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Null-field BIEM for solving a scattering problem from a point source to a two-layer prolate spheroid

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Abstract

In this work, the acoustic scattering problem from a point source to a two-layer prolate spheroid is solved by using the null-field boundary integral equation method (BIEM) in conjunction with degenerate kernels. To fully utilize the spheroid geometry, the fundamental solutions and the boundary densities are expanded by using the addition theorem and spheroidal harmonics in the prolate spheroidal coordinates, respectively. For the confocal structure, the analytical solution can be analytically derived by using the null-field BIEM. Besides, it is interesting that the kidney-stone system can be simulated by a two-layer spheroid structure. Finally, an example is considered for the parameter study. Also, a special case of the acoustic scattering problem of a point source by a rigid scatterer is also done by setting the density of inner medium to be infinity.

Problem description

The governing equation of the scattering problem of a point source is the non-homogeneous three-dimensional Helmholtz equation as follows:

$$(\nabla^2+k^2)u(\mathbf{x})=-4\pi\delta(\mathbf{x}-\mathbf{r}')$$

The boundary condition on the surface of the igid scatterer is

$$\frac{\partial u_k(\mathbf{x})}{\partial n_{\mathbf{x}}}=0,\ \mathbf{x}\in S_1$$

The interface conditions on the surface are

$$\begin{cases} u_{in}(\mathbf{x})+u_{sc}(\mathbf{x})=u_k(\mathbf{x}),\ \mathbf{x}\in S_0 \\ \dfrac{1}{\rho_{ext}}\dfrac{\partial(u_{in}(\mathbf{x})+u_{sc}(\mathbf{x}))}{\partial n_{\mathbf{x}}}=\dfrac{1}{\rho_k}\dfrac{\partial u_k(\mathbf{x})}{\partial n_{\mathbf{x}}},\ \mathbf{x}\in S_0 \end{cases}$$

Null-field boundary integral formulations-the present version

$$4\pi u(\mathbf{x})=\int_S T(\mathbf{s},\mathbf{x})u(\mathbf{s})dS(\mathbf{s})-\int_S U(\mathbf{s},\mathbf{x})t(\mathbf{s})dS(\mathbf{s}),\ \mathbf{x}\in V\cup S$$

$$0=\int_S T(\mathbf{s},\mathbf{x})u(\mathbf{s})dS(\mathbf{s})-\int_S U(\mathbf{s},\mathbf{x})t(\mathbf{s})dS(\mathbf{s}),\ \mathbf{x}\in V^c\cup S$$

Expansion for boundary densities

$$u(\mathbf{s})=\sum_{v=0}^{\infty}\sum_{w=0}^{v}g_{wv}S_{wv}(q,\eta_{\mathbf{s}})\cos(w\phi_{\mathbf{s}})+\sum_{v=1}^{\infty}\sum_{w=1}^{v}h_{wv}S_{wv}(q,\eta_{\mathbf{s}})\sin(w\phi_{\mathbf{s}}),\ \mathbf{s}\in S$$

$$t(\mathbf{s})=\frac{\sqrt{\xi_{\mathbf{s}}^2-1}}{c\sqrt{\xi_{\mathbf{s}}^2-\eta_{\mathbf{s}}^2}}\left[\sum_{v=0}^{\infty}\sum_{w=0}^{v}p_{wv}S_{wv}(q,\eta_{\mathbf{s}})\cos(w\phi_{\mathbf{s}})+\sum_{v=1}^{\infty}\sum_{w=1}^{v}q_{wv}S_{wv}(q,\eta_{\mathbf{s}})\sin(w\phi_{\mathbf{s}})\right],\ \mathbf{s}\in S,$$

The closed-form fundamental solution

$$U(\mathbf{s},\mathbf{x})=-\frac{e^{ikr}}{r}=-ikh_0^{(1)}(kr),\ r=|\mathbf{x}-\mathbf{s}|$$

Degenerate kernel

$$U(\mathbf{s},\mathbf{x})=\begin{cases} U_N^E(\mathbf{s},\mathbf{x})=-2ik\sum_{n=0}^{N}\sum_{m=0}^{n}\dfrac{\varepsilon_m}{\Lambda_{mn}}je_{mn}(q,\xi_{\mathbf{s}})he_{mn}(q,\xi_{\mathbf{x}}) \\ \qquad S_{mn}(q,\eta_{\mathbf{s}})S_{mn}(q,\eta_{\mathbf{x}})\cos[m(\phi_{\mathbf{s}}-\phi_{\mathbf{x}})],\ \xi_{\mathbf{x}}\geq\xi_{\mathbf{s}}, \\ U_N^I(\mathbf{s},\mathbf{x})=-2ik\sum_{n=0}^{N}\sum_{m=0}^{n}\dfrac{\varepsilon_m}{\Lambda_{mn}}je_{mn}(q,\xi_{\mathbf{x}})he_{mn}(q,\xi_{\mathbf{s}}) \\ \qquad S_{mn}(q,\eta_{\mathbf{s}})S_{mn}(q,\eta_{\mathbf{x}})\cos[m(\phi_{\mathbf{s}}-\phi_{\mathbf{x}})],\ \xi_{\mathbf{x}}<\xi_{\mathbf{s}}, \end{cases}$$

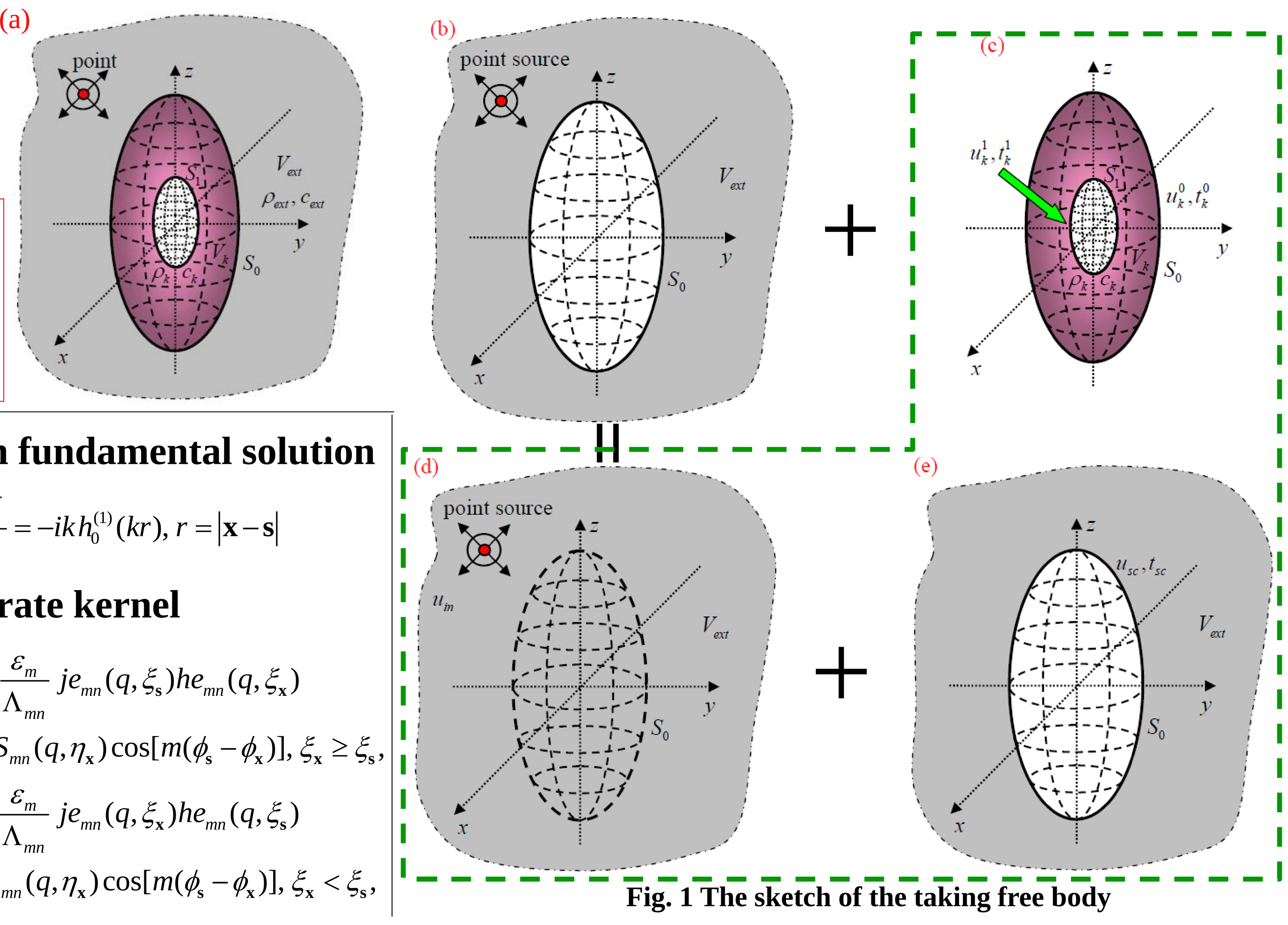
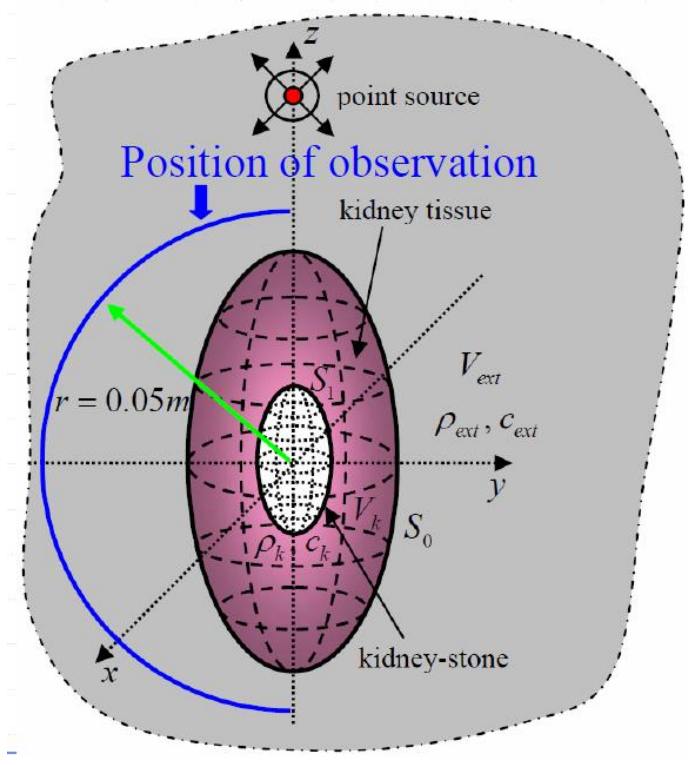


Fig. 1 The sketch of the taking free body

Results and discussion



$\rho_k = 1022\ Kg/m^3$
$c_k = 1533\ m/\text{sec}$
$\rho_{ext} = 1000\ Kg/m^3$
$c_{ext} = 1493\ m/\text{sec}$
$r_0 = 0.1m$
$r = 0.05m$
$a_1 = 0.01m$
$b_1 = 0.005m$
$\xi_0 = 4\xi_1$

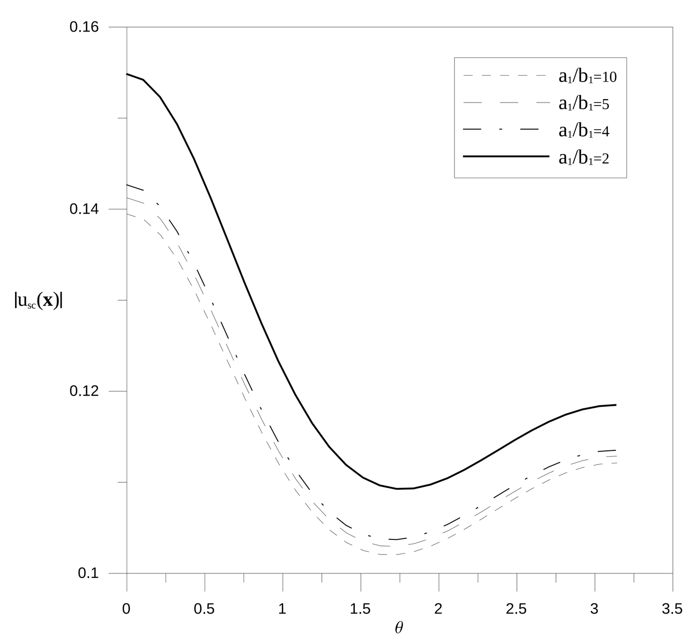


Fig. 3 Scattering field versus by changing the ratio of kidney-stone radii

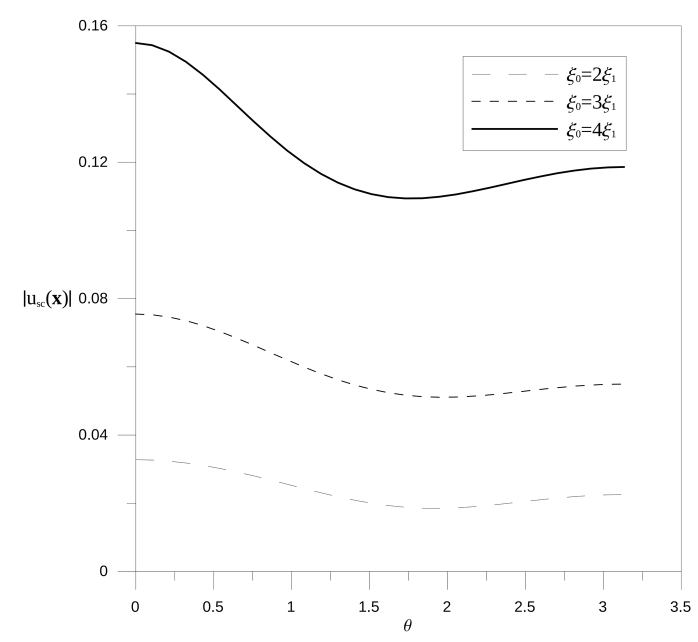


Fig. 5 Scattering field versus by changing the radial parameter of the kidney surface

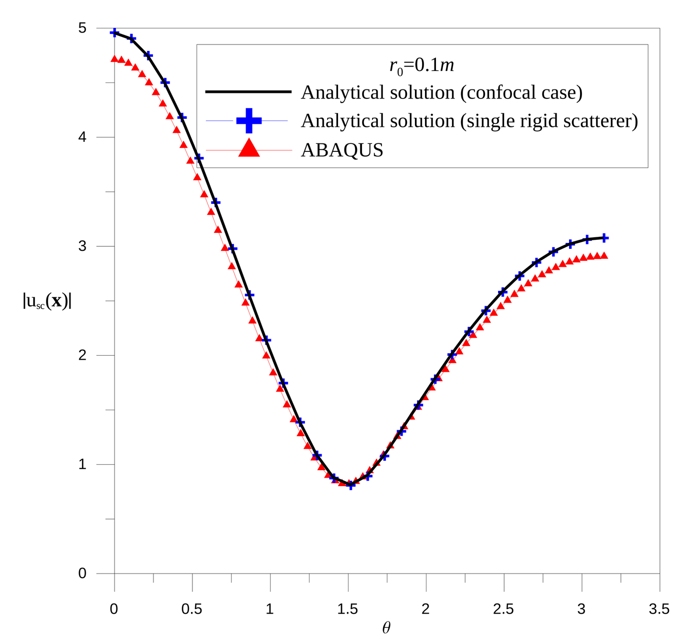


Fig. 7 Scattering field versus for a case of a single rigid scatterer (r_0=0.1m)

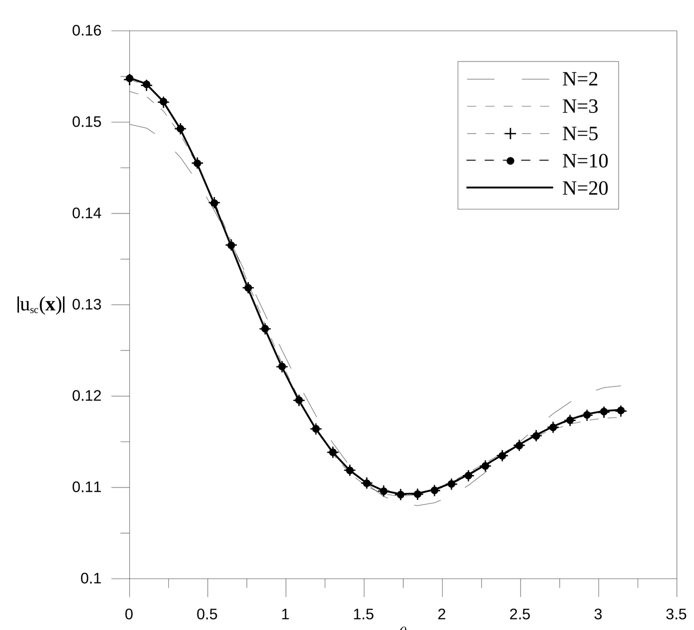


Fig. 2 Scattering field using different number of truncation terms

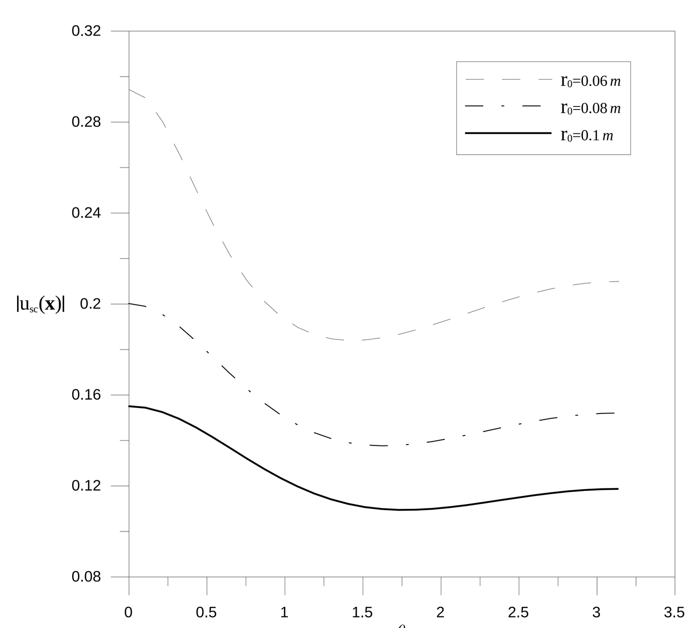


Fig. 4 Scattering field versus by varying the position of the point source

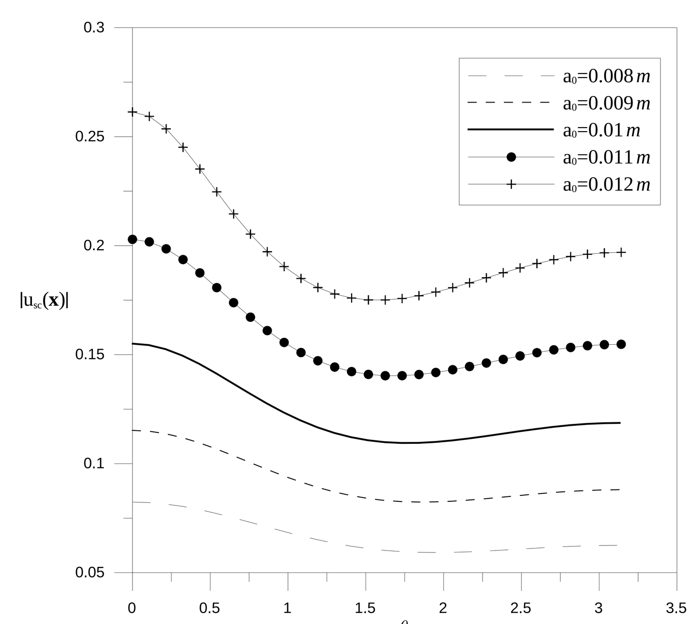


Fig. 6 Scattering field versus by changing the length of semi-major axis

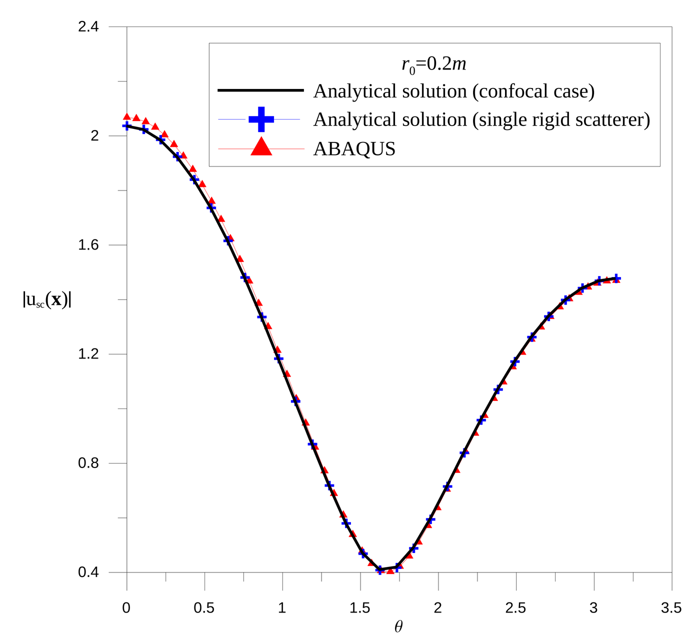


Fig. 8 Scattering field versus for a case of a single rigid scatterer (r_0=0.2m)

Conclusions

1. Based on the addition theorem, the closed-form fundamental solution is expanded into the degenerate kernel in the prolate spheroidal coordinates.
2. The size of kidney-stone can be predicted by the parameter study.
3. Numerical results can be used as a reference for clinical medical treatment.

References

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