1$, which can be thought of as a continuum limit of a chain of a large number of beads. Each point of the chain moves randomly as a Brownian motion with variance given by the temperature. At the same time the string feels a force coming from its curvature, which corresponds to the spring force between two neighboring beads in the discrete case. A pairwise potential V is present between every pairs of points on the same string or on different strings. [4] discussed the case of one single string for $V \equiv 0$. We are mainly concerned with the case of over-damped dynamics, although Newtonnian dynamics will also be briefly discussed in Section 4.

Mathematically at any time $t \geqslant 0$, each polymer $X_i(t)$ is a map from $[0,1]$ to $\mathbb{R}^3$, $i = 1, \ldots, N$. The dynamics of this system is described by the following stochastic partial differential equations:

$$dX_i(t,\sigma) = \tau dW_i(t) + \kappa \frac{\partial^2}{\partial \sigma^2} X_i(t,\sigma)dt + \frac{1}{N}\sum_{j=1}^{N}\int_0^1 f(X_i(t,\sigma), X_j(t,\sigma'))d\sigma' dt, \tag{1.1}$$

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Nonconforming finite element methods on quadrilateral meshes

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract It is well known that it is comparatively difficult to design nonconforming finite elements on quadrilateral meshes by using Gauss-Legendre points on each edge of triangulations. One reason lies in that these degrees of freedom associated with these Gauss-Legendre points are not all linearly independent for usual expected polynomial spaces, which explains why only several lower order nonconforming quadrilateral finite elements can be found in literature. The present paper proposes two families of nonconforming finite elements of any odd order and one family of nonconforming finite elements of any even order on quadrilateral meshes. Degrees of freedom are given for these elements, which are proved to be well-defined for their corresponding shape function spaces in a unifying way. These elements generalize three lower order nonconforming finite elements on quadrilaterals to any order. In addition, these nonconforming finite element spaces are shown to be full spaces which is somehow not discussed for nonconforming finite elements in literature before.

Keywords nonconforming finite element, rectangle

MSC(2010) 65N30, 65N15, 35J25

Citation: Hu J, Zhang S Y. Nonconforming finite element methods on quadrilateral meshes. Sci China Math, 2013, 56: 2599–2614, doi: 10.1007/s11425-013-4741-7

1 Introduction

Because of their flexibility and stability when compared with conforming finite element methods, nonconforming finite element methods have become very important and effective discretization methods for numerically solving, among others, high order elliptic problems, Stokes-like problems and Reissner-Mindlin plate bending problems.

Quadrilateral meshes are very important in scientific and engineering computing. Indeed, many popular softwares for computations of the fluid mechanics in two dimensions are defined on quadrilateral meshes. However, most of nonconforming finite element methods for second order problems are defined on triangles [3–6, 8] while there are only a few nonconforming finite element methods on quadrilaterals.

Compared with nonconforming triangular finite elements, it is more difficult to construct nonconforming quadrilateral finite elements. In fact, a sufficient condition for convergence of consistency error terms is to require nonconforming functions to be continuous at Gauss-Legendre points on interior edges of

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Anisotropic mesh generation methods based on ACVT and natural metric for anisotropic elliptic equation

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract Anisotropic meshes are known to be well-suited for problems which exhibit anisotropic solution features. Defining an appropriate metric tensor and designing an efficient algorithm for anisotropic mesh generation are two important aspects of the anisotropic mesh methodology. In this paper, we are concerned with the natural metric tensor for use in anisotropic mesh generation for anisotropic elliptic problems. We provide an algorithm to generate anisotropic meshes under the given metric tensor. We show that the inverse of the anisotropic diffusion matrix of the anisotropic elliptic problem is a natural metric tensor for the anisotropic mesh generation in three aspects: better discrete algebraic systems, more accurate finite element solution and superconvergence on the mesh nodes. Various numerical examples demonstrating the effectiveness are presented.

Keywords anisotropic mesh, metric tensor, superconvergence, anisotropic elliptic equation

MSC(2010) 65N30, 65N50

Citation: Huang Y Q, Su Y F, Wei H Y, et al. Anisotropic mesh generation methods based on ACVT and natural metric for anisotropic elliptic equation. Sci China Math, 2013, 56: 2615–2630, doi: 10.1007/s11425-013-4728-4

1 Introduction

Anisotropic problems are common in mathematical modeling of physical problems. They appear in various fields of application, such as flows in porous media [4, 27], quasi-neutral plasma simulations [17], semiconductor modeling [38], image processing [47], and so on. The anisotropic problems exhibit a common anisotropic feature that their solutions change more significantly in one direction than others. From the approximation theory point of view, it is well known that it is advantageous to use a properly chosen anisotropic mesh in resolving the anisotropy in numerical approximations.

In this paper, we consider the anisotropic elliptic equation in two dimensions:

$$\begin{cases} -\nabla \cdot (A\nabla u) = f, & \text{in } \Omega, \\ u = g, & \text{on } \Gamma, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with boundary Γ, the anisotropic diffusion $A \in \mathbb{R}^{2\times 2}$ is symmetric positive definite matrix, f is a given function in $L^2(\Omega)$, and $g \in H^{\frac{1}{2}}(\Gamma)$ is a given boundary function.

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Approximate acoustic cloaking in inhomogeneous isotropic space

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We consider the approximate acoustic cloaking in an inhomogeneous isotropic background space. By employing transformation media, together with the use of a sound-soft layer lining right outside the cloaked region, we show that one can achieve the near-invisibility by the "blow-up-a-small-region" construction. This is based on novel scattering estimates corresponding to multiple multi-scale obstacles located in an isotropic space. We develop a novel system of integral equations to decouple the nonlinear scattering interaction among the small obstacle components, the regular obstacle components and the inhomogeneous background medium.

Keywords transformation optics, approximate invisibility cloaking, scattering estimates, integral operator equations

MSC(2010) 35Q60, 35R30, 78A46, 46N20

Citation: Li J Z, Liu H Y, Mao S P. Approximate acoustic cloaking in inhomogeneous isotropic space. Sci China Math, 2013, 56: 2631–2644, doi: 10.1007/s11425-013-4665-2

1 Introduction

A region is said to be *cloaked* if its contents together with the cloak are indistinguishable from the background space to certain exterior wave detections. Blueprints for making objects invisible to electromagnetic waves were proposed by Pendry et al. [22] and Leonhardt [16] in 2006. In the case of electrostatics, the same idea was discussed by Greenleaf et al. [9] in 2003. The key ingredient for those constructions is that optical parameters have transformation properties and could be *push-forwarded* to form new material parameters. The obtained materials/media are called *transformation media*, which we shall examine in the current work for the approximate acoustic cloaking in an inhomogeneous isotropic space.

The transformation media proposed in [9, 22] are rather singular. This poses much challenge to both theoretical analysis and practical fabrication. In order to avoid the singular structures, several regularized approximate cloaking schemes are proposed and investigated in [1,3,4,7,8,13,14,17–20,23]. The basic idea is to introduce regularization into the singular transformation underlying the ideal cloaking, and instead of the perfect invisibility, one considers the "near-invisibility" depending on an asymptotic regularizer. The

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Stability analysis and a priori error estimate of explicit Runge-Kutta discontinuous Galerkin methods for correlated random walk with density-dependent turning rates

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract In this paper, we analyze the explicit Runge-Kutta discontinuous Galerkin (RKDG) methods for the semilinear hyperbolic system of a correlated random walk model describing movement of animals and cells in biology. The RKDG methods use a third order explicit total-variation-diminishing Runge-Kutta (TVDRK3) time discretization and upwinding numerical fluxes. By using the energy method, under a standard Courant-Friedrichs-Lewy (CFL) condition, we obtain L^2 stability for general solutions and a priori error estimates when the solutions are smooth enough. The theoretical results are proved for piecewise polynomials with any degree $k \geqslant 1$. Finally, since the solutions to this system are non-negative, we discuss a positivity-preserving limiter to preserve positivity without compromising accuracy. Numerical results are provided to demonstrate these RKDG methods.

Keywords discontinuous Galerkin method, explicit Runge-Kutta method, stability, error estimates, correlated random walk, positivity-preserving

MSC(2010) 65M60, 65M15

Citation: Lu J F, Shu C W, Zhang M P. Stability analysis and a priori error estimate of explicit Runge-Kutta discontinuous Galerkin methods for correlated random walk with density-dependent turning rates. Sci China Math, 2013, 56: 2645–2676, doi: 10.1007/s11425-013-4739-1

1 Introduction

Aggregation and coordinated movement are common behaviors that can be observed in many species such as schools of fish and flocks of birds, as well as in some cell populations. These behaviors lead to a variety of forms and patterns and they serve different purposes. For example, the school of fish moving in a highly organized way is considered a strategy against predation [15], and under starvation conditions the cells aggregate and form stalks to become fruiting bodies [17]. There are two kinds of factors which play important roles in influencing these behaviors. One is external signals, such as chemicals, light, temperature and humidity. The other is interaction between individuals (self-organized movements) that causes group formation. In this paper, we focus on models describing self-organized movements.

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Multi-neighboring grids schemes for solving PDE eigen-problems

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract Instead of most existing postprocessing schemes, a new preprocessing approach, called multi-neighboring grids (MNG), is proposed for solving PDE eigen-problems on an existing grid $\mathbb{G}(\Delta)$. The linear or multi-linear element, based on box-splines, are taken as the first stage $\boldsymbol{K}_1^h U^h = \lambda_1^h \boldsymbol{M}_1^h U^h$. In this paper, the j-th stage neighboring-grid scheme is defined as $\boldsymbol{K}_j^h U^h = \lambda_j^h \boldsymbol{M}_j^h U^h$, where $\boldsymbol{K}_j^h := \boldsymbol{M}_{j-1}^h \otimes \boldsymbol{K}_1^h$ and $\boldsymbol{M}_j^h U^h$ is to be found as a better mass distribution over the j-th stage neighboring-grid $\mathbb{G}(\Delta)$, and $\boldsymbol{K}_j^h$ can be seen as an expansion of $\boldsymbol{K}_1^h$ on the j-th neighboring-grid with respect to the $(j-1)$-th mass distribution $\boldsymbol{M}_{j-1}^h$. It is shown that for an ODE model eigen-problem, the j-th stage scheme with $2j$-th order B-spline basis can reach $2j$-th order accuracy and even $(2j+2)$-th order accuracy by perturbing the mass matrix. The argument can be extended to high dimensions with separable variable cases. For Laplace eigen-problems with some 2-D and 3-D structured uniform grids, some $2j$-th order schemes are presented for $j \leqslant 3$.

Keywords PDE eigen-problem, discrete Rayleigh quotient, multi-neighboring grids schemes, B-splines

MSC(2010) 65N35, 65F22, 65N25

Citation: Sun J C. Multi-neighboring grids schemes for solving PDE eigen-problems. Sci China Math, 2013, 56: 2677–2700, doi: 10.1007/s11425-013-4731-9

1 Introduction

As is well-known, the Rayleigh quotient of PDE eigen-equation $Lu = \lambda u$ is defined as

$$R[v] := \frac{(Lv, v)}{(v, v)} = \frac{a(v, v)}{(v, v)} \tag{1.1}$$

and the Petrov-Galerkin discrete Rayleigh-quotient can be expressed as

$$R[v^h] := \frac{a(v^h, \tilde{v}^h)}{(v^h, \tilde{v}^h)}, \quad \forall\, v^h \in S^h, \quad \tilde{v}^h \in V^h, \tag{1.2}$$

where the trial space S^h and test space V^h are two finite-dimensional approximation subspaces in the PDE admissible space in (1.1).

For a given grid and its two node bases $\{\phi_1^h, \ldots, \phi_N^h\}$, $\{\tilde{\phi}_1^h, \ldots, \tilde{\phi}_N^h\}$, if $v^h = \sum U_j^h \phi_j^h$, then the discrete Rayleigh-quotient becomes

$$R[v^h] = \frac{(U^h)^{\mathrm{T}} \boldsymbol{K}^h U^h}{(U^h)^{\mathrm{T}} \boldsymbol{M}^h U^h}, \tag{1.3}$$

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A phase model for point spread function estimation in ground-based astronomy

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract In ground-based astronomy, images of objects in outer space are acquired via ground-based telescopes. However, the imaging system is generally interfered by atmospheric turbulence and hence images so acquired are blurred with unknown point spread function (PSF). To restore the observed images, aberration of the wavefront at the telescope's aperture, i.e., the phase, is utilized to derive the PSF. However, the phase is not readily available. Instead, its gradients can be collected by wavefront sensors. Thus the usual approach is to use regularization methods to reconstruct high-resolution phase gradients and then use them to recover the phase in high accuracy. Here, we develop a model that reconstructs the phase directly. The proposed model uses the tight frame regularization and it can be solved efficiently by the Douglas-Rachford alternating direction method of multipliers whose convergence has been well established. Numerical results illustrate that our new model is efficient and gives more accurate estimation for the PSF.

Keywords point spread function, astronomical imaging, phase model, tight frame, alternating direction method of multipliers

MSC(2010) 90C33, 94A08, 97M50

Citation: Chan R H, Yuan X M, Zhang W X. A phase model for point spread function estimation in ground-based astronomy. Sci China Math, 2013, 56: 2701–2710, doi: 10.1007/s11425-013-4742-6

1 Introduction

In ground-based astronomy, images of objects in outer space are acquired via ground-based telescopes. However, the imaging system is generally interfered by atmospheric turbulence and the resulting images are usually blurred. The mathematical model of the imaging system in ground-based astronomy is

$$\boldsymbol{g}(x,y) = \boldsymbol{k}(x,y) * \boldsymbol{f}(x,y) + \varepsilon(x,y), \tag{1.1}$$

where $\boldsymbol{f}(x,y)$ is the true object in outer space, $\boldsymbol{g}(x,y)$ is its observation by the ground-based telescope, $\boldsymbol{k}(x,y)$ is the *point spread function* (PSF) caused by the atmospheric turbulence, "$*$" is the convolutional operation, and $\varepsilon(x,y)$ is zero-mean white noise. When the PSF $\boldsymbol{k}$ is known, the problem (1.1), i.e., finding the true $\boldsymbol{f}$ from the observation $\boldsymbol{g}$, is an ill-posed inverse problem arising from the area of image

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Extrapolation cascadic multigrid method on piecewise uniform grid

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract The triangular linear finite elements on piecewise uniform grid for an elliptic problem in convex polygonal domain are discussed. Global superconvergence in discrete H^1-norm and global extrapolation in discrete L^2-norm are proved. Based on these global estimates the conjugate gradient method (CG) is effective, which is applied to extrapolation cascadic multigrid method (EXCMG). The numerical experiments show that EXCMG is of the global higher accuracy for both function and gradient.

Keywords linear finite element, piecewise uniform grid, superconvergence, extrapolation, extrapolation cascadic multigrid method

MSC(2010) 65N30

Citation: Chen C M, Hu H L. Extrapolation cascadic multigrid method on piecewise uniform grid. Sci China Math, 2013, 56: 2711–2722, doi: 10.1007/s11425-013-4732-8

1 Introduction

In modern science and engineering computations people faces to several main difficulties: large scale, high efficiency, high accuracy, posterior estimates and so on, in which a key difficulty is to guarantee the accuracy desired. We are interested in increasing high accuracy by superconvergence and extrapolation, which increases work less and is more efficient for high dimension.

On the other hand, to solve N-order linear system of equation derived by the finite difference method or finite element method for PDE's, multigrid method (MG) is an optimal algorithm, solving N-order linear system only by work $O(N)$. There are two important types:

1) (Classical) Multigrid method (MG) has used three operators between different levels of grid, i.e., the interpolation, restriction and iteration. There are V-cycle and W-cycle, whose codes seem a bit complicated.

2) Cascadic multigrid method (CMG) was proposed by Borneman-Deunfhard [3] in 1996 and Shaidurov [22, 24] during 1996 and 1999, where only the interpolation and iteration from coarse grids to fine grids are used, whose code is easily realized. Soon, Shi et al. [26, 28, 29] made further analysis and extension [14, 15, 21, 23, 25, 27, 30–33].

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Uniformly convergent $H(\mathrm{div})$-conforming rectangular elements for Darcy-Stokes problem

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We consider a singular perturbation problem which describes 2D Darcy-Stokes flow. An $H(\mathrm{div})$-conforming rectangular element, DS-R14, is proposed and analyzed first. This element has 14 degrees of freedom for velocity and is proved to be uniformly convergent with respect to perturbation constant. We then simplify this element to get another $H(\mathrm{div})$-conforming rectangular element, DS-R12, which has 12 degrees of freedom for velocity. The uniform convergence is also obtained for this element. Finally, we construct a de Rham complex corresponding to DS-R12 element.

Keywords Darcy-Stokes problem, $H(\mathrm{div})$-conforming rectangular elements, uniform convergence

MSC(2010) 65N30

Citation: Chen S C, Dong L N, Qiao Z H. Uniformly convergent $H(\mathrm{div})$-conforming rectangular elements for Darcy-Stokes problem. Sci China Math, 2013, 56: 2723–2736, doi: 10.1007/s11425-013-4692-z

1 Introduction

We consider the Darcy-Stokes problem. Find the velocity $\boldsymbol{u}$ and the pressure p such that

$$\begin{cases} (I-\varepsilon^2\Delta)\boldsymbol{u}-\mathbf{grad}p=\boldsymbol{f} & \text{in } \Omega,\\ \mathrm{div}\boldsymbol{u}=g & \text{in } \Omega,\\ \boldsymbol{u}=0 & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

Here $\Omega\subset\mathbb{R}^2$ is a bounded, convex and connected polygonal domain with boundary $\partial\Omega$, $\varepsilon\in(0,1]$ is a parameter, Δ is the standard Laplace operator. The vector field $\boldsymbol{f}$ and scalar field g are given data. The problem (1.1) admits a unique solution if g satisfies $\int_\Omega g\,dx=0$ and p is determined only up to addition of a constant. When $\varepsilon=1$ and $g=0$, this problem is simply a standard Stokes problem, but with an additional non-harmful lower order term. When $\varepsilon=0$, the first equation of (1.1) has the form of Darcy's law for flow in a homogeneous porous medium. When the singular perturbation parameter ε varies between 0 and 1, the partial differential equations (1.1) change the type. This problem arises in various physical models, for examples, subsurface flow problem [15, 26], heat and mass transfer in pipes [19, 23],

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Application of auxiliary space preconditioning in field-scale reservoir simulation

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We study a class of preconditioners to solve large-scale linear systems arising from fully implicit reservoir simulation. These methods are discussed in the framework of the auxiliary space preconditioning method for generality. Unlike in the case of classical algebraic preconditioning methods, we take several analytical and physical considerations into account. In addition, we choose appropriate auxiliary problems to design the robust solvers herein. More importantly, our methods are user-friendly and general enough to be easily ported to existing petroleum reservoir simulators. We test the efficiency and robustness of the proposed method by applying them to a couple of benchmark problems and real-world reservoir problems. The numerical results show that our methods are both efficient and robust for large reservoir models.

Keywords reservoir simulation, black-oil model, fully implicit method, auxiliary space preconditioning, algebraic multigrid method, Krylov subspace iterative method

MSC(2010) 65N22, 65F08, 65N55

Citation: Hu X Z, Xu J C, Zhang C S. Application of auxiliary space preconditioning in field-scale reservoir simulation. Sci China Math, 2013, 56: 2737–2751, doi: 10.1007/s11425-013-4737-3

1 Introduction

Petroleum reservoir simulation is a tool for predicting hydrocarbon reservoir performance under various operating conditions. It helps petroleum engineers to obtain information pertaining to the processes that take place within oil reservoirs—information that can be used to maximize recovery and minimize environmental damage. The fully implicit method (FIM), also referred to as simultaneous solution (SS) (see [11]), is a natural numerical discretization scheme that is widely-used in academic and commercial reservoir simulators at present. This method provides better stability than other often-used methods, such as implicit pressure explicit saturation (IMPES). However, FIM gives rise to coupled, non-symmetric, ill-conditioned Jacobian systems, which are usually difficult to solve.

The most time-consuming part of FIM is solving the Jacobian system $Ax = b$ at each Newton iteration. Very often, solving such linear systems with direct or iterative solvers takes more than 80% of the computational time in reservoir simulation. Furthermore, ever-increasing demand for more accurate numerical simulation has led to larger and more heterogenous discrete field-scale reservoir models. And

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An adaptive time stepping method with efficient error control for second-order evolution problems

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract This work is concerned with time stepping finite element methods for abstract second order evolution problems. We derive optimal order a posteriori error estimates and a posteriori nodal superconvergence error estimates using the energy approach and the duality argument. With the help of the a posteriori error estimator developed in this work, we will further propose an adaptive time stepping strategy. A number of numerical experiments are performed to illustrate the reliability and efficiency of the a posteriori error estimates and to assess the effectiveness of the proposed adaptive time stepping method.

Keywords a posteriori error analysis, adaptive algorithm, reconstruction, evolution problems

MSC(2010) 65M60, 65M15

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1 Introduction

Adaptive time stepping methods are very important in developing efficient algorithms for solving evolution problems arising from fluid dynamics, epitaxial growth and many other applied sciences (cf. [11,13,18,21, 22]). Such methods enable us to adopt feasible time steps to carry out time discretization, so that we are able to take much less computational cost to get numerical solutions with desired accuracy. The strategy for choosing time steps adaptively is very technical and problem-oriented, and one typical approach is based on the a posteriori error estimator corresponding to the underlying problem [5, 7]. Hence, a posteriori error analysis in time plays an important role in constructing adaptive time stepping methods.

As far as we know, there has existed a very sophisticated investigation on a posteriori error analysis for abstract first order evolution problems (cf. [1–4,10]). Precisely speaking, with the help of higher order appropriate reconstructions of the approximate solutions, the optimal order a posteriori error estimates of some time discretization methods were established in [1–3,10]. Furthermore, a posteriori superconvergence estimates for the error at the nodes for Galerkin and Runge-Kutta methods were derived in [4]. However,

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l^1-error estimates on the immersed interface upwind scheme for linear convection equations with piecewise constant coefficients: A simple proof

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract A linear convection equation with discontinuous coefficients arises in wave propagation through interfaces. An interface condition is needed at the interface to select a unique solution. An upwind scheme that builds this interface condition into its numerical flux is called the immersed interface upwind scheme. An l^1-error estimate of such a scheme was first established by Wen et al. (2008). In this paper, we provide a simple analysis on the l^1-error estimate. The main idea is to formulate the solution to the underline initial-value problem into the sum of solutions to two convection equations with constant coefficients, which can then be estimated using classical methods for the initial or boundary value problems.

Keywords l^1-error estimates, linear convection equation with discontinuous coefficients, immersed interphase method

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1 Introduction

We are interested in the linear convection equation

$$\begin{cases} u_t + (c(x)u)_x = 0, & t > 0, \quad x \in \mathbb{R}, \\ u(x,0) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.1}$$

with a piecewise constant coefficient

$$c(x) = \begin{cases} c^- > 0, & x < 0, \\ c^+ > 0, & x > 0. \end{cases} \tag{1.2}$$

Such equations arise in modeling wave propagation through interfaces, where jumps in $c(x)$ correspond to interfaces between different media. This is also the simplest example of hyperbolic conservations laws

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Abstract principal component analysis

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We present the basic idea of abstract principal component analysis (APCA) as a general approach that extends various popular data analysis techniques such as PCA and GPCA. We describe the mathematical theory behind APCA and focus on a particular application to mode extractions from a data set of mixed temporal and spatial signals. For illustration, algorithmic implementation details and numerical examples are presented for the extraction of a number of basic types of wave modes including, in particular, dynamic modes involving spatial shifts.

Keywords abstract principal component analysis, pattern recognition, mode extraction, reduced order modeling, traveling waves

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1 Introduction

Principal component analysis (PCA) [11, 12] is among the most widely used techniques for dimension reduction, feature identification, pattern recognition and other types of exploratory data analysis. For different application disciplines, it is also named as principal orthogonal decomposition (POD), Karhunen-Loève transform (KLT) and empirical orthogonal function (EOF) analysis. PCA manages to find a linear subspace with principal orthogonal components which are linear combinations of original variables from the given data. The order of principal components is determined by their significance reflected from the variance of combined variables. PCA is usually implemented in a straightforward way with the singular value decomposition (SVD). There are various extensions of the classic PCA sharing the same spirit. For example, probabilistic PCA (PPCA) [19] provides a probabilistic formulation from a Gaussian latent variable model based on the maximum-likelihood estimation of parameters. Nonlinear PCA (NLPCA) [14] and kernel PCA (KPCA) [18] identify nonlinear components or manifolds by an embedding of given sample data points into a high-dimensional feature space using an unknown nonlinear map which may not have an explicit form; the classic PCA is then performed in the feature space with a pre-selected kernel for the description of nonlinearities. Generalized PCA (GPCA) [20, 22] is developed with the motivation of grouping given sample points to several linear subspaces with the number and dimensions

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Initial-boundary value problem of three phase flow in porous media

Dedicated to Professor Shi Zhong-Ci on the Occasion of his 80th Birthday

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Abstract We study the mathematical model of three phase compressible flows through porous media. Under the condition that the rock, water and oil are incompressible, and the compressibility of gas is small, we present a finite element scheme to the initial-boundary value problem of the nonlinear system of equations, then by the convergence of the scheme we prove that the problem admits a weak solution.

Keywords porous media, three phase flow, weak solution, finite element method

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1 Introduction

We study the mathematical model of three phase flows through porous media. It is known that the rigorous mathematical theory on the existence of a weak solution to two phase incompressible flows is complete [1]. In [3], we studied the model of two phase compressible flows under the condition of slightly compressibility. We study three phase immiscible flows here. Under the conditions that the rock, the oil and the water are incompressible, we present a finite element scheme to the initial-boundary value problem of the nonlinear system of equations, then by the convergence of the scheme we prove that the problem admits a weak solution. The governing system of equations reads

$$\frac{\partial}{\partial t}(\sigma\phi B_j s_j) + \nabla \cdot \phi_j = 0, \quad j = 1, 2, 3, \tag{1.1}$$

where the first flow represents water, and the second and third flows represent oil and gas, respectively. The expressions for the volumetric flow vector ϕ_j is

$$\phi_j = -\sigma K \frac{k_{rj} B_j}{\mu_j}(\nabla p_j - \rho_j g \nabla z). \tag{1.2}$$

Notation. s_j: saturation, $s_j = (\text{volume of fluid } j)/\ (\text{volume of fluid } 1+2+3)$. $\sigma(x)dx$ = 3-dimensional volume; p_j: pressure in fluid number j; $\phi = \phi(x, p) \in [0, 1]$: porosity; $B_j(p) = \frac{\rho_j(p)}{\rho_j(p_{\text{ref}})}$: the volume factor, ρ_j: density, where p is the global pressure, and p_{ref} is a reference pressure; $K(x)$: the permeability of the

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