International Journal of Computational Methods
Vol. 3, No. 3 (2006) 321–336

THREE-DIMENSIONAL ANALYSIS OF ARBITRARY-SHAPED CRACKS IN PIEZOELECTRIC SOLIDS UNDER SHEAR LOADING

HE-GEN YU and MENG-CHENG CHEN*

School of Civil Engineering, East China Jiaotong University
Nanchang 330013, Jiangxi, P. R. China
**chenmch@ecjtu.jx.cn*

Received September 2005
Accepted December 2005

This paper is a sequel of the author's previous work [Chen (2004)]. It deals with some basic linear electro-elastic fracture problems for an arbitrary-shaped planar crack in a three-dimensional infinite transversely isotropic piezoelectric material subjected to shear loading that is antisymmetric with respect to the crack. The finite-part integral concept is used to derive hypersingular integral equations for the crack from available solutions of the point force and charge for an infinite transversely isotropic piezoelectric solid. Closed-type solutions for the full electro-elastic fields, for the stress and electric displacement K-fields and the energy release rate G are obtained. In particular, under uniform shear loading, exact expressions for an elliptical crack are derived with introducing the ellipsoidal coordinates. Finally, numerical examples for some typical crack problems are also demonstrated in table and graphic forms.

Keywords: Transverse isotropy; piezoelectric material; arbitrary-shaped planar crack; hypersingular integral equation; shear loading.

1. Introduction

With the increasing applications of piezoelectric materials as actuating and sensing devices in active control systems, much attention has been paid to electroelastic interaction by more and more researchers. Piezoelectric ceramic are ones kinds of transversely isotropic materials that have perfect piezoelectricity and its application is more extensive. Because of their intrinsic natural brittle property, when subjected to severe loading, these materials can fail prematurely as a result of the propagation of flaws or defects induced during the manufacturing process and by the in-service electromechanical loading. Hence, it is important to understand and be able to analyze the fracture characteristics of piezoelectric materials so that reliable service life predictions of the pertinent devices can be conducted.

In the last decade, considerable works on cracks in piezoelectric materials are devoted to two-dimensional (2D) problems [Zhang *et al.* (2002); Qin (2001)].

*Corresponding author.

Suo *et al.* [1992] used the extended Stroh's formulism to investigate the interface crack problems of piezoelectric ceramics. Pak [1992] used the method of distributed locations and electric dipoles to calculate the electroelastic fields for anti-plane and plane strain in infinite piezoelectric materials subjected to far-field electromechanical loadings. Park and Sun [1995] invoked the impermeable crack assumption and presented the full field closed-form solutions for all the three modes of fracture for an infinite piezoelectric medium. Zhang and Meguid [1997] analyzed a circular arc-crack in piezoelectric materials.

As for three-dimensional (3D) aspect of crack problems in piezoelectric materials, there are comparatively few works except those of specially shaped cracks [Wang (1992, 1994); Wang and Huang (1995); Kokan *et al.* (1996); Zhao *et al.* (1997); Chen and Shioya (1999, 2000); Chen *et al.* (2000)]. Shang *et al.* [2003] and the author [Chen (2003a,b; 2004)] have addressed the non-elliptical crack problems in three-dimensional piezoelectric structures. However, to the author's best knowledge, there is no any research work discussing arbitrary-shaped piezoelectric crack problems subjected to shear loading except those of penny-shaped cracks [Chen and Shioya (2000); Karapetian *et al.* (2000)].

It is noted that, one powerful method for 3D crack problems is to utilize Somigliana identity to reduce these problems to finite-part integral equations [Ioakimididis (1982)], as hypersingular integral equations. Thereafter, this method has been generalized into solving general crack problems in isotropic materials [Chen *et al.* (2002a,b)], bimaterials [Chen *et al.* (1999)], transversely isotropic materials [Chen (2004a)] as well as piezoelectric materials [Chen (2003a,b; 2004b)].

The purpose of this paper is to make more systematic theoretical and numerical studies of a three-dimensional arbitrary-shaped planar crack embedded in an infinite transversely isotropic piezoelectric solid under shear loading by the use of the aforementioned method. To this end, the recently derived by Ding *et al.* [(2004)] point-force and point-charge fundamental solutions for a transversely isotropic piezoelectric solid and the sense of the finite-part integral introduced by Hadamard [1923] are employed. 2D hypersingular equations with unknown displacements satisfying the boundary conditions are obtained for the arbitrary-shaped planar crack. It will be shown that the equations have the same structures as the ones for elasticity. The only difference is the definition of the involved material constants, which has no effect on the form of the solutions. Thus, the theoretical results and the numerical method presented by the author [Chen *et al.* (2002b)] can be utilized to analyze the problem. In particular, for uniform shear loading, exact solutions are firstly perfectly obtained in terms of the singular behavior near the crack front edge and with introducing the ellipsoidal coordinates. In the special case of the penny-shaped crack, the exact solutions are reduced to those existing in the literatures [Kokan *et al.* (1996); Chen and Shioya (2000); Karapetian *et al.* (2000)]. Some numerical examples for typical crack problems are given to demonstrate the versatilities of the present method.

2. Electroelastic Fields due to Crack Disturbances

A fixed rectangular cartesian coordinate system (x, y, z) is used. We consider an infinite transversely isotropic piezoelectric solid containing an arbitrary-shaped planar crack. Suppose, that the whole space is occupied with an electro-elastic media with elastic constants c_{ijkl}, piezoelectric constants e_{ijk} and dielectric permittivities $\in_{ij}$. The planar crack is assumed to lie in the plane of symmetry, i.e. in x-y and the polling direction be along with z (see Fig. 1). Note that the displacement and potential jumps on the crack surfaces $u_i(\xi,\eta) = u_i^+(\xi,\eta) - u_i^-(\xi,\eta)$ $(i = x, y, z,$ similarly hereinafter without declaration) and $\phi(\xi,\eta) = \phi^+(\xi,\eta) - \phi^-(\xi,\eta)$, thus in terms of the point-force and point-charge fundamental solutions for a transversely isotropic piezoelectric solid [Ding *et al.* (2004)] and the sense of the finite-part integral [Hadamard (1923)], the stress and electric displacement fields at a point $\mathbf{P}(x, y, z)$ in the space region due to the impermeable crack disturbances can be written as

$$\begin{aligned}\sigma_{xz}(x,y,z) = \frac{1}{4\pi}\Bigg\{\int_{S^+}\Bigg[\chi_0\omega_{01}\left[\frac{2}{r_0^3} - \frac{3\left((x-\xi)^2 + s_0^2 z^2\right)}{r_0^5}\right] \\ + \sum_{i=1}^{3}\chi_i\omega_{i1}\left[-\frac{1}{r_i^3} + \frac{3(x-\xi)^2}{r_i^5}\right]\Bigg]u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\ + \int_{S^+} 3\left[\sum_{i=1}^{3}\frac{\chi_i\omega_{i1}}{r_i^5} - \frac{\chi_0\omega_{01}}{r_0^5}\right](x-\xi)(y-\eta)u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\ + \int_{S^+} 3\sum_{i=1}^{3}\frac{\chi_i\vartheta_{i1}(x-\xi)s_i z}{r_i^5}u_z(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\ + \int_{S^+} 3\sum_{i=1}^{3}\frac{\chi_i\vartheta_{i2}(x-\xi)s_i z}{r_i^5}\phi(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta\Bigg\},\end{aligned} \tag{1}$$

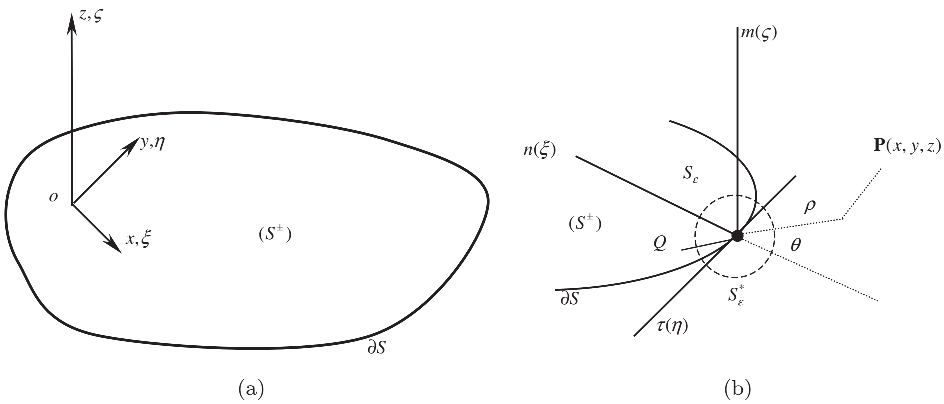

Fig. 1. (a) Configuration of a planar crack in an infinite piezoelectrics and (b) a local intrinsic coordinate system.

$$
\begin{aligned}
\sigma_{yz}(x,y,z) = \frac{1}{4\pi}\Bigg\{ & \int_{S^+} 3\left[\sum_{i=1}^{3}\frac{\chi_i\omega_{i1}}{r_i^5} - \frac{\chi_0\omega_{01}}{r_0^5}\right](x-\xi)(y-\eta)u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+}\left[\chi_0\omega_{01}\left[\frac{2}{r_0^3} - \frac{3\left((y-\eta)^2 + s_0^2z^2\right)}{r_0^5}\right]\right. \\
& \left. + \sum_{i=1}^{3}\chi_i\omega_{i1}\left[-\frac{1}{r_i^3} + \frac{3(y-\eta)^2}{r_i^5}\right]\right] u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+} 3\sum_{i=1}^{3}\frac{\chi_i\vartheta_{i1}(y-\eta)s_iz}{r_i^5}u_z(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+} 3\sum_{i=1}^{3}\frac{\chi_i\vartheta_{i2}(y-\eta)s_iz}{r_i^5}\phi(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta\Bigg\}, \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{zz}(x,y,z) = -\frac{1}{4\pi}\Bigg\{ & \int_{S^+} 3\sum_{i=1}^{3}\frac{\eta_i\omega_{i1}(x-\xi)s_iz}{r_i^5}u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& - \int_{S^+} 3\sum_{i=1}^{3}\frac{\eta_i\omega_{i1}(y-\eta)s_iz}{r_i^5}u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+}\sum_{i=1}^{3}\eta_i\vartheta_{i1}\left[\frac{1}{r_i^3} - \frac{3s_i^2z^2}{r_i^5}\right]u_z(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+}\sum_{i=1}^{3}\eta_i\vartheta_{i2}\left[\frac{1}{r_i^3} - \frac{3s_i^2z^2}{r_i^5}\right]\phi(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta\Bigg\}, \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
D_z(x,y,z) = -\frac{1}{4\pi}\Bigg\{ & \int_{S^+} 3\sum_{i=1}^{3}\frac{\mu_i\omega_{i1}(x-\xi)s_iz}{r_i^5}u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& - \int_{S^+} 3\sum_{i=1}^{3}\frac{\mu_i\omega_{i1}(y-\eta)s_iz}{r_i^5}u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+}\sum_{i=1}^{3}\mu_i\vartheta_{i1}\left[\frac{1}{r_i^3} - \frac{3s_i^2z^2}{r_i^5}\right]u_z(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \\
& + \int_{S^+}\sum_{i=1}^{3}\mu_i\vartheta_{i2}\left[\frac{1}{r_i^3} - \frac{3s_i^2z^2}{r_i^5}\right]\phi(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta\Bigg\}, \qquad (4)
\end{aligned}
$$

where $i = 1, 2, 3$; χ_0, χ_i, ω_{01}, ω_{i1}, ϑ_{i1}, ϑ_{i2}, η_i, and μ_i are constants related to the elastic constants, piezoelectric constants and dielectric permittivities and they are respectively given by the author [Chen (2003)]; $r_0 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + s_0^2z^2}$, $r_i = \sqrt{(x-\xi)^2 + (y-\eta)^2 + s_i^2z^2}$; $s_0^2 = c_{66}/c_{44}$ and s_i^2 are the roots of the following characteristic equation:

$$as^6 - bs^4 + cs^2 - d = 0, \tag{5}$$

in which

$$\begin{gathered} a = c_{44}(e_{33}^2 + c_{33} \in_{33}), \quad b = c_{33}m_3 + \in_{33}[c_{44}^2 - (c_{13}+c_{44})^2] + e_{33}(2m_4 - c_{11}e_{33}), \\ c = c_{44}m_3 + \in_{11}[c_{11}c_{33} - (c_{13}+c_{44})^2] + e_{15}(2m_4 - c_{44}e_{15}), \\ d = c_{11}(e_{15}^2 + c_{44} \in_{11}), \end{gathered} \tag{6}$$

$$\begin{gathered} m_1 = \in_{11} (c_{13}+c_{44}) + e_{15}(e_{15}+e_{31}), \quad m_2 = \in_{33} (c_{13}+c_{44}) + e_{33}(e_{15}+e_{31}), \\ m_3 = c_{11} \in_{33} + c_{44} \in_{11} + (e_{15}+e_{31})^2, \\ m_4 = c_{11}e_{33} + c_{44}e_{15} - (c_{13}+c_{44})(e_{15}+e_{31}). \end{gathered} \tag{7}$$

3. Hypersingular Integral Equations and Natures of Their Solutions

In terms of the concept of finite-part integrals [Hadamard (1923)] and the boundary conditions on the upper crack surfaces, the problems for an arbitrary-shaped 3D planar crack in piezoelectricity under shear loading can be reduced to the solution of the following 2D hypersingular integral equations:

$$\begin{aligned} \frac{1}{4\pi}\Bigg\{ & \int\!\!\!\!\!\!=_{S^+} \frac{\left(2\chi_0\omega_{01} - \sum_{i=1}^3 \chi_i\omega_{i1}\right) u_x(\xi,\eta)}{r^3} \mathrm{d}\xi\,\mathrm{d}\eta \\ & + \int\!\!\!\!\!\!=_{S^+} \frac{3\left(\sum_{i=1}^3 \chi_i\omega_{i1} - \chi_0\omega_{01}\right)(x-\xi)^2 u_x(\xi,\eta)}{r^5} \mathrm{d}\xi\,\mathrm{d}\eta \\ & + \int\!\!\!\!\!\!=_{S^+} \frac{3\left(\sum_{i=1}^3 \chi_i\omega_{i1} - \chi_0\omega_{01}\right)(x-\xi)(y-\eta) u_y(\xi,\eta)}{r^5} \mathrm{d}\xi\,\mathrm{d}\eta \Bigg\} = -p_x(x,y), \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{1}{4\pi}\Bigg\{ & \int\!\!\!\!\!\!=_{S^+} \frac{\left(2\chi_0\omega_{01} - \sum_{i=1}^3 \chi_i\omega_{i1}\right) u_y(\xi,\eta)}{r^3} \mathrm{d}\xi\,\mathrm{d}\eta \\ & + \int\!\!\!\!\!\!=_{S^+} \frac{3\left(\sum_{i=1}^3 \chi_i\omega_{i1} - \chi_0\omega_{01}\right)(y-\eta)^2 u_y(\xi,\eta)}{r^5} \mathrm{d}\xi\,\mathrm{d}\eta \\ & + \int\!\!\!\!\!\!=_{S^+} \frac{3\left(\sum_{i=1}^3 \chi_i\omega_{i1} - \chi_0\omega_{01}\right)(x-\xi)(y-\eta) u_x(\xi,\eta)}{r^5} \mathrm{d}\xi\,\mathrm{d}\eta \Bigg\} = -p_y(x,y), \end{aligned} \tag{9}$$

where $x, y \in S^+$; $\int\!\!\!\!\!\!=_{S^+}$ means finite-part integral of Hadamard [1923] and $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$. It can be seen from Eqs. (8) and (9) that the displacement jumps $u_i(\xi,\eta)$ are decoupled with the electric potential jumps $\phi(\xi,\eta)$ under shear loading. The natures of the unknown displacement jumps $u_i(\xi,\eta)$ near the front smooth periphery ∂S^+ of the planar crack, in terms of the author's work [Chen (2003); Chen *et al.* (1999)], can be expressed as

$$u_i(\xi,\eta) = C_i(Q)\xi^{\lambda_i}, \tag{10}$$

Here, the repeated indices are not summed; $C_i(Q)$ are real constants concerning with the position of the point Q, and λ_i are unknown singularities that take the values within range $0 < \lambda_i < 1$.

Inserting Eq. (10) into Eqs. (8) and (9) and using 2D dominant analysis of hypersingular integrals, one has

$$\lambda_x = \lambda_y = \lambda_z = 1/2. \tag{11}$$

4. Singular Stress Fields Under Shear Loading

According to Eqs. (10) and (11) as well as calculating the dominant-parts of integrals in Eqs. (1)–(4) at a point $\mathbf{P}(-\rho\cos\theta, y, \rho\sin\theta)$ ahead of the periphery of the planar crack [see Fig. 1(b)], we can get the singular stress displacement fields under shear loading. When the point $\mathbf{P}$ approaches the point Q, with the two-dimensional dominant analysis, the following relationships are obtained:

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\left\{\frac{1}{r_i^3}-\frac{3(x-\xi)^2}{r_i^5}\right\}u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta = -\frac{\pi C_x(Q)\cos\frac{\Theta_i}{2}}{\sqrt{R_i}}, \tag{12}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\left\{\frac{1}{r_i^3}-\frac{3(y-\eta)^2}{r_i^5}\right\}u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta = 0, \tag{13}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\left\{\frac{2}{r_0^3}-\frac{3\left[(x-\xi)^2+s_0^2z^2\right]}{r_0^5}\right\}u_x(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta = 0, \tag{14}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\left\{\frac{2}{r_0^3}-\frac{3\left[(y-\eta)^2+s_0^2z^2\right]}{r_0^5}\right\}u_y(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta = \frac{\pi C_y(Q)\cos\frac{\Theta_0}{2}}{\sqrt{R_0}}, \tag{15}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\frac{3(x-\xi)(y-\eta)}{r_0^5}u_x(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = \int_{S_\varepsilon^*}\frac{3(x-\xi)(y-\eta)}{r_0^5}u_y(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = 0, \tag{16}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\frac{3(x-\xi)(y-\eta)}{r_i^5}u_x(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = \int_{S_\varepsilon^*}\frac{3(x-\xi)(y-\eta)}{r_i^5}u_y(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = 0, \tag{17}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\frac{3(x-\xi)s_iz}{r_i^5}u_z(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = -\frac{\pi C_z(Q)\sin\frac{\Theta_i}{2}}{\sqrt{R_i}}, \tag{18}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\frac{3(x-\xi)s_i z}{r_i^5}\phi(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = -\frac{\pi C_4(Q)\sin\frac{\Theta_i}{2}}{\sqrt{R_i}}, \tag{19}$$

$$\lim_{S_\varepsilon^*\to 0}\int_{S_\varepsilon^*}\frac{3(y-\eta)s_i z}{r_i^5}u_z(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = \int_{S_\varepsilon^*}\frac{3(y-\eta)s_i z}{r_i^5}\phi(x,\xi)\mathrm{d}\xi\,\mathrm{d}\eta = 0, \tag{20}$$

in which

$$\Theta_0 = \arctan\frac{s_0\sin\theta}{\cos\theta},\quad \Theta_i = \arctan\frac{s_i\sin\theta}{\cos\theta}, \tag{21}$$

$$R_0 = \rho\sqrt{\cos^2\theta + s_0^2\sin^2\theta},\quad R_i = \rho\sqrt{\cos^2\theta + s_i^2\sin^2\theta}. \tag{22}$$

From Eqs. (1), (2), and (12)–(20), the singular stress fields under shear loading can be expressed as

$$\sigma_{xz}(\rho,\theta) = \sum_{i=1}^{3}\frac{\chi_i\omega_{i1}C_x(Q)}{4\sqrt{R_i}}\cos\frac{\Theta_i}{2} - \sum_{i=1}^{3}\frac{\chi_i\vartheta_{i1}C_z(Q)+\chi_i\vartheta_{i2}C_4(Q)}{4\sqrt{R_i}}\sin\frac{\Theta_i}{2}, \tag{23}$$

$$\sigma_{yz}(\rho,\theta) = \frac{\chi_0\omega_{01}C_y(Q)}{4\sqrt{R_0}}\cos\frac{\Theta_0}{2}. \tag{24}$$

Equations (23) and (24) are strictly obtained from 3D linear theory of piezoelectricity. It can be seen that the angular distribution functions for the stress fields ahead of the planar crack front edge are very simple and compact. From Eq. (23), we also see that the electric field has an effect on the stress $\sigma_{xz}(\rho,\theta)$ ahead of the crack front edge.

5. Fracture Parameters for Shearing Crack Problems

5.1. *Stress intensity factors*

Analogous to the definitions of stress intensity factors for three-dimensional elasticity, we define the stress intensity factors near the point Q on the smooth periphery ∂S^+ of the three-dimensional planar crack [see Fig. 1(b)] as follows:

$$K_{\mathrm{II}}(Q) = \lim_{\rho\to 0}\sqrt{2r}\sigma_{xz}(\rho,\theta)\Big|_{\theta=0},\quad K_{\mathrm{III}}(Q) = \lim_{\rho\to 0}\sqrt{2r}\sigma_{yz}(\rho,\theta)\Big|_{\theta=0}. \tag{25}$$

Therefore the stress intensity factors can be given as

$$K_{\mathrm{II}}(Q) = \lim_{\xi\to 0}\left[\frac{\sum_{i=1}^{3}\chi_i\omega_{i1}}{2\sqrt{2}}\frac{u_x(\xi,0)}{\sqrt{\xi}}\right],\quad K_{\mathrm{III}}(Q) = \lim_{\xi\to 0}\left[\frac{\chi_0\omega_{01}}{2\sqrt{2}}\frac{u_y(\xi,0)}{\sqrt{\xi}}\right]. \tag{26}$$

Equation (26) is almost identical to that of linear elastic fracture mechanics [Chen *et al.* (2002)] but material constants.

5.2. *The local energy release rate*

As shown in Fig. 1(b), supposed that the planar crack S has extended an infinitesimal crack area S_ε^* (it along with its neighboring region S_ε is mutually complementary to be a small circle with a radius ε) along the crack plane through the point Q, by way of virtual work, the total local energy release rate $G(Q)$ due to the crack increment is readily computed from the work done by the released stresses

$$G(Q) = \lim_{S_\varepsilon^* \to 0} \frac{1}{2S_\varepsilon^*} \int_{S_\varepsilon^*} [\sigma_{xz}(\xi,\eta)u_x^*(\xi,\eta) + \sigma_{yz}(\xi,\eta)u_y^*(\xi,\eta)]\mathrm{d}\xi\,\mathrm{d}\eta \tag{27}$$

where $\sigma_{xz}(\xi,\eta)$ and $\sigma_{yz}(\xi,\eta)$ are the stress components ahead of the planar crack front edge when the increment S_ε^* of the crack is zero, and they can be calculated from Eqs. (23) and (24); $u_x^*(\xi,\eta)$ and $u_y^*(\xi,\eta)$ are the displacement jumps when the crack has increased an infinitesimal area of S_ε^*, and they can be determined from Eq. (26). Substitution of Eqs. (23), (24) and (26) into (27), $G(Q)$ is rewritten as

$$G(Q) = \lim_{S_\varepsilon^* \to 0} \frac{1}{2S_\varepsilon^*} \int_{S_\varepsilon^*} \left(\frac{2K_{\mathrm{II}}^2(Q)}{\mathrm{i}\sum_{i=1}^{3}\chi_i\omega_{i1}} + \frac{2K_{\mathrm{III}}^2(Q)}{\mathrm{i}\chi_0\omega_{01}} \right) \frac{\sqrt{\varepsilon+\xi}}{\sqrt{\xi}}\mathrm{d}\xi\,\mathrm{d}\eta. \tag{28}$$

Using the definition of Beta function, we have

$$G(Q) = \frac{\pi K_{\mathrm{II}}^2(Q)}{2\sum_{i=1}^{3}\chi_i\omega_{i1}} + \frac{\pi K_{\mathrm{III}}^2(Q)}{2\chi_0\omega_{01}}. \tag{29}$$

Equation (29) is rigorously obtained from the three-dimensional theory of piezoelectrics.

6. Exact Solutions for Elliptical Cracks

6.1. *The displacement jumps*

In the case of elliptical cracks subjected to uniform shear loading, we can obtain exact solutions of hypersingular integral Eqs. (8) and (9). We were not aware of any published research on the exact solutions of elliptical cracks in piezoelectrics under uniform shear loading. To this end, the hypersingular integral Eqs. (8) and (9) should be modified in form.

As shown in Fig. 2(a), analogous to the method used in Chen [2004], consider the following limit operation:

$$\begin{aligned}\frac{\partial^2}{\partial x^2}\int_{S^+} \frac{f(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta \\ = -\lim_{\substack{\rho\to 0\\ \sigma\to 0}} \frac{\partial}{\partial x}\left\{ \left[\int_{S^+} \frac{x-\xi}{r^3} f(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta - \int_{S_\sigma} \frac{x-\xi}{r^3} f(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \right] \right. \\ \left. + f(x,y)\int_{S_\sigma} \frac{x-\xi}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta \right\},\end{aligned} \tag{30}$$

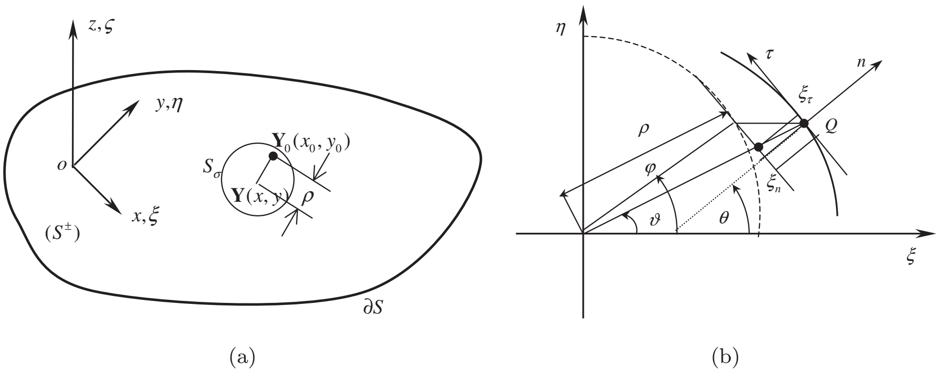

Fig. 2. (a) Configuration of problem proof and (b) configuration of stress intensity factor determination.

in which $\fint$ is the Cauchy integral of principal value; S_σ is a small circle with a radius σ centered at $\mathbf{Y}(x, y)$; ρ is the distance between the point $\mathbf{Y}(x, y)$ and $\mathbf{Y}_0(x_0, y_0)$, i.e. $\rho = \left|\mathbf{Y} - \mathbf{Y}_0\right| < \sigma$. By the use of the concept of finite-part integrals, Eq. (30) is rewritten as

$$\begin{aligned}
&\frac{\partial^2}{\partial x^2}\int_{S^+}\frac{f(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta \\
&\quad= -\lim_{\substack{\rho\to 0\\ \sigma\to 0}}\left\{ \oint\!\!\!\!\!\!=_{S^+}\left[\frac{1}{r^3}-\frac{3(x-\xi)^2}{r^5}\right] f(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta \right.\\
&\qquad+\frac{\partial f(x,y)}{\partial x}\fint_{S_\sigma}\frac{x-\xi}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta + f(x,y)\oint\!\!\!\!\!\!=_{S_\sigma}\left[\frac{1}{r^3}-\frac{3(x-\xi)^2}{r^5}\right]\mathrm{d}\xi\,\mathrm{d}\eta \\
&\qquad\left.-\frac{\partial f(x,y)}{\partial x}\fint_{S_\sigma}\frac{x-\xi}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta - f(x,y)\frac{\partial}{\partial x}\fint_{S_\sigma}\frac{x-\xi}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta\right\}. \qquad (31)
\end{aligned}$$

In terms of the definitions of finite-part integrals and Cauchy integrals, one has

$$\frac{1}{2}\lim_{\rho\to 0}\oint\!\!\!\!\!\!=_{S_\sigma}\frac{1}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta = \lim_{\rho\to 0}\oint\!\!\!\!\!\!=_{S_\sigma}\frac{(x-\xi)^2}{r^5}\mathrm{d}\xi\,\mathrm{d}\eta = -\lim_{\rho\to 0}\frac{\partial}{\partial x}\fint_{S_\sigma}\frac{x-\xi}{r^3}\mathrm{d}\xi\,\mathrm{d}\eta = -\frac{\pi}{\sigma}. \qquad (32)$$

Thus, Eq. (30) is simplified as

$$\frac{\partial^2}{\partial x^2}\int_{S^+}\frac{f(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta = -\oint\!\!\!\!\!\!=_{S^+}\left[\frac{1}{r^3}-\frac{3(x-\xi)^2}{r^5}\right] f(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta. \qquad (33)$$

Similarly,

$$\frac{\partial^2}{\partial y^2}\int_{S^+}\frac{f(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta = -\oint\!\!\!\!\!\!=_{S^+}\left[\frac{1}{r^3}-\frac{3(y-\eta)^2}{r^5}\right] f(\xi,\eta)\mathrm{d}\xi\,\mathrm{d}\eta, \qquad (34)$$

$$\frac{\partial^2}{\partial x \partial y} \int_{S^+} \frac{f(\xi,\eta)}{r} \mathrm{d}\xi\, \mathrm{d}\eta = \neq\!\!\!\!\int_{S^+} \frac{3(x-\xi)(y-\eta)}{r^5} f(\xi,\eta) \mathrm{d}\xi\, d\eta. \tag{35}$$

Substitution of Eqs. (33)–(35) into (8) and (9) leads to

$$\frac{1}{4\pi}\left\{\chi_0\omega_{01}\Delta\int_{S^+}\frac{u_x(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta - \left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x^2}\int_{S^+}\frac{u_x(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta \right.$$
$$\left. - \left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x\partial y}\int_{S^+}\frac{u_y(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right\} = -p_x(x,y), \tag{36}$$

$$\frac{1}{4\pi}\left\{\chi_0\omega_{01}\Delta\int_{S^+}\frac{u_y(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta - \left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial y^2}\int_{S^+}\frac{u_y(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta \right.$$
$$\left. - \left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x\partial y}\int_{S^+}\frac{u_x(\xi,\eta)}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right\} = -p_y(x,y). \tag{37}$$

What follows, we will discuss the solutions of Eqs. (36) and (37). From Eqs. (10) and (11), the displacement jumps across the crack surfaces under uniform shear loading can be expressed as

$$u_x(\xi,\eta) = A_x\sqrt{1-(\xi/a)^2-(\eta/b)^2}, \quad u_y(\xi,\eta) = A_y\sqrt{1-(\xi/a)^2-(\eta/b)^2}, \tag{38}$$

where A_x and A_y are constants to be determined. Substituting (38) into (36) and (37), we have

$$\frac{1}{4\pi}\left\{A_x\chi_0\omega_{01}\Delta\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right.$$
$$- A_x\left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x^2}\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta$$
$$\left. - A_y\left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x\partial y}\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right\}$$
$$= -p_x(x,y), \tag{39}$$

$$\frac{1}{4\pi}\left\{A_y\chi_0\omega_{01}\Delta\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right.$$
$$- A_y\left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial y^2}\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta$$
$$\left. - A_x\left(\chi_0\omega_{01}-\sum_{i=1}^{3}\chi_i\omega_{i1}\right)\frac{\partial^2}{\partial x\partial y}\int_{S^+}\frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta\right\}$$
$$= -p_y(x,y). \tag{40}$$

According to Eshelby [1946], one has

$$\int_{S^+} \frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta = \frac{\pi ab}{2}\int_0^\infty \left\{1-\frac{x^2}{a^2+s}-\frac{y^2}{b^2+s}\right\}\frac{ds}{\sqrt{Q(s)}}, \qquad (41)$$

in which s is the integral variable, and $Q(s) = (a^2+s)(b^2+s)s$. If setting $s = t^2$, the closed-type solution of Eq. (41) is obtained as [Gradshteyn and Ryzhik (1969)]

$$\int_{S^+} \frac{\sqrt{1-(\xi/a)^2-(\eta/b)^2}}{r}\mathrm{d}\xi\,\mathrm{d}\eta = \pi b\left[K(k)-\frac{x^2}{a^2}\frac{K(k)-E(k)}{k^2}-\frac{y^2}{a^2}\frac{E(k)-(1-k^2)K(k)}{(1-k^2)k^2}\right] \qquad (42)$$

where $k^2 = 1-(b/a)^2$; $K(k)$ and $E(k)$ are the complete elliptic integrals of the first kind and that of the second kind, respectively.

From Eqs. (39), (40), and (42), we have

$$A_x = \frac{2bk^2}{B(k)}p_{x0}, \quad A_y = \frac{2bk^2}{C(k)}p_{y0}, \qquad (43)$$

here p_{x0} and p_{y0} are respectively uniform shear loadings along the x- and y-axis, and

$$\begin{aligned} B(k) &= \left(\chi_0\omega_{01} - k'^2\sum_{i=1}^3 \chi_i\omega_{i1}\right)E(k) + \left(\sum_{i=1}^3 \chi_i\omega_{i1} - \chi_0\omega_{01}\right)k'^2K(k), \\ C(k) &= \left(\sum_{i=1}^3 \chi_i\omega_{i1} - k'^2\chi_0\omega_{01}\right)E(k) + \left(\chi_0\omega_{01} - \sum_{i=1}^3 \chi_i\omega_{i1}\right)k'^2K(k), \end{aligned} \quad k' = b/a \le 1.$$

Substitution of (43) into (38) yields

$$\begin{aligned} u_x(\xi,\eta) &= \frac{2bk^2}{B(k)}p_{x0}\sqrt{1-(\xi/a)^2-(\eta/b)^2}, \\ u_y(\xi,\eta) &= \frac{2bk^2}{C(k)}p_{y0}\sqrt{1-(\xi/a)^2-(\eta/b)^2}. \end{aligned} \qquad (44)$$

We have not been aware of any reports similar to Eq. (44) in the literature for a piezoelectric elliptical crack.

6.2. *The stress intensity factors*

As shown in Fig. 2(b), we introduce a local triple orthogonal intrinsic coordinate system: (τ, n, z) at a point Q along the smooth periphery ∂S of the planar crack, then the expressions for the stress intensity factors in Eq. (26) are rewritten as

$$\begin{aligned} K_{\mathrm{II}}(Q) &= \lim_{\xi_n\to 0}\left[\frac{\sum_{i=1}^3 \chi_i\omega_{i1}}{2\sqrt{2}}\frac{u_n(\xi_n,0)}{\sqrt{\xi_n}}\right], \\ K_{\mathrm{III}}(Q) &= \lim_{\xi_n\to 0}\left[\frac{\chi_0\omega_{01}}{2\sqrt{2}}\frac{u_\tau(\xi_n,0)}{\sqrt{\xi_n}}\right]. \end{aligned} \qquad (45)$$

Using Eqs. (44) and (45) as well as the following relationship of coordinate transformation:

$$u_n(\xi,\eta) = u_x(\xi,\eta)\cos\theta + u_y(\xi,\eta)\sin\theta$$

$$u_\tau(\xi,\eta) = -u_x(\xi,\eta)\sin\theta + u_y(\xi,\eta)\cos\theta$$

$$\tan\theta = \frac{b\sin\varphi}{a\cos\varphi} \quad \text{and} \quad \xi_n = \frac{ab(1-\rho)}{\sqrt{a^2\sin^2\varphi + b^2\cos^2\varphi}},$$

we have

$$K_{\rm II}(Q) = \sum_{i=1}^{3}\chi_i\omega_{i1}\left[\frac{bp_{x0}}{B(k)}\cos\varphi + \frac{ap_{y0}}{C(k)}\sin\varphi\right]k^2\sqrt{\frac{b}{a}}\bigg/\bar{K}(Q), \tag{46}$$

$$K_{\rm III}(Q) = \chi_0\omega_{01}\left[\frac{bp_{y0}}{C(k)}\cos\varphi - \frac{ap_{x0}}{B(k)}\sin\varphi\right]k^2\sqrt{\frac{b}{a}}\bigg/\bar{K}(Q), \tag{47}$$

where $\bar{K}(Q) = (a^2\sin^2\varphi + b^2\cos^2\varphi)^{1/4}$. To the author's best knowledge, exact derivations of Eqs. (46) and (47) for a piezolectric elliptical crack subjected to uniform pressures are firstly perfectly obtained by the present author. In the case of penny-shaped cracks, $E(k) = K(k) = \pi/2$ and Eqs. (46) and (47) are reduced to

$$K_{\rm II}(Q) = \frac{4\sqrt{a}}{\pi}\frac{\sum_{i=1}^{3}\chi_i\omega_{i1}}{\left(\sum_{i=1}^{3}\chi_i\omega_{i1} + \chi_0\omega_{01}\right)}(p_{x0}\cos\varphi + p_{y0}\sin\varphi), \tag{48}$$

$$K_{\rm III}(Q) = \frac{4\sqrt{a}}{\pi}\frac{\chi_0\omega_{01}}{\left(\sum_{i=1}^{3}\chi_i\omega_{i1} + \chi_0\omega_{01}\right)}(p_{y0}\cos\varphi - p_{x0}\sin\varphi). \tag{49}$$

The above results of Eqs. (48) and (49) can be combined into those obtained by Chen and Shioya [2000] and Karapetian *et al.* [2000].

7. Numerical Results and Discussion

In this section, using the previous numerical procedure suggested by the author (Chen [2003b]), several numerical examples are considered based on the hypersingular integral Eqs. (8) and (9). These numerical results for elliptical crack problems will be compared with the exact solutions (46)–(49).

For convenience, it is assumed that the crack is loaded with a uniform shear loading $p_x(x,y) = q$ and $p_y(x,y) = p_{y0}(= 0)$ in all examples. The piezoelectric material PZT-4 is used for all computations. The electroelastic material constants are

$$c_{11} = 139.0,\quad c_{12} = 74.3,\quad c_{13} = 77.8,\quad c_{33} = 113.0,\quad c_{44} = 25.6\ \text{GPa}$$

$$e_{15} = 13.44,\quad e_{31} = -6.98,\quad e_{33} = 13.84\ \text{C/m}^2,$$

$$\in_{11} = 60.0\times10^{-10},\quad \in_{33} = 54.7\times10^{-10}\ \text{C/Vm}.$$

Table 1. Comparisons of numerical dimensionaless stress intensity factors Y_{II} and Y_{III} with exact solutions for a penny shaped crack.

φ	0	22.5	45	67.5	90
Y_{II}					
Numerical results	0.76975	0.71115	0.54429	0.29457	0.0000
Equation (48)	0.76975	0.71115	0.54429	0.29457	0.0000
Y_{III}					
Numerical results	0.0000	−0.19268	−0.35602	−0.46517	−0.50349
Equation (49)	0.0000	−0.19268	−0.35602	−0.46517	−0.50349

7.1. *Penny-shaped crack*

Under uniform shear loading, the exact solutions for a penny-shaped crack in an infinite piezoelectric ceramics material have been derived by the author in the proceeding section [see Eqs. (48) and (49)]. They can be rewritten as

$$\begin{aligned} K_{\text{II}}(Q) &= \frac{4\sqrt{a}}{\pi}\frac{\sum_{i=1}^{3}\chi_i\omega_{i1}}{\left(\sum_{i=1}^{3}\chi_i\omega_{i1}+\chi_0\omega_{01}\right)}(p_{x0}\cos\varphi+p_{y0}\sin\varphi) \\ &= Y_{\text{II}}(\chi_0\omega_{01},\chi_i\omega_{i1},a,\varphi)q\sqrt{a}, \end{aligned} \tag{50}$$

$$\begin{aligned} K_{\text{III}}(Q) &= \frac{4\sqrt{a}}{\pi}\frac{\chi_0\omega_{01}}{\left(\sum_{i=1}^{3}\chi_i\omega_{i1}+\chi_0\omega_{01}\right)}(p_{y0}\cos\varphi-p_{x0}\sin\varphi) \\ &= Y_{\text{III}}(\chi_0\omega_{01},\chi_i\omega_{i1},a,\varphi)q\sqrt{a}. \end{aligned} \tag{51}$$

Table 1 lists the present numerical results and the exact solutions. It can be seen that both are identical to the fifth significant digit.

7.2. *Elliptical crack*

The exact solution of an elliptical crack embedded in an infinite piezoelectric media subjected to uniform shear loading is shown in Eqs. (46) and (47). The expressions of stress intensity factors take the form

$$\begin{aligned} K_{\text{II}}(Q) &= \sum_{i=1}^{3}\chi_i\omega_{i1}\left[\frac{bp_{x0}}{B(k)}\cos\varphi+\frac{ap_{y0}}{C(k)}\sin\varphi\right]k^2\sqrt{\frac{b}{a}}\Big/\bar{K}(Q) \\ &= Y_{\text{II}}(\chi_0\omega_{01},\chi_i\omega_{i1},a,b,\varphi)q\sqrt{b} \end{aligned} \tag{52}$$

$$\begin{aligned} K_{\text{III}}(Q) &= \chi_0\omega_{01}\left[\frac{bp_{y0}}{C(k)}\cos\varphi-\frac{ap_{x0}}{B(k)}\sin\varphi\right]k^2\sqrt{\frac{b}{a}}\Big/\bar{K}(Q) \\ &= Y_{\text{III}}(\chi_0\omega_{01},\chi_i\omega_{i1},a,b,\varphi)q\sqrt{b} \end{aligned} \tag{53}$$

Table 2 gives comparisons of present numerical stress intensity factors for an elliptical crack with an aspect ratio $a/b = 2$ with theoretical solutions. It is shown

Table 2. Comparisons of numerical dimensionaless stress intensity factors Y_{II} and Y_{III} with exact solutions for an ellipitical crack.

φ	0	22.5	45	67.5	90
Y_{II}					
Numerical results	0.78466	0.66185	0.44125	0.21860	0.0000
Equation (46)	0.78466	0.66184	0.44125	0.21859	0.0000
Y_{III}					
Numerical results	0.0000	−0.35866	−0.57725	−0.69039	−0.72585
Equation (47)	0.0000	−0.35864	−0.57724	−0.69038	−0.72584

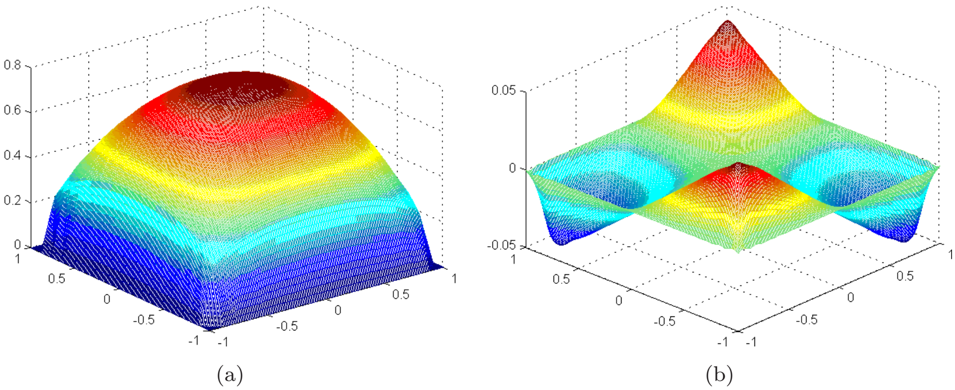

Fig. 3. (a) Displacement jumps u_x along x-axis and (b) displacement jumps u_y along x-axis.

that the present numerical results yield a good convergence to the fifth significant figure compared with the exact solutions.

7.3. *Rectangular crack*

The hypersingular integral Eqs. (8) and (9) are analyzed for a rectangular crack. Since no exact solutions are available for this crack configuration, only numerical analysis is performed with an aspect ratio of $a/b = 1$ as an example. Figures 3(a) and 3(b) plot the displacement jumps u_x on the crack surfaces under uniform loading along x-axis and the displacement jumps u_y along y-axis, respectively.

8. Conclusion

In this paper, starting from the fundamental solution of a point force and electric charge for three-dimensional transversely isotropic piezoelectric solid, we have performed rigorous theoretical analyses of some basic problems of fracture mechanics by using the hypersingular integral equation method and obtained a system of hypersingular integral Eqs. (8) and (9) for solving arbitrarily shaped planar crack problems under shear loading. Exact solutions are presented for a piezoelectric elliptical crack subjected to uniform shear loading. Modes II and III stress intensity factors also derived in an exact manner. These solutions are undoubtedly helpful for us to study 3D piezoelectric crack problems and complements of 3D fracture mechanics of piezoelectrics. In the final discussion of the present paper, as examples of applications, we have also given numerical results for elliptical cracks and rectangular cracks under uniform shear loading and compared with theoretical solutions.

Acknowledgments

The author wishes to express his thanks to the Natural Science Foundation of Jiangxi Province under Grants No. 0112001, P. R. China, and to Prof. N. A. Noda of Mechanical Engineering Department, Kyushu Institute of Technology, Japan for his helpful suggestion. This research was supported in part by the Japan Society for the Promotion of Science Postdoctoral Fellowship.

References

Chen, J. T., Kuo, S. R. & Lin, J. H. [2002] "Analytical study and numerical experiments for degenerate scale problems in the boundary element method for two-dimensional elasticity," *Int. J. Numer Methods Engrg.* **54**, 1669–1681.

Chen, M.-C., Noda, N.-A. & Tang, R.-J. [1999] "Application of finite-part integral to planar interfacial fracture problems in 3D bimaterails," *ASME J. Appl. Mech.* **66**, 885–890.

Chen, M.-C., Yu, H.-G. & Tang, R.-J. [2002] "A new numerical method with high accuracy for 3D crack problems," *Solida Mech. Sinica* **23**, 207–212 (in Chinese).

Chen, M.-C. [2003a] "Application of finite-part integrals to three-dimensional fracture problems for piezoelectric media, Part I: Hypersingular integral equation and theoretical analysis," *Int. J. Fract.* **121**, 133–148.

Chen, M.-C. [2003b] "Application of finite-part integrals to three-dimensional fracture problems for piezoelectric media, Part II: Numerical analysis," *Int. J. Fract.* **121**, 149–161.

Chen, M.-C. [2004a] "A method of hypersingular integral equation for three-dimensional cracks in transversely isotropic materials," *Symposium of the IXth Conf. Chinese Modern Mathematics and Mechanics*, October 4–7, Shanghai.

Chen, M.-C. [2004b] "3D fracture mechanics for a mode I crack in piezoelectric media," *Int. J. Comp. Methods* **1**, 445–456.

Chen, W. Q. & Shioya, T. [1999] "Fundamental solution for a penny-shaped crack in a piezoelectric medium," *J. Mech. Phys. Solids* **47**, 1459–1475.

Chen, W. Q. & Shioya, T. [2000] "Complete and exact solutions of a penny-shaped crack in piezoelectric solid: Antisymetric shear loadings," *Int. J. Solids Struct.* **37**, 2603–2619.

Chen, W. Q., Shioya, T. & Ding, H. J. [2000] "A penny-shaped crack in piezoelectrics: Resolved," *Int. J. Fract.* **105**, 49–56.

Ding, H. J. *et al.* [2004] "Green's functions and boundary element method for transversely isotropic piezoelectric materials," *Engineering Analysis with Boundary Elements* **28**(8), 975–987.

Eshelby, J. D. [1946] "The elastic field outside ellipsoidal inclusion," *Proc. Royal Soc. London A* **252**, 561–569.

Gradshteyn, I. S. & Ryzhik, I. M. [1969] *Table of Integrals, Series and Products* (Wiley, New York).

Hadamard, J. [1923] *Lectures on Cauchy's Problem in Linear Partial Differential Equations* (Dover, New York).

Ioakimidis, N. I. [1982] "Application of finite-part integrals to the singular integral equations of crack problems in plane and three-dimensional elasticity," *Acta Mech.* **45**, 31–47.

Karapetian, E., Sevostianov, I. & Kachanov, M. [2000] "Penny-shaped and half-plane cracks in a transversely isotropic piezoelectric solid under arbitrary loading," *Arch. Appl. Mech.* **70**, 201–229.

Kokan, I., Hui, C. Y. & Molkov, V. [1996] "Stress and induction field of a spherodial inclusion or a penny-shaped crack in a transversely isotropic piezoelectric material," *Int. J. Solids Struct.* **33**, 2719–2737.

Pak, Y. E. [1992] "Linear electro-elastic fracture mechanics of piezoelectric materials," *Int. J. Fract.* **54**, 79–100.

Park, S. B. & Sun, C. T. [1995] "Effect of electric field on fracture of piezoelectric ceramics," *Int. J. Fract.* **70**, 203–216.

Qin, H. Y. [2001] *Fracture Mechanics of Piezoelectric Materials* (WIT Press, Southhampton, Boston).

Suo, Z., Kuo, C. M., Barnett, D. M. & Wills, J. R. [1992] "Fracture mechanics for piezoelectric ceramics," *J. Mech., Phys. Solids* **40**, 739–756.

Shang, F. L., Kuna, M. & Abendroth, M. [2003] "Finite element analyses of three-dimensional crack problems in piezoelectric structures," *Engrg. Fract. Mech.* **70**, 143–160.

Wang, B. [1992] "Three-dimensional analysis of a flat elliptical crack in a piezoelectric material," *Int. J. Engrg. Sci.* **30**, 781–791.

Wang, Z .K. [1994] "Penny-shaped crack in transversely isotropic piezoelectric materials," *Acta Mech. Sinica* **10**, 49–60.

Wang, Z. K. & Huang, S. H. [1995] "Stress intensification near an elliptical border," *Theor. Appl. Fract. Mech.* **22**, 229–237.

Zhang, T. Y., Zhao, M. H. & Tong, P. [2002] "Fracture of piezoelectric ceramics," *Adv. Appl. Mech.* **38**, 147–289.

Zhang, Z & Meguid, S. A. [1997] "Analysis of a circular arc-crack in piezoelectric materials," *Int. J. Fract.* **84**, 143–158.

Zhao, M. H., Shen, Y. P., Liu, J. Y. & Liu, G. N. [1997] "Isolated crack in three-dimensional piezoelectric solid, Part II: Stress intensity factors for circular crack," *Theor. Appl. Fract. Mech.* **26**, 141–149.