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Nonuniqueness in the BEM/BIEM and its treatment

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Abstract

Four treatment of nonuniqueness of degenerate scale occurring in the BEM/BIEM by NTOU/MSV group is reviewed first. In this research, we examine the sufficient and necessary boundary integral formulation for the 2D Laplace problem subject to the Dirichlet boundary condition. Both analytical study and BEM implementation is addressed. For the analytical study, we employ the degenerate kernel in the polar and elliptical coordinates to prove the unique solution of Fichera’s formulation for any size of circle and ellipse, respectively. In numerical implementation, the BEM program developed by NTOU/MSV group is employed to see the validity of the above formulation. Finally, an ellipse case is demonstrated by using five regularization techniques hypersingular formulation, method of adding a rigid body mode, rank promotion by adding the boundary flux equilibrium, CHEEF method and the percent Fichera’s method. Beside, an arbitrary shape is numerically implemented to check the uniqueness solution of BEM.

Problem description

In civil and hydraulic engineering practice, seepage and torsion problems can be modeled by using the 2D Laplace equation. It is well known that BEM is an efficient approach to deal with these problems. However, the nonuniqueness solution may occur.

In this study, we examine the sufficient and necessary boundary integral formulation for the 2D Laplace problem subject to the Dirichlet boundary condition. The governing equation and boundary condition of the Laplace problem subject to the Dirichlet boundary condition are shown below:

$$\nabla^2 u(\mathbf{x}) = 0, \ \mathbf{x} \in D$$

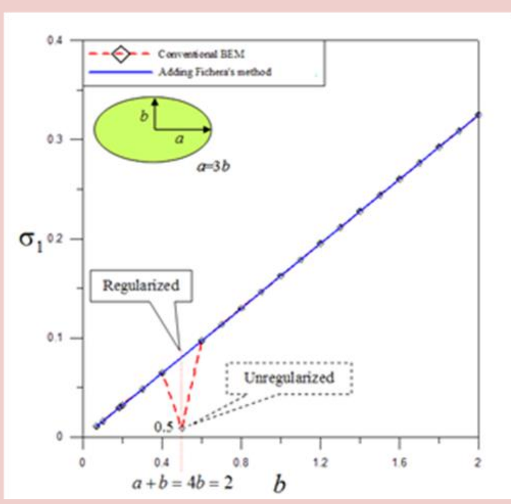
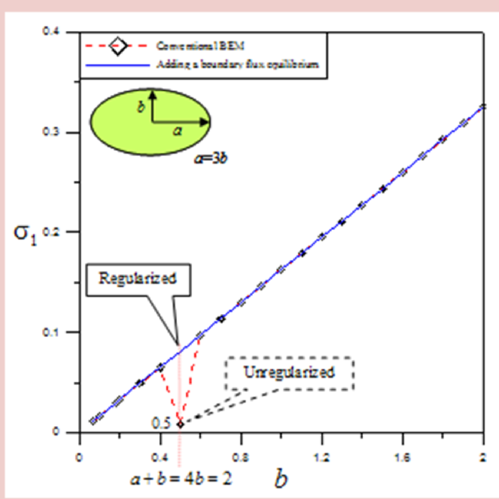
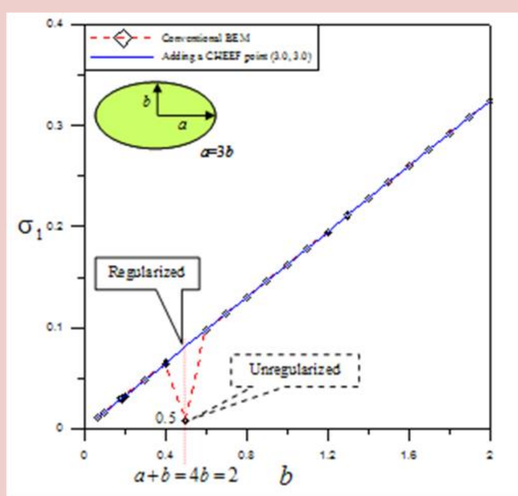
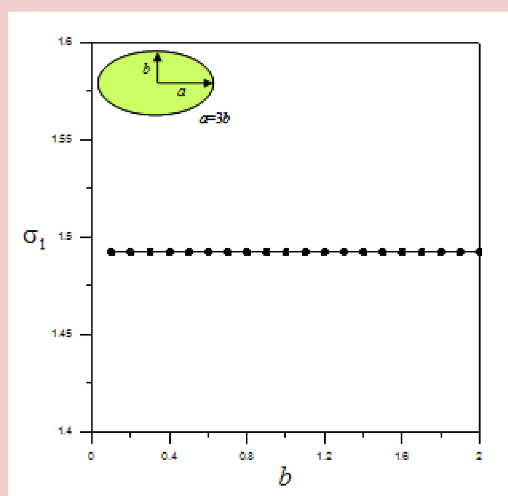
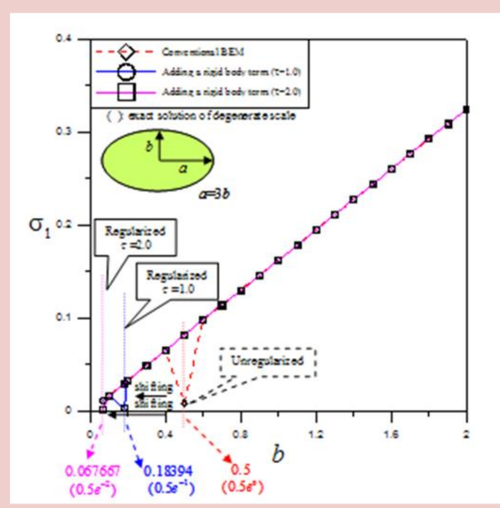
$$u(\mathbf{x}) = f(\mathbf{x}), \ \mathbf{x} \in B$$

Five regularization techniques for nonuniqueness in the BEM/BIEM

Method	Integral formulation	Extra constraint
Fichera’s method	$\int_B U(\mathbf{s},\mathbf{x})\alpha(\mathbf{s})dB(\mathbf{s}) + \gamma = f(\mathbf{x}), \ \mathbf{x} \in B$	$\int_B \alpha(\mathbf{s})dB(\mathbf{s}) = 0$
The boundary flux equilibrium	$2\pi u(\mathbf{x}) = \int_B T(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s}), \ \mathbf{x} \in D$	$\int_B t(\mathbf{s})dB(\mathbf{s}) = 0$
The CHEEF method		CHEEF point
The hypersingular formulation		LM formulation
The method of adding a rigid body mode		$U(\mathbf{s},\mathbf{x}) = \ln r + 1$

Results and discussion

The first minimum singular value versus scale after regularization

Method	Fichera’s method	The boundary flux equilibrium	The CHEEF method	The hypersingular formulation	The method of adding a rigid body mode
Formulation	$\int_B U(\mathbf{s},\mathbf{x})\alpha(\mathbf{s})dB(\mathbf{s}) + \gamma = f(\mathbf{x}), \ \mathbf{x} \in B$ $0 = \int_B \alpha(\mathbf{s})dB(\mathbf{s})$	$2\pi u(\mathbf{x}) = \int_B T(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s})$ $0 = \int_B t(\mathbf{s})dB(\mathbf{s})$	$0 = \int_B T(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s}), \ \mathbf{x} \in D^c$	$2\pi u(\mathbf{x}) = \int_B T(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s})$ $2\pi t(\mathbf{x}) = \int_B M(\mathbf{s},\mathbf{x})u(\mathbf{s})dB(\mathbf{s}) - \int_B L(\mathbf{s},\mathbf{x})t(\mathbf{s})dB(\mathbf{s})$	$U(\mathbf{s},\mathbf{x}) = \ln r + 1$
Results					

Conclusions

Both two formulations, the indirect BEM using the Fichera’s idea as well as the direct BEM with the flux equilibrium, yield the unique solution for any scale size of domain.

References

[1] J.T.Chen, C.S.Wu, K.H.Chen and Y.T.Lee, 2006, Degenerate scale for analysis of circular plate using the boundary integral equations and boundary elements method, Computational Mechanics, Vol.38, pp.33-49.

[2] J.T.Chen, S.R.Lin and K.H.Chen, 2005, Degenerate scale problem when solving Laplace equation by BEM and its treatment, Int.J.Numer.Meth.Engng., Vol.62, NO.2, pp.233-261.

[3] J.T.Chen, W.C.Chen, S.R.Lin and I.L.Chen, 2003, Rigid body mode and spurious mode in the dual boundary element formulation for the Laplace equation, Computers and Structures, Vol.81,NO.13,pp.1395-1404.

[4] G. Fichera, Linear elliptic equations of higher order in two independent variables and singular integral equation, Proc. Conference on partial Differential Equations and Continuum Mechanics (Madison, Wis.), Univ. of Wisconsin Press, Madison, 1961.