

Conservative schemes of the null field method for Dirichlet problems of Laplace's equation in circular domains with circular holes

Hung-Tsai Huang*, Zi-Cai Li†, Ming-Gong Lee‡

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Abstract

Recently, the null-field method (NFM) has been proposed by Chen and his co-researchers for solving boundary value problems involving circular domains with circular holes. The explicit algebraic equations of the NFM are derived in our recent paper. However, even for the Dirichlet problem of Laplace's equation, when the logarithmic capacity (transfinite diameter) $C_\Gamma = 1$ is given, the solutions may not exist, or not unique if existing, to cause a singularity of the discrete algebraic equations, called the degenerate scale problems. In this paper, the new conservative schemes of NFM are proposed. The conservative schemes can always bypass the degenerate scale problems; though numerically it causes a severe instability. A new pseudo-singularity property is discovered that only the minimal singular value σ_{\min} of the discrete matrices is infinitesimal to cause the instability. To restore good stability of the conservative schemes, the over-determined systems and the truncated singular value decomposition (TSVD) are proposed.

*Speaker, Department of Applied Mathematics, I-Shou University, Kaohsiung, Taiwan 84001, huanght@isu.edu.tw.

†Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung, Taiwan 80424.

‡Department of Leisure and Recreation Management, Chung Hua University, Hsin-Chu, Taiwan 30012.