INTERNATIONAL UNION OF THEORETICAL AND
APPLIED MECHANICS

MECHANICS

AT THE TURN OF THE CENTURY

1900
1990
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The Generalized Ray Theory and Transient Responses of Layered Elastic Solids

Y.-H. Pao and R.R. Gajewski

In: Physical Acoustics, ed. P. Mason & R.N Thurston, 13 (1977), 183-265.

Proposed by *Franz Ziegler (Vienna).*

Motivation:

Research interests continuously focused on wave phenomena in elastic solids since d`Alembert. A State-of-the-Art Review is given in the famous book by W.M. Ewing, W.S. Jardetzky and Frank Press, Elastic Waves in Layered Media, McGraw-Hill, 1957, where the Weyl-Sommerfeld integral expansion of cylindrical or spherical waves is mentioned as a mathematical possibility of a representation but with no hope of fruitful application. Twenty years later, Pao reformulated the boundary value problem of wave refraction on plane interfaces between two dissimilar elastic bodies and rigorously derived the plane wave expansion by combining the Laplace- and Fourier transformations. Thereby, the generalized ray integrals were systematically constructed and source (emittance) functions of various point sources identified, receiver functions for displacement, velocity, acceleration and stress components at buried and surface observatory points derived, the phase function of the plane waves geometrically determined for all possible paths between source and receiver, reflection and transmission coefficients of displacement potentials explored, etc. The limiting process from a buried source to a surface source is also performed rendering the surface-emittance functions at once.

These ray integrals are transformed such that the inverse Laplace transforms render a finite numerical integration in the complex slowness plane. They are ordered consecutively by their arrival times in a fixed source-receiver configuration in a layered solid. Also the numerical implementations are fully worked in the paper.

The paper is a landmark in mechanics and in geophysics. For the first time generalized ray integrals were constructed systematically for any source-receiver configuration in a layered solid. Body waves, the Rayleigh wave and refracted waves were identified. Further, presenting the detailed integration procedure (even for parallel computing in a matrix notation) made the ray integrals a competitive working tool. Summation of the ray integrals gives the exact transient solution for any finite observation time. Thus, a major progress was achieved in the solution technique for a large class of problems, reaching from the homogeneous and layered half-space to layered plates.

Based on this paper, Pao explored the divergence effect of waves propagating in a dipping layer in 1982.