

Degenerate scale problems of null-field methods for Dirichlet problems of Laplace's equation

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Abstract

For circular domains with circular holes, the null field method (NFM) is proposed by J.T. Chen and his coresearchers when solving boundary integral equation [1]. The explicit algebraic equations of the NFM are recently given in [2], and their conservative schemes are proposed in [3]. However, even for the Dirichlet problem of Laplace's equation, when the logarithmic capacity (transfinite diameter) $C_\Gamma = 1$, the solutions may not exist, or not being unique if existing, to cause a singularity of the discrete algebraic equations or the original boundary integral equation, and is discussed in Christiansen [4] and many research articles, called the degenerate scale problems. This talk is devoted to a complete and comprehensive analysis on the degenerate scale problem of the

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NFM for Dirichlet problems. We have found that the conservative schemes can always bypass the degenerate scale problems; though numerically it causes a severe instability. A new pseudo-singularity property is discovered that only the minimal singular value σ_{\min} of the discrete matrices is infinitesimal to cause the instability. However, even for the Dirichlet problem of Laplace's equation, there also exist several kinds of pitfalls of field nodes possibly leading to degenerate scale problems, which will be revealed in detail. To remove singularity of discrete matrices and to restore good stability, several effective techniques are proposed, the singular value decomposition (TSVD) is recommended. Numerical experiments are carried out to verify the theoretical analysis made. Based on the analysis and computation, the trouble of degenerate scale problems when solving by the NFM which have bothered researchers for a long time, see [5, 6], can be bypassed.

Keywords: *Null field method, the conservative scheme, pseudo-singularity, degenerate scale problem, the logarithmic capacity (transfinite diameter), circular domains, Dirichlet problem, truncated singular value decomposition.*

References

- [1] J. T. Chen and W. C. Shen, *Null-field approach for Laplace problems with circular boundaries using degenerate kernels*, Numer. Meth. PDE., Vol. 25, pp. 63 – 86, 2009.
- [2] Z.C. Li, H. T. Huaug, C. P. Liaw and M. G. Lee, *The null-field method of Dirichlet problems of Laplace's equation on circular domains with circular holes*, Engineering Analysis with Boundary Elements, Vol. 36, pp. 477-491, 2012.

- [3] M. G. Lee, Z.C. Li, H. T. Huaug, and J. Y. Chiang, *Conservative schemes and degenerate scale problems of null-field methods for Dirichlet problems of Laplace's equation*, accepted by Engineering Analysis with Boundary Elements, 2012.
- [4] S. Christiansen, *Integral equations without a unique solution can be made useful for solving some plane harmonic equations*, J. Inst Math.Appl., Vol. 143–159, 1975.
- [5] J. T. Chen, J. H. Lin, S. R. Kuo and Y. P. Chui, *Analytical study and numerical experiments for degenerate scale problems in the boundary element method using degenerate kernel and circulants*. Engineering Analysis with Boundary Elements, Vol. 25, pp. 819 – 828,2001.
- [6] J. T. Chen and W. C. Shen, *Degenerate scale for multiply connected Laplace problems*, Mechanics, Research Communication, Vol. 34, pp. 69 – 77, 2007.