A method to extract the lateral and normal components of motion from the capacitance change of a moving MEMS comb drive

W Merlijn van Spengen¹,² and Erwin C Heeres¹

¹ Leiden University, Kamerlingh Onnes Laboratory, PO Box 9504, 2300RA Leiden, The Netherlands
² Falco Systems, Gelderlandplein 75L, 1082LV Amsterdam, The Netherlands

E-mail: spengen@physics.leidenuniv.nl

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Abstract
MEMS comb drives made in surface micromachining can suffer from a parasitic out-of-plane motion (levitation) in addition to the intended lateral motion. We have developed a model that accurately describes the capacitance changes of an actuated comb drive that suffers from levitation. We show that the model can be used to very accurately extract the lateral motion as a function of actuation voltage. This enables us to use the comb drive as a position sensor with very high accuracy, which does not suffer from levitation-induced nonlinearities.

Introduction
In surface micromachining, comb drives are the most commonly used actuators for moderate in-plane displacements (microns) and forces (micro-Newton). They are also often used as motion sensing elements in accelerometers and gyroscopes, among others.

Tang et al [1] have already discussed in 1992 the parasitic out-of-plane motion of actuated comb drives, an effect that causes trouble in many MEMS devices, e.g. gyroscopes [2] and torsional oscillators [3]. They found that the effect can be predicted and, in some cases, counteracted by a special design. A more comprehensive finite element modeling (FEM) of the levitation effect has been performed more recently [4–6]. The effect can also be put to use, as shown in [7], where it is used for vertical actuation.

The levitation effect is one of the reasons (among others) why more recently people have increasingly been using thick layers and DRIE (deep reactive ion etching) to make the structures stiffer in the out-of-plane direction. This prevents the out-of-plane motion to a large extent, but may, on a smaller scale, still introduce nonlinearities in the response. For other research, especially using complex structures in multiple thin layers, levitation is an important issue to reckon with.

The levitation force shows up as a detrimental effect if one is interested in measuring the displacement of actuated comb drives by monitoring the capacitance changes. We will show that by a careful measurement of the capacitance versus actuation voltage, the lateral and parasitic normal components of the comb drive motion can be very accurately separated by a simple analytical model.

The test structure: typical C/V curve of an actuated comb drive

We have developed a comb drive structure that has both actuation and displacement measurement comb drives, which can be used to measure surface forces. We have called it the ‘nano-battering ram’ [8] (figures 1 and 2). The device has a head and a counter-surface with a gap of 2.0 µm between the two, which provides a motion calibration opportunity. When the ram is moved by applying an actuation voltage, it will move only until the gap is closed.

We have also designed a sensitive electronic measurement system to monitor the comb drive capacitance changes with high sensitivity [9]. This enables us to see the capacitance changes of the comb drive actuator upon actuation. In figure 3, the result of pulling the measurement combs out of each other (thus decreasing the capacitance) by an actuation voltage on...
Figure 1. Schematic top view of the test vehicle: the nano-battering ram.

Figure 2. SEM image of the nano-battering ram (produced in the MEMSCap PolyMUMPs process).

The Vact combs is given. We see that at 54 V the head touches the counter-surface because the gap is closed. At this point the comb drive has moved 2.0 μm. When moving back, the stiction force holds the ram in contact until at 52 V the restoring force of the comb drive springs is so large that the head and the counter-surface separate again.

Theoretically, with only lateral motion of the comb drive, the curve should only show a parabola of decreasing capacitance with the maximum at 0 V, because the lateral comb drive motion is given by the equations of figure 4. However, we see a significant capacitance increase at the lower voltage side of the curve, which is caused by levitation of the comb drive. In addition, the curves open up with some hysteresis at the lower end of the curve. The latter is caused by the relatively high speed at which the structure is actuated as compared to its resonance in this measurement and will not be further discussed here. To use the capacitance change for position monitoring, a model is required that describes both the in-plane and out-of-plane motions and their combined influence on the capacitance change.

Levitation

Comb drive levitation [1] is caused by the fact that a ground plane is required under the grounded moving structure to prevent it from collapsing due to small voltage variations between the moving part and the chip substrate. The fringe field lines from the top cannot be compensated by field lines from the bottom (figure 5), causing a net upward motion.

This effect is easily observable in figure 6, where a comb drive structure actuated in the SEM (scanning electron microscope) is shown. A darkening of the actuation electrodes occurs, caused by a modification of the path of the SEM electrons by the actuation voltage (voltage contrast). We see that the comb drive beams move both up and sideways.

Tang et al [1] have provided a model for the out-of-plane motion. They found with 2D electrostatic simulations that an equilibrium height exists, toward which the actuated comb drive wants to move. This height is defined by a maximum number of field lines between the actuation electrode and the moving structure (the highest effective capacitance). They call this height \( z_0 \). The actuation voltage at which this height \( z_0 \) is approached depends both on the spring coefficient of the comb drive in the \( z \)-direction \( k_z \) and on the vertical drive capacity \( \gamma_z \), a proportionality constant to obtain the out-of-plane force. This \( \gamma_z \) depends on the overlap of the actuation comb fingers and the moving comb fingers. They derive that the actual levitation \( z \) at a certain voltage of a comb drive can be given by

\[
 z = z_0 \frac{V^2}{k_z z_0 + \gamma_z V^2}. \tag{1}
\]

Figure 7 shows the calculated height change \( z \) as a function of the actuation voltage for a device with \( z_0 = 0.5 \mu m \), \( k_z = 1 \text{ N m}^{-1} \) and \( \gamma_z = 1.5 \text{ nN} \mu m^{-1} \). Tang et al [1] did not calculate the effect of this levitation on the lateral motion and the capacitance of the comb drive. Also, the other FEM
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\[ F_x = \frac{\varepsilon n (p + z)}{g} V^2 \]

\[ F_{act} = n F_x \]

\[ F_{act} = k_x \delta_x \]

\[ \delta_x = \frac{en (p + z)}{k_x g} V^2 \]

\[ C = \frac{2 n \varepsilon_0 l}{g} \]

**Figure 4.** Typical comb drive actuator with the important geometric variables and equations indicated [10]. \( \varepsilon \) = permittivity of free space, \( V \) = applied actuation voltage, \( n \) = number of comb fingers, \( t \) = thickness of combs, \( F_x \) = electrostatic force in the \( x \)-direction, \( \delta_x \) = displacement of the comb drive in the \( x \)-direction, \( k_x \) = spring constant of supporting beam in the \( x \)-direction, \( C \) = the comb drive capacitance (factor 2 for both sides of the finger).

**The model for combined lateral and normal motions**

For the calculation of the total capacitance change as a function of voltage, we model the comb drive as a system where the capacitance increases linearly with increasing \( z \) until the equilibrium height is reached. This means that the total force in the \( x \)-direction (lateral motion) is

\[ F_x = \frac{en (p + z)}{g} V^2, \quad (2) \]

where \( p \) is the ‘effective overlap’ in the vertical direction at zero bias and \( z \) is the extra ‘overlap’ between the fingers caused by the levitation. In fact, purely geometrically speaking, the overlap decreases with increasing \( z \), but due to the non-uniform field line distribution, electrically the overlap effectively increases. This is the reason that levitation occurs at all (figures 8(a) and (b)).

Because \( F_x = k_x \delta_x \), with \( k_x \) as the lateral spring constant and \( \delta_x \) as the lateral deflection, the total deflection in the \( x \)-direction is

\[ \delta_x = \frac{en (p + z)}{k_x g} V^2. \quad (3) \]

**Figure 5.** Cross-section of a levitated comb drive. The fringe field lines from the top cause a net force upward.

Simulation studies [4-6] do not contain an investigation of the total comb drive capacitance change as a function of the applied driving voltage, but only as a function of the change in the height of the comb drive. Therefore, it is difficult to directly use the results from these studies for correcting the capacitance change like the one given in figure 3 for the levitation effect.

**Figure 6.** Two frames of a movie made in the SEM (scanning electron microscope), clearly showing the levitation of the comb drive, as well as the lateral motion, upon actuation. This is best observed in the encircled area. Also, note the voltage contrast that darkens the actuation combs due to the actuation voltage that changes the speed of the electrons. (a) No actuation, (b) actuation voltage: 60 V.
overlap length of the fingers at 0 V (figure 4), is a recursive relation in the model, we assume
influenced by the levitation
V
the actuation voltage
in full, we can see the capacitance change
of the opposite comb next to them. If we write out the equation
comb fingers contribute to the total capacitance with the finger
in which the factor 2 is introduced because both sides of the
overlap is incomplete (only over a height
p
less. This has been modeled by a ‘virtual’ system where the initial
increases because the screening effect of the ground plane becomes
the effective overlap (side view) that the fingers electrostatically feel
and
p
going to
z
progressively better upon actuation by an extra overlap height

Figure 7. Calculated comb drive height change as a function of actuation voltage.

(a)
\[ \text{Movable comb finger} \]
\[ \text{Fixed comb finger} \]

(b)
\[ \text{Movable finger moving up with actuation voltage} \]
\[ \text{Fixed comb finger} \]
\[ \text{Movable comb finger} \]

Figure 8. (a) Side view of the real geometrical variation upon actuation. The movable finger gets progressively less overlap with the fixed finger as the levitation effect increases. (b) Definition of \( z \) and \( p \). Although the fingers in reality are moving out of each other, the effective overlap (side view) that the fingers electrostatically feel increases because the screening effect of the ground plane becomes less. This has been modeled by a ‘virtual’ system where the initial overlap is incomplete (only over a height \( p \)) and becomes progressively better upon actuation by an extra overlap height \( z \), going to \( z_0 \) for perfect overlap.

Here, we see that the efficiency of the drive voltage \( V \) partially depends on the levitation \( z \). The lateral deflection \( \delta \) is hence influenced by the levitation \( z \). In fact, the deflection \( \delta \) changes the overlap length (\( =l \) at 0 V) of the fingers, and hence also slightly changes \( y_z \), which depends on this overlap. To prevent a recursive relation in the model, we assume \( y_z \) to be constant, which is a very good approximation, as will be shown later by fitting the model to the measured data. However, in the case where the comb drive moves over (almost) the full length of the combs so that this effect cannot be neglected, this approximation should be reinvestigated.

If we assume a parallel plate type of geometry, the corresponding capacitance change \( \Delta C \), with \( l \) the equilibrium overlap length of the fingers at 0 V (figure 4), is

\[ \Delta C = \frac{2\varepsilon n (p + z)(l - \delta)}{g}, \quad \text{(4)} \]
in which the factor 2 is introduced because both sides of the comb fingers contribute to the total capacitance with the finger of the opposite comb next to them. If we write out the equation in full, we can see the capacitance change \( \Delta C \) as a function of the actuation voltage \( V \):

\[ \Delta C(V) = \frac{2\varepsilon n (p + z)(l - \delta)}{g} \left( \frac{p + z_0 y_z V^2}{k_z z_0 + y_z V^2} \right) \cdot \left( l - n \varepsilon \left( \frac{p + z_0 y_z V^2}{k_z g} \right) \right). \quad \text{(5)} \]

In this model all parameters are defined by the geometry and basic physics, except \( y_z \), which can be used as a fit parameter. If we use \( n = 128 \), \( \varepsilon = 8.85 \times 10^{-12} \text{ F m}^{-1} \), \( p = 1.5 \times 10^{-6} \text{ m} \), \( l = 7.0 \times 10^{-6} \text{ m} \), \( z_0 = 0.5 \times 10^{-6} \text{ m} \), \( k_z = 1 \text{ N m}^{-1} \), \( k_z = 1 \text{ N m}^{-1} \) and \( g = 2.0 \times 10^{-6} \text{ m} \), and the fit parameter \( y_z = 1.5 \times 10^{-6} \text{ N V}^{-2} \text{ m} \), the curve fits the measured capacitance as a function of voltage perfectly (figure 9).

Extracting the lateral and normal motion components

The lateral and normal components of the comb drive motion can now be extracted by using the fitted curve. The lateral motion is given by equation (3) and the normal motion (levitation) by equation (1). The capacitance change caused by the normal motion \( \Delta C_n(V) \) alone is, to a very good approximation, equal to

\[ \Delta C_n(V) = \frac{2n \varepsilon}{g} z l, \quad \text{(6)} \]

where we have again used the approximation that the \( z \)-drive force does not suffer from a shortening of the effective \( l \) due to the motion as discussed in the text after equation (3). This approximation cannot be used when edge effects of very short comb fingers cannot be neglected, in which case the calculation becomes much more complex. By subtracting this capacitance change \( \Delta C_n(V) \) from the total measured capacitance change, one obtains the capacitance change caused by the lateral
motion only (figure 10). By a similar method, the capacitance change due to the levitation motion only can be extracted as well (not shown).

We see from figure 10 that the lateral motion (normally the intended motion of a comb drive) behaves very well, like the parabolic dependence on actuation voltage as predicted by the simple theory of figure 4.

Because in this device the total motion until the gap is closed is known, an accurate scale in micrometers can be added to the measured data. This enables us to see the calibrated lateral motion with high accuracy (figure 11). Now, also note to the measured data. This enables us to see the calibrated closed is known, an accurate scale in micrometers can be added.

Figure 10. Capacitance change of the intended lateral motion after correction for the levitation.

Figure 11. Calibrated lateral motion as a function of the actuation voltage by using the corrected measured capacitance data. In addition, a true parabola is shown. Its 0 V position is lower than 0 μm, because the lateral driving force is smaller than the expected true parabola at low actuation voltages. The comb drive is not fully levitated there, and hence does not yet experience the full driving force. The discrepancy between approach and retract at the low voltage end is caused by the fact that the speed of motion in this measurement was too high for the device to follow accurately due to its low resonance frequency (not further discussed in the text).

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Because in this device the total motion until the gap is closed is known, an accurate scale in micrometers can be added to the measured data. This enables us to see the calibrated lateral motion with high accuracy (figure 11). Now, also note that the small capacitance increase after the gap is closed at higher voltages has disappeared due to the correction. A true parabola has also been added to see the effect of the levitation on the electrostatic driving force: the position change at low voltages is 30 nm smaller than that expected for a true parabola, because the comb drive does not experience the full lateral driving force when it is not yet fully levitated. However, the effect here is small, and comparable to a deviation due to the finite time response of the system that also gives an inaccuracy on the lower end of the curve (not further discussed here).

Conclusion

We have presented a model for both the regular lateral motion and the parasitic out-of-plane levitation motion of an actuated comb drive, and have described their combined influence on the capacitance change as a function of actuation voltage. It has been shown that the model offers an accurate description of a real comb drive in which the two motions are present. By using the model, one can correct for the aberrations of the expected parabolic curve that arises when monitoring the position of an actuated comb drive by its capacitance change, and the result matches the data very well. Using this model, it is possible to design a very accurate position monitoring system based on the capacitance change of an actuated comb drive.

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References