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A special crack tip displacement discontinuity element

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Abstract

Based on the analytical solution to the problem of a constant discontinuity in displacement over a finite line segment in the x, y plane of an infinite elastic solid and the note of the crack tip element by Crouch, in the present paper, the special crack tip displacement discontinuity element is developed. Further the analytical formulas for the stress intensity factors of crack problems in general plane elasticity are given. In the boundary element implementation the special crack tip displacement discontinuity element is placed locally at each crack tip on top of the non-singular constant displacement discontinuity elements that cover the entire crack surface. Numerical results show that the displacement discontinuity modeling technique of a crack presented in this paper is very effective.

Keywords: Crack tip element; Stress intensity factor; Crack; Displacement discontinuity

1. Introduction

Among several elastic two-dimensional crack modeling strategies by the boundary element method, there exist the multi-domain formulation (Blandford et al., 1981), the stress formulation with regularization (Balas et al., 1989), and the dual boundary element method (Hong and Chen, 1988; Portela and Aliabadi, 1992). For each formulation, options are available such as building in the crack tip stress singularity (Tanaka and Itoh, 1987), using the quarter-point boundary element (Blandford et al., 1981), and strategically refining the near-crack-tip non-singular element. Further details on elastic crack analysis by the boundary element method are given in (Cruse, 1989; Aliabadi and Rooke, 1991).

Even though much achievement has been made in crack modeling techniques, both simple and practical crack modeling technique is still needed, in particular for complex multiple crack growth problems (Cotterel and Rice, 1980; Khan and Paul, 1988). The displacement discontinuity method (Scouch et al., 1983), as a boundary element method, is very well used to analyze the crack problems in plane elasticity because, physically, one may imagine a displacement discontinuity as a line crack whose opposing surfaces have been displaced relative to one another. Based on the analytical solution (Scouch, 1976; Scouch et al., 1983) to the problem of a constant discontinuity in displacement over a finite line segment in the x, y plane of an infinite elastic solid and the note of the crack tip element, in the present paper, the special crack tip displacement discontinuity elements are developed to compute the stress intensity factors of crack problems in general plane elasticity. In the boundary element implementation the special crack tip displacement

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discontinuity element is placed locally at each crack tip on top of the ordinary non-singular displacement discontinuity elements that cover the entire crack surface. Numerical results show that the displacement discontinuity modeling technique of a crack presented in this paper is very effective.

2. Theoretical foundation of constant displacement discontinuity method

The problem of a constant displacement discontinuity over a finite line segment in the x, y plane of an infinite elastic solid is specified by the condition that the displacements be continuous everywhere except over the line segment in question. The line segment may be chosen to occupy a certain portion of the x axis, say the portion $|x| < a$, $y = 0$. If we consider this segment to be a line crack, we can distinguish its two surfaces by saying that one surface is on the positive side of $y = 0$, denoted $y = 0_+$, and the other is on the negative side, denoted $y = 0_-$. In crossing from one side of the line segment to the other, the displacements undergo a *constant* specified change in value $D_i = (D_x, D_y)$.

The displacement discontinuity D_i is defined as the difference in displacement between the two sides of the segment:

$$\begin{aligned} D_x &= u_x(x, 0_-) - u_x(x, 0_+) \\ D_y &= u_y(x, 0_-) - u_y(x, 0_+) \end{aligned} \tag{1}$$

Because u_x and u_y are positive in the positive x and y coordinate directions, it follows that D_x and D_y are positive as illustrated in Fig. 1.

The solution to the subject problem is given by Scouch (1976). The displacements and stresses can be written as

$$\begin{aligned} u_x &= D_x[2(1-\nu)f_{,y} - yf_{,xx}] + D_y[-(1-2\nu)f_{,x} - yf_{,xy}] \\ u_y &= D_x[(1-2\nu)f_{,x} - yf_{,xy}] + D_y[2(1-\nu)f_{,y} - yf_{,yy}] \end{aligned} \tag{2}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GD_x[2f_{,xy} + yf_{,xyy}] + 2GD_y[f_{,yy} + yf_{,yyy}] \\ \sigma_{yy} &= 2GD_x[-yf_{,xyy}] + 2GD_y[f_{,yy} - yf_{,yyy}] \\ \sigma_{xy} &= 2GD_x[f_{,yy} + yf_{,yyy}] + 2GD_y[-yf_{,xyy}] \end{aligned} \tag{3}$$

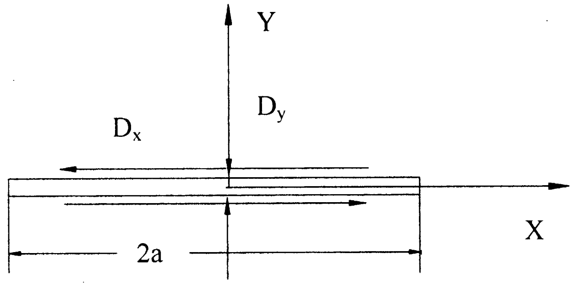

Fig. 1. Schematic of constant displacement discontinuity components D_x and D_y.

Function $f(x,y)$ in these equations are

$$\boldsymbol{f}\ (\boldsymbol{x},\boldsymbol{y}) = \frac{-1}{4\pi(1-v)}\left\{\boldsymbol{y}\left(\arctan\frac{\boldsymbol{y}}{\boldsymbol{x}-\boldsymbol{a}} - \arctan\frac{\boldsymbol{y}}{\boldsymbol{x}+\boldsymbol{a}}\right) - (\boldsymbol{x}-\boldsymbol{a})\ln\sqrt{(\boldsymbol{x}-\boldsymbol{a})^2+\boldsymbol{y}^2} + (\boldsymbol{x}+\boldsymbol{a})\ln\sqrt{(\boldsymbol{x}+\boldsymbol{a})^2+\boldsymbol{y}^2}\right\} \tag{4}$$

and its derivatives are given as follows:

$$\begin{aligned}
f_{,x} &= F_2(x,y) = \frac{1}{4\pi(1-v)}\left[\ln\sqrt{(x-a)^2+y^2} - \ln\sqrt{(x+a)^2+y^2}\right]\\
f_{,y} &= F_3(x,y) = -\frac{1}{4\pi(1-v)}\left[\arctan\frac{y}{x-a} - \arctan\frac{y}{x+a}\right]\\
f_{,xy} &= F_4(x,y) = \frac{1}{4\pi(1-v)}\left[\frac{y}{(x-a)^2+y^2} - \frac{y}{(x+a)^2+y^2}\right]\\
f_{,xx} &= -f_{,yy} = F_5(x,y) = \frac{1}{4\pi(1-v)}\left[\frac{x-a}{(x-a)^2+y^2} - \frac{x+a}{(x+a)^2+y^2}\right]\\
f_{,xyy} &= -f_{,xxx} = F_6(x,y) = \frac{1}{4\pi(1-v)}\left[\frac{(x-a)^2-y^2}{\{(x-a)^2+y^2\}^2} - \frac{(x+a)^2-y^2}{\{(x+a)^2+y^2\}^2}\right]\\
f_{,yyy} &= -f_{,xxy} = F_7(x,y) = \frac{2y}{4\pi(1-v)}\left[\frac{x-a}{\{(x-a)^2+y^2\}^2} - \frac{x+a}{\{(x+a)^2+y^2\}^2}\right]
\end{aligned} \tag{5}$$

G and v in these equations are shear modulus and the Poisson's ratio, respectively.

Eqs. (2)–(5) are used by Scouch et al. (1983) to set up a constant displacement discontinuity boundary element method.

3. Basic formulas required to set up higher displacement discontinuity element

Now, consider arbitrary displacement discontinuity distributions along element length *2a*, as shown in Fig. 2:

$$\boldsymbol{D}_i = \boldsymbol{D}_i(\xi) \quad (\boldsymbol{i} = 1,2) \tag{6a}$$

or

$$\begin{aligned}\boldsymbol{D}_x &= \boldsymbol{D}_x(\xi)\\ \boldsymbol{D}_y &= \boldsymbol{D}_y(\xi)\end{aligned} \tag{6b}$$

Based on the solution of constant discontinuity in displacement given by Scouch (1976), i.e., formulas (2)–(5), the displacements and stresses at domain point (x,y) due to a differential element (with length $2\mathrm{d}\xi$ and the center (source point)) displacement discontinuity can be obtained from the differential viewpoint:

$$\begin{aligned}
\mathrm{d}u_x &= D_x(\xi)[2(1-v)T_3(x,y,\xi,\mathrm{d}\xi) - yT_5(x,y,\xi,\mathrm{d}\xi)] + D_y(\xi)[-(1-2v)T_2(x,y,\xi,\mathrm{d}\xi) - yT_4(x,y,\xi,\mathrm{d}\xi)]\\
\mathrm{d}u_y &= D_x(\xi)[(1-2v)T_2(x,y,\xi,\mathrm{d}\xi) - yT_4(x,y,\xi,\mathrm{d}\xi)] + D_y(\xi)[2(1-v)T_3(x,y,\xi,\mathrm{d}\xi) - yT_5(x,y,\xi,\mathrm{d}\xi)]
\end{aligned} \tag{7}$$

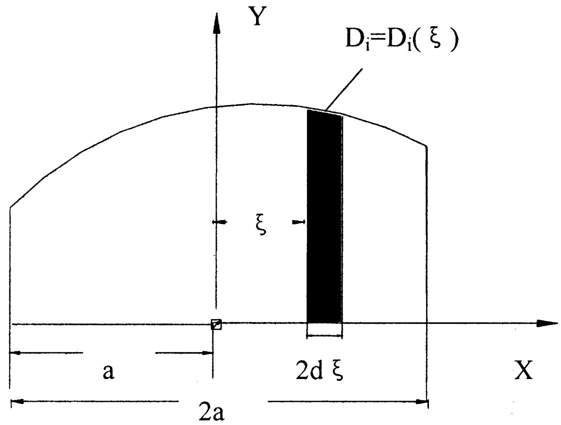

Fig. 2. Schematic of an arbitrary displacement discontinuity function and its differential element.

and

$$\begin{aligned}
d\sigma_{xx} &= 2GD_x(\xi)[2T_4(x,y,\xi,d\xi) + yT_6(x,y,\xi,d\xi)] + 2GD_y(\xi)[-T_5(x,y,\xi,d\xi) + yT_7(x,y,\xi,d\xi)] \\
d\sigma_{yy} &= 2GD_x(\xi)[-yT_6(x,y,\xi,d\xi)] + 2GD_y(\xi)[-T_5(x,y,\xi,d\xi) - yT_7(x,y,\xi,d\xi)] \\
d\sigma_{xy} &= 2GD_x(\xi)[-T_5(x,y,\xi,d\xi) + yT_7(x,y,\xi,d\xi)] + 2GD_y(\xi)[-yT_6(x,y,\xi,d\xi)]
\end{aligned} \tag{8}$$

Functions T_2, T_3, T_4, T_5, T_6, T_7 in these equations are given by

$$\begin{aligned}
T_2(x,y,\xi,d\xi)/d\xi &= V_2(x,y,\xi) = -\frac{1}{4\pi(1-v)}\frac{x-\xi}{(x-\xi)^2+y^2} \\
T_3(x,y,\xi,d\xi)/d\xi &= V_3(x,y,\xi) = -\frac{1}{4\pi(1-v)}\frac{y}{(x-\xi)^2+y^2} \\
T_4(x,y,\xi,d\xi)/d\xi &= V_4(x,y,\xi) = \frac{2y}{4\pi(1-v)}\frac{x-\xi}{\{(x-\xi)^2+y^2\}^2} \\
T_5(x,y,\xi,d\xi)/d\xi &= V_5(x,y,\xi) = \frac{1}{4\pi(1-v)}\frac{(x-\xi)^2-y^2}{\{(x-\xi)^2+y^2\}^2} \\
T_6(x,y,\xi,d\xi)/d\xi &= V_6(x,y,\xi) = \frac{2}{4\pi(1-v)}\left\{\frac{(x-\xi)^3}{[(x-\xi)^2+y^2]^3} - \frac{3(x-\xi)y^2}{[(x-\xi)^2+y^2]^3}\right\} \\
T_7(x,y,\xi,d\xi)/d\xi &= V_7(x,y,\xi) = \frac{2y}{4\pi(1-v)}\left\{\frac{3(x-\xi)^2}{[(x-\xi)^2+y^2]^3} - \frac{y^2}{[(x-\xi)^2+y^2]^3}\right\}
\end{aligned} \tag{9}$$

Obviously, if the following integrals are obtained

$$U_{ij}(x,y) = \int_{-a}^{a} D_j(\xi)V_i(x,y,\xi)d\xi \quad (i = 2,3,\ldots,7;\ j = 1,2) \tag{10}$$

then the displacements and stresses at domain point (x,y) due to the whole element displacement discontinuity can be written as

$$\begin{aligned}
u_x &= [2(1-v)U_{3x}(x,y) - yU_{5x}(x,y)] + [-(1-2v)U_{2y}(x,y) - yU_{4y}(x,y)] \\
u_y &= [(1-2v)U_{2x}(x,y) - yU_{4x}(x,y)] + [2(1-v)U_{3y}(x,y) - yU_{5y}(x,y)]
\end{aligned} \tag{11}$$

and

$$\begin{aligned}\sigma_{xx} &= 2G[2U_{4x}(x,y) + yU_{6x}(x,y)] + 2G[-U_{5y}(x,y) + yU_{7y}(x,y)]\\ \sigma_{yy} &= 2G[-yU_{6x}(x,y)] + 2G[-U_{5y}(x,y) - yU_{7y}(x,y)]\\ \sigma_{xy} &= 2G[-U_{5x}(x,y) + yU_{7x}(x,y)] + 2G[-yU_{6y}(x,y)]\end{aligned} \tag{12}$$

The formulas (9)–(12) are the basic formulas required to set up higher displacement discontinuity element.

4. Special crack tip displacement discontinuity element

In this section, the basic formulas (9)–(12) required to set up higher displacement discontinuity element will be used to set up a special crack tip displacement discontinuity element to deal with crack problems in general plane elasticity. Referred to the crack tip element suggested by Scouch et al. (1983), the special crack tip displacement discontinuity element is developed here in order to analyze crack problems in general plane elasticity. The schematic of the special displacement discontinuity element at the left tip of crack is shown in Fig. 3. Its displacement discontinuity functions are chosen as

$$\begin{aligned}\boldsymbol{D}_x &= \boldsymbol{H}_s\left(\tfrac{a+\xi}{a}\right)^{\frac{1}{2}}\\ \boldsymbol{D}_y &= \boldsymbol{H}_n\left(\tfrac{a+\xi}{a}\right)^{\frac{1}{2}}\end{aligned} \tag{13}$$

where H_s and H_n are the tangential and normal displacement discontinuity quantities at the center of the special element, respectively. Here, it is noted that the special element has the same unknowns as the two-dimensional constant displacement discontinuity element It can be seen that the displacement discontinuity functions defined according to (13) can model the displacement fields around the crack tip. Therefore, The stress field determined by the displacement discontinuity functions (13) possesses $r^{-1/2}$ singularity around the crack tip.

After substituting (13) into (10), one has

$$U_{ij}(x,y) = H_j \int_{-a}^{a} \left(\frac{a+\xi}{a}\right)^{1/2} V_i(x,y,\xi)\mathrm{d}\xi = H_j B_i(x,y) \quad (i = 2,3,\ldots,7;\ j = 1,2) \tag{14}$$

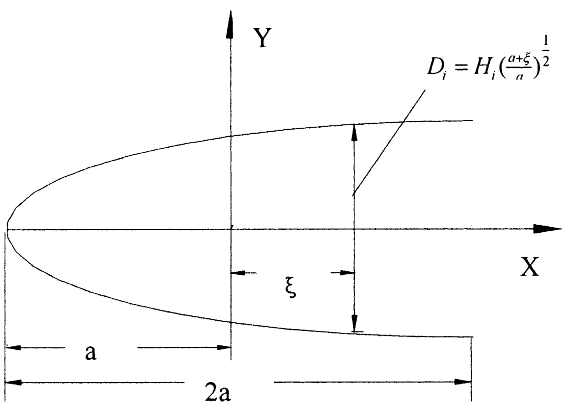

Fig. 3. Schematic of the special displacement discontinuity at left crack tip.

where

$$B_i(x,y) = \int_{-a}^{a} \left(\frac{a+\xi}{a}\right)^{1/2} V_i(x,y,\xi)\mathrm{d}\xi \quad (i = 2,3,\ldots,7) \tag{15}$$

After substituting (14) into (11) and (12), one can obtain

$$\begin{aligned} u_x &= H_s[2(1-v)B_3(x,y) - yB_5(x,y)] + H_n[-(1-2v)B_2(x,y) - yB_4(x,y)] \\ u_y &= H_s[(1-2v)B_2(x,y) - yB_4(x,y)] + H_n[2(1-v)B_3(x,y) - yB_5(x,y)] \end{aligned} \tag{16}$$

and

$$\begin{aligned} \sigma_{xx} &= 2GH_s[2B_4(x,y) + yB_6(x,y)] + 2GH_n[-B_5(x,y) + yB_7(x,y)] \\ \sigma_{yy} &= 2GH_s[-yB_6(x,y)] + 2GH_n[-B_5(x,y) - yB_7(x,y)] \\ \sigma_{xy} &= 2GH_s[-B_5(x,y) + yB_7(x,y)] + 2GH_n[-yB_6(x,y)] \end{aligned} \tag{17}$$

It can be seen by comparing (16) and (17) with (2) and (3) that the displacements and stresses due to the special crack tip element possess the same forms as those due to a constant displacement discontinuity element, with $F_i(x,y)$ $(i = 2,3,\ldots,7)$ in (2) and (3) being replaced by $B_i(x,y)$ $(i = 2,3,\ldots,7)$, D_x and D_y by H_s and H_n, respectively. This is very importance to the boundary element implementation. It enables the boundary element implementation to be easy.

The computation of B_i $(i = 2,3,\ldots,7)$ will be taken into account in the following from four respects.

(1) For an arbitrary domain point $P(x,y)$ $(y \neq 0)$, generally, the integrals (15) are difficultly solved analytically. In this paper, the Gauss numerical integration is used to calculate them. The following transformation is made:

$$\xi = \boldsymbol{at} \tag{18}$$

then one has

$$B_i(x,y) = \int_{-a}^{a} \left(\frac{a+\xi}{a}\right)^{1/2} V_i(x,y,\xi)\mathrm{d}\xi = a\int_{-1}^{1} V_i(x,y,at)(1+t)^{1/2}\mathrm{d}t \quad (i = 2,3,\ldots,7) \tag{19}$$

Therefore, $B_i(x,y)$ can be given by

$$B_i(x,y) = a\sum_j V_i(x,y,a\zeta_j)(1+\zeta_j)^{1/2} w_j \quad (i = 2,3,\ldots,7) \tag{20}$$

where ζ_i and w_i are the Gauss point coordinates and corresponding weighed factors, respectively.

(2) For an arbitrary domain point $P(x,y)$ $(y = 0)$, the integrals B_2, B_4, B_5, B_6, B_7 in (14) can be solved analytically. For $x > -a$, one can obtain

$$\begin{aligned} \boldsymbol{B}_2(\boldsymbol{x},0) &= \frac{-1}{4\pi(1-v)}\left\{-2\sqrt{2} + \sqrt{\frac{\boldsymbol{x}+\boldsymbol{a}}{\boldsymbol{a}}}\ln\left|\frac{\sqrt{\boldsymbol{x}+\boldsymbol{a}}+\sqrt{2\boldsymbol{a}}}{\sqrt{\boldsymbol{x}+\boldsymbol{a}}-\sqrt{2\boldsymbol{a}}}\right|\right\} \\ \boldsymbol{B}_4(\boldsymbol{x},0) &= 0 \\ \boldsymbol{B}_5(\boldsymbol{x},0) &= \frac{1}{4\pi(1-v)}\left\{\frac{\sqrt{2}}{\boldsymbol{x}-\boldsymbol{a}} - \frac{1}{2\sqrt{\boldsymbol{a}(\boldsymbol{x}+\boldsymbol{a})}}\ln\left|\frac{\sqrt{\boldsymbol{x}+\boldsymbol{a}}+\sqrt{2\boldsymbol{a}}}{\sqrt{\boldsymbol{x}+\boldsymbol{a}}-\sqrt{2\boldsymbol{a}}}\right|\right\} \\ \boldsymbol{B}_6(\boldsymbol{x},0) &= \frac{1}{4\pi(1-v)}\left\{\frac{\sqrt{2}}{(\boldsymbol{x}-\boldsymbol{a})^2} - \frac{\sqrt{2}}{2(\boldsymbol{x}^2-\boldsymbol{a}^2)} - \frac{1}{4\sqrt{\boldsymbol{a}}(\boldsymbol{x}+\boldsymbol{a})^{3/2}}\ln\left|\frac{\sqrt{\boldsymbol{x}+\boldsymbol{a}}+\sqrt{2\boldsymbol{a}}}{\sqrt{\boldsymbol{x}+\boldsymbol{a}}-\sqrt{2\boldsymbol{a}}}\right|\right\} \\ \boldsymbol{B}_7(\boldsymbol{x},0) &= 0 \end{aligned} \tag{21}$$

While for $x < -a$, if one lets r denote the distance from the crack tip along the crack extension line, i.e.,

$$r = |x| - a \tag{22}$$

one has

$$\begin{aligned}
B_2(x,0) &= \frac{-1}{4\pi(1-\nu)}\left\{-2\sqrt{2} + 2\sqrt{\frac{r}{a}}\arctan\sqrt{\frac{2a}{r}}\right\}\\
B_4(x,0) &= 0\\
B_5(x,0) &= \frac{1}{4\pi(1-\nu)}\left\{-\frac{\sqrt{2}}{r+2a} + \frac{1}{\sqrt{ar}}\arctan\sqrt{\frac{2a}{r}}\right\}\\
B_6(x,0) &= \frac{1}{4\pi(1-\nu)}\left\{\frac{\sqrt{2}}{(r+2a)^2} - \frac{\sqrt{2}}{2r(r+2a)} - \frac{1}{2\sqrt{a}r^{3/2}}\arctan\sqrt{\frac{2a}{r}}\right\}\\
B_7(x,0) &= 0
\end{aligned} \tag{23}$$

(3) For an arbitrary domain point $P(x,y)$ $(y=0)$, the integral B_3 in (14) is

$$B_3(x,0) = \begin{cases} 0 & |x| \succ a \\ +\frac{1}{4(1-\nu)} & y = 0_+ \quad |x| \prec a \\ -\frac{1}{4(1-\nu)} & y = 0_- \quad |x| \prec a \end{cases} \tag{24}$$

(4) From (21) and (24), one can easily obtain the element-self effects

$$\begin{aligned}
B_2(0,0) &= \frac{-1}{4\pi(1-\nu)}\left[-2\sqrt{2} + \ln\left|\frac{1+\sqrt{2}}{1-\sqrt{2}}\right|\right]\\
B_3(0,0) &= \begin{cases} +\frac{1}{4(1-\nu)} & y = 0_+ \\ -\frac{1}{4(1-\nu)} & y = 0_- \end{cases}\\
B_4(0,0) &= 0\\
B_5(0,0) &= \frac{1}{4\pi(1-\nu)}\left[-\sqrt{2} - \frac{1}{2}\ln\left|\frac{1+\sqrt{2}}{1-\sqrt{2}}\right|\right]\Big/ a\\
B_6(0,0) &= \frac{1}{4\pi(1-\nu)}\left[-\frac{3\sqrt{2}}{2} - \frac{1}{4}\ln\left|\frac{1+\sqrt{2}}{1-\sqrt{2}}\right|\right]\Big/ a^2\\
B_7(0,0) &= 0
\end{aligned} \tag{25}$$

For the special crack tip displacement discontinuity element at the right tip of crack, the similar formulas can be obtained.

5. Computation formulas for stress intensity factors

Based on the displacement field around the crack tip and the definition of the displacement discontinuity functions (13), we can obtain the calculation formulas of stress intensity factors K_{I} and K_{II}:

$$K_{\mathrm{I}} = -\frac{\sqrt{2\pi}GH_n}{4(1-\nu)\sqrt{a}} \tag{26}$$

$$K_{\mathrm{II}} = -\frac{\sqrt{2\pi}GH_s}{4(1-v)\sqrt{a}} \tag{27}$$

6. Examples

An infinite plate with a through crack of length $2a$ which is subjected to uniform stress normal to the crack plane at distances sufficiently far away from the crack is taken for example to compute the stress intensity factor K_{I}. Owing to its symmetry, only half is taken for the analysis. Table 1 gives that the ratio of the numerical solution to the analytical one for stress intensity factor K_{I} is varied with the number of elements. In this calculation, the special element and constant elements are taken to possess the equal size. Table 2 gives that the ratio of the numerical solution to the analytical one for stress intensity factor K_{I} is varied with the ratio of the size of the special element to the one of constant elements. Here, the sizes of constant elements are taken to be equal and the number of total elements is 11. It can be seen from Table 1 that a good result for the stress intensity factor K_{I} can be obtained using the special crack element placed at the crack tip. It can be seen from Table 2 that the ratio of the size of the special element to that of constant elements is necessarily taken to be from 0.9 to 1.3 to obtain a good result. This can be regarded as the limitation to the approach presented in the present paper.

As another example, an inclined crack plate with a through crack of length $2a$ which is subjected to uniform stress at distances sufficiently far away from the crack is used to compute the stress intensity factors K_{I} and K_{II}. Some numerical results are given in Table 3. In this calculation, the special elements and constant elements are taken to possess the equal size and the number of total elements is taken to be 20, i.e., two special elements and 18 constant elements. It can be seen that from Table 3 that no matter how large or

Table 1
Variation of the ratio of the numerical solution to the analytical one for stress intensity factor K_{I} for an infinite center crack plate with the number of elements

Number of elements	3	5	7	10	15	25
$K_{\mathrm{I}}/K_{\mathrm{I(analytical)}}$	0.9621	0.9775	0.9838	0.9885	0.9921	0.995

Table 2
Variation of the ratio of the numerical solution to the analytical one for stress intensity factor K_{I} for an infinite center crack plate with the ratio of the size of special element to the one of constant elements

$a_{\mathrm{special}}/a_{\mathrm{constant}}$	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
$K_{\mathrm{I}}/K_{\mathrm{I(analytical)}}$	1.2048	1.1690	1.1394	1.1143	1.0928	1.0742	1.0578	1.0433	1.0303
$a_{\mathrm{special}}/a_{\mathrm{constant}}$	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45
$K_{\mathrm{I}}/K_{\mathrm{I(analytical)}}$	1.0186	1.0080	0.9984	0.9896	0.9815	0.9741	0.9671	0.9607	0.9547

Table 3
Variation of the ratio of the numerical solution to the analytical one for stress intensity factors K_{I} and K_{II} for an infinite inclined center crack plate with the angle β between crack plane and load

Angle β	5	10	20	30	40	45	50	60	70	80	85
$K_{\mathrm{I}}/K_{\mathrm{I(analytical)}}$	0.9895	0.9898	0.9896	0.9898	0.9898	0.9885	0.9897	0.9897	0.9898	0.9897	0.9896
$K_{\mathrm{II}}/K_{\mathrm{II(analytical)}}$	0.9896	0.9897	0.9897	0.9897	0.9897	0.9885	0.9897	0.9897	0.9897	0.9897	0.9896

small is the angle β between the load and the crack plane, the numerical solutions of the stress intensity factors K_I and K_{II} are in good agreement with the analytical ones.

7. Conclusions

Based on the analytical solution to the problem of a constant discontinuity in displacement over a finite line segment in the x, y plane of an infinite elastic solid and the note of the crack tip element by Crouch, in the present paper, the special crack tip displacement discontinuity element is developed. Further the analytical formulas for the stress intensity factors of crack problems in general plane elasticity are given. Numerical results show that the displacement discontinuity modeling technique of a crack presented in this paper is very effective for computing stress intensity factors.

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