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Neural network model for solving integral equation of acoustic scattering using wavelet basis

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SUMMARY

In this work, neural networks (NNs) are trained in order to obtain a fast and efficient solution of the integral equation of acoustic scattering. Wavelets sparsification methods are utilized to reduce the NN size and complexity. The non-uniqueness problem which arises in solving this integral equation at characteristic frequencies can, also, be solved using such network without any additional computational load. Experimental results show excellent agreement between the NN-based solution and the analytical solution of a spherical scatterer. Copyright © 2006 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The moment method approach is a powerful tool for solving the integral equation of acoustic scattering problems. The moment method is essentially a discretization scheme whereby a general operator equation is transformed into a matrix equation. The resulting matrix is always dense when the conventional expansion and testing functions are used. Also, the moment matrix suffers from non-uniqueness problem at the characteristic wave numbers of the corresponding interior eigenequation [1, 2]. Recently, the mechanism by which the modal participation factor dominates the numerical instability at such frequencies is presented [3]. The solution of such a problem requires excessive computations. One of the main methods used to overcome non-uniqueness is the addition of the constraints on internal fields at a finite number of points (M, $M \ll N$ where N is the number of unknowns). This technique is known as CHIEF method [1]. Applying the

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discretization of the scattering surface to the CHIEF equations leads to an overdetermined system of equations for the surface field. A potential problem with this approach is the choice of appropriate interior points. The selected interior point must not be a nodal point of the corresponding interior eigenmode at the considered natural frequency. Another approach for solving the non-uniqueness problem was introduced by Burton and Miller [4]. They formed a linear combination of the Helmholtz integral equation used by CHIEF and its normal derivative which combines singular and hypersingular equations. This formulation is valid for all wave numbers [1]. A major drawback of this approach is the need to evaluate of the hypersingular integrals involving a double normal derivative of the free-space Green's function. The use of wavelet basis to sparsify a dense matrix can, significantly, reduce the necessary computations to find its inverse [5, 6]. A recent attempt is introduced in [7] to reduce the size and computational burden of the acoustic scattering problem using wavelets. The saving of additional computations required for solving the non-uniqueness problem is addressed in [8].

The neural networks (NNs) are, recently, employed in the calculation of the moment matrix elements [9]. The NN is, also, used in calculating Green's functions in the integral equations of electromagnetic-wave propagation in layered media [10]. Alternatively, the NN is trained to approximate the mathematical expression of an acoustic scattering on an acoustically hard sphere [11]. The NN can, also, be utilized to solve the system of equations resulting from discretizing the integral equation of acoustic scattering. However, the NN size becomes intractable due to the large number of the moment matrix elements. As a result, selection of the elements that characterize the moment matrix appears to be very crucial. The optimum selection of these elements is instrumental in reducing the size of the NN and consequently the training time. Dimensionality reduction of the inputs is, widely, addressed in the literature [12]. It is, basically, divided into two main categories: feature extraction and feature selection. Feature extraction algorithms create new features based on transformations or combinations of the original feature space to another space of reduced dimensions with some desired properties. On the other hand, feature selection algorithms select the best subset of the original features discarding redundant and irrelevant features. The wavelet analysis proved to be very effective as a feature extraction algorithm [7, 13–15].

In this work, the acoustic scattering field on a surface is expanded into Helmholtz integral equation in terms of wavelet basis functions. The substitution of such expansion into the integral equation results in a new matrix which can be thresholded appropriately to obtain a sparse matrix. The obtained sparse matrix exhibits, also, symmetrical patterns along its columns. NN-based solution of the problem is tried utilizing the sparsified moment matrix. The sparse and symmetrical properties of this matrix are exploited to reduce the NN size. This could drastically reduce the number of inputs to the NN, and consequently, minimize the training computational load. Exploiting the symmetrical pattern of the sparsified moment matrix, only a subset of the matrix elements could be selected which can uniquely represent it. This subset of moment matrix elements along with the incident field are considered as the set of inputs to the NN. Different training sets are constructed for different wavelengths of the incident field. The trained NN is, then, tested at different wave numbers including characteristic numbers at which a non-uniqueness problem is encountered with traditional methods. All solutions are compared with the analytical solution of the scattered field and show excellent match.

The paper is organized as follows. Section 2 summarizes the multiresolution wavelet analysis. The wavelet multiresolution analysis is applied in solving acoustic scattering in Section 3. A brief description of the NN model is given in Section 4. The NN design is discussed in Section 5. In Section 6, the numerical results are presented. Conclusions are drawn in Section 7.

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2. WAVELET MULTIRESOLUTION ANALYSIS

A set of subspaces $\{S_j\}$ where $j \in Z$ is said to be a multiresolution approximation of $L^2(R)$ if the following relations are satisfied:

$$
\begin{gathered}
S_j \subset S_{j+1} \quad \forall j \in Z \\
\bigcup_{j \in Z} S_j \quad \text{is dense in } L^2(R) \quad \forall j \in Z \\
\bigcap_{j \in Z} S_j = \{0\} \\
\psi(2x) \in S_{j-1} \quad \Leftrightarrow \quad \psi(x) \in S_j \\
\psi(x) \in S_j \;\Leftrightarrow\; \psi(x - 2^{-j} n) \in S_j \;\; \forall j, \;\; n \in Z
\end{gathered} \tag{1}
$$

where Z is the set of integers.

The properties in Equation (1) state that $\{S_j\}_{j \in Z}$ is a nested sequence of subspaces that effectively covers $L^2(R)$. That is, every square integrable function can be approximated as closely as desired by a function that belongs to at least one of the subspaces S_j.

Accordingly, a wavelet family can be generated from what is called mother wavelet. All wavelets of a family share the same properties and their collection constitutes a complete basis. A wavelet ψ_{jk} is defined as follows:

$$\psi_{jk}(x) = 2^{j/2} \psi(2^j x - k) \tag{2}$$

where j and k are indices indicating scale and location of a particular wavelet. As a result, the wavelet family is a collection of wavelet functions $\psi_{jk}(x)$ that are translated along the real axis x, then dilated by 2^j times and the new dilated wavelet is translated along the real line again. The wavelet function is necessarily to have local (or almost local) support in both a real dimension and frequency domains.

The decomposition of a discrete function in orthonormal bases functions is called a multi-resolution analysis. An approximation of a function $f(x) \in L^2(R)$, at a resolution of 2^{-j}, can be defined as the projection on different wavelet functions

$$f(x) = \sum_j \sum_k a_{jk} \psi_{jk}(x), \quad j = 1, \ldots, M \text{ and } k = 1, \ldots, N \tag{3}$$

where a_{jk} is the amplitude of each wavelet at different locations and resolutions (scales). M is the number of discretization points in space and N is the number of wavelet decompositions.

A formal approximation of an unknown function at a given resolution, with a finite number of successive length scales, requires both scaling and translation operators across the expansion dimension. From a practical point of view, the approximate solution of an integral equation can be expressed as a summation of an approximate function $C(x)$ and a series of orthogonal wavelets for the finer details.

The approximation function $C(x)$ can represent a constant value along the solution domain [16]. More general form of Equation (3) can be written as follows:

$$f(x) = a_0 C(x) + \sum_{j=n1}^{n2} \sum_{k=m1}^{m2} a_{jk} \psi_{jk}(x) \tag{4}$$

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where a_0 and a_{jk} are yet to be determined coefficients and

$$C(x) = \begin{cases} 1, & x \in \text{problem boundary} \\ 0 & \text{otherwise} \end{cases}$$

The summation in (4) is over values of j ranging from $n1$, which corresponds to the larger characteristic length scale, to $n2$, which corresponds to the desired resolution in scaling. Recalling that with reference to $\psi_{00}(x)$, the effective width of $\psi_{jk}(x)$ is changed gradually by a factor of 2^{-m} and its centre—a point on the wavelet grid—is shifted by the distance $m2^{-n}$. For a given value of m, the number of wavelet functions, $k(n) = m_2 - m_1$, is set so that their centres fall within the problem domain and outside. Hence, as j increases, more wavelets and more grid points are involved at each resolution level.

The discrimination between two classes of wavelet families should be considered in the selection of wavelet family for multiresolution analysis besides that it should be orthonormal. The following two examples exhibit the localization properties of wavelets:

$$\text{Shannon family} \quad \psi(x) = \frac{\sin(\pi x/2)}{\pi x/2} \cos\left(\frac{3\pi x}{2}\right) \tag{5}$$

$$\text{Haar family} \quad \psi(x) = \begin{cases} 1, & 0 \leqslant x < 0.5 \\ -1, & 0.5 \leqslant x \leqslant 1 \\ 0, & x > 1 \end{cases} \tag{6}$$

The above two wavelet family examples are opposite to each other in terms of their localization properties. The Haar wavelet has good space localization but poor space frequency localization. Its spectrum is non-zero when the frequency tends to ∞. It does not have compact support in the space–frequency domain. In contrast, the Shannon wavelet has non-compact support in space, hence it has poor space localization [17].

3. WAVELET ANALYSIS OF THE ACOUSTIC SCATTERING PROBLEM

For a surface S on which a unit normal $\mathbf{n}$, pointing outward, when a harmonic acoustic wave φ^i impinges upon an acoustically hard body V enclosed by that surface, the resulting integral equation for smooth surface has the following form:

$$4\pi\varphi^i(x') + \int_S \frac{\partial G(x, x')}{\partial n} \varphi(x') \, \mathrm{d}S(x') = 2\pi\varphi(x), \quad x' \in S \tag{7}$$

where G is the free-space Green's function which is given by $G(x, x') = \mathrm{e}^{-\mathrm{i}kR}/R$ where R is the distance between the field point x and a source point x' while k is the wave number and $\varphi(x)$ is the acoustic field at a point x. If we consider only the fully axisymmetric bodies, scattering equation (7) reduces to one-dimensional form.

The unknown field function can be expanded in terms of wavelet base functions as in Equation (4). The approximation of $\varphi(x) \in L^2(R)$, at a resolution of 2^{-j}, can be defined as

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the projection on different wavelet functions as follows:

$$\varphi(x) = a_0 + \sum_j \sum_k a_{jk} \psi_{jk}(x), \quad j = 1, \ldots, M \text{ and } k = 1, \ldots, N \tag{8}$$

where a_{jk} is the amplitude of each wavelet at different resolutions (scales) and locations while a_0 represents a constant bias in the function. M are the discretization points on the scatterer surface.

Substituting (8) into the Galerkin's discretized form of the integral equation, it reduces to [7]

$$\mathbf{A}_{M\times M} \mathbf{W}_{M\times M} \boldsymbol{\phi}_{M\times 1} = \mathbf{Y}_{M\times 1} \tag{9}$$

where $\mathbf{A}$ is the Galerkin's moment matrix, $\boldsymbol{\phi}$ is the unknown wavelet amplitudes vector, $\mathbf{Y}$ is the incident field vector defined at the surface points. The effective support (non-zero elements), in the matrix $\mathbf{W}$ of a wavelet ψ_{mn}, is the interval outside of which the wavelet is practically zero. Using Equation (7), the moment matrix can expressed in the following way:

$$\mathbf{A} = \begin{bmatrix} G'(R11) & G'(R12) & \cdots & G'(R1M) \\ G'(R21) & G'(R22) & \cdots & G'(R2M) \\ \vdots & & & \vdots \\ G'(RM1) & G'(RM2) & \cdots & G'(RMM) \end{bmatrix}$$

where $G'(Rij)$ is the first derivative of Green's function in Equation (7) for the distance between the discretized points i and j. Moreover, the matrix W has the following structure:

$$\mathbf{W}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & \cdots & & & & & & & 1 & 1 \\ b_0 & b_1 & & & & & & & & & & \\ & & b_0 & b_1 & & & & & & & & \\ & & & & & & & & & & b_0 & b_1 \\ b_0 & b_0 & b_1 & b_1 & & & & & & & & \\ & & & & b_0 & b_0 & b_1 & b_1 & & & & \\ & & & & \cdots & \cdots & \cdots & & & & & \\ b_0 & b_0 & b_0 & \cdots & b_0 & b_0 & \cdots & \cdots & b_1 & b_1 & b_0 & b_1 \end{bmatrix}$$

If the Haar wavelet or Daubechies [18, 19] of order 2 is substituted in the matrix $\mathbf{W}$, its size becomes $M \times M$ where b_0 and b_1 are the Haar wavelet coefficients.

As such, Equation (9) can be rewritten as

$$\mathbf{B}\boldsymbol{\phi} = \mathbf{Y} \tag{10}$$

where

$$\mathbf{B} = \mathbf{AW}$$

Each element in matrix $\mathbf{B}$ can be set to zero if it does not exceed what can be called sparsification threshold. Accordingly, $\mathbf{B}$ will be highly sparse and shows some symmetrical properties like what is found in matrix $\mathbf{W}$.

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The proper choice of the mother wavelet plays a crucial role in the signal preprocessing. Several families have proven to be useful in signal and matrix processing (Daubechies, Biorthogonal, Haar, Shanon, etc.). Each family has specific properties which make them suitable for certain applications. There are many orthonormal wavelets that give acceptable localizations both in time and frequency [15, 20]. Wavelet decomposition using Daubechies wavelet proves to be a success along a wide range of integral-equation solutions as presented in [7, 21–24].

4. NEURAL NETWORK MODEL

NNs are computing systems that work in an analogy of the nervous system of the brain, in which connections organize neurons into networks. NNs are computational structures that can be configured by examples, and can improve their performance by a dynamic adaptation process. The adaptation process of an NN is performed once, and only on a finite subset of all possible input instances, also called the training set, which consists of input–output vector pairs. The NN is trained to map the applied input vectors into corresponding output vectors.

Feed-forward NN is a network of nodes (neurons) organized into layers: an input layer, one or more hidden layers and an output layer of nodes. The input layer is made up of M nodes, where M is the dimension of the input vector. The task of the input layer is to pass the inputs of the network to the hidden layer. The hidden layer, in turn, performs a nonlinear mapping from the input space to a new space. A number of hidden nodes are set empirically to achieve the best network performance. The hidden nodes have a hyperbolic-tangent transfer function. In general, the nodes of the output layer have a linear transfer function.

Accurate mapping might require a large number of input vectors which will yield an NN of a complex structure. Therefore, further preprocessing of the input vectors applied to NN might significantly enhance its performance and reduce both complexity of the network and training time [15]. In this work, the employment of wavelet basis functions is adopted to allow a significant reduction of the dimension of the input vector without degrading the performance of the network.

5. NEURAL NETWORK DESIGN

The remarkable ability of NNs to synthesize complicated nonlinear relationships through learning from examples is exploited to obtain a solution for Equation (7). In this section, we will examine the selection of NN inputs as well as its architecture, design and training criteria.

5.1. Input set selection

The input to the network will be some representative elements of the sparsified moment matrix $\mathbf{B}$, and the incident wave (excitation) $\mathbf{Y}$, while the corresponding output is the scattered field $\boldsymbol{\phi}$ on the scattering surface. Training sets that characterize this relation are constructed.

Formally, the input vectors to the network are the unique patterns chosen from matrix $\mathbf{B}$. It turns out that the $\mathbf{B}$ matrix has similar cyclic and symmetrical structure like the matrix $\mathbf{W}$ in (9) and (10). These properties can be attributed to the symmetry of the moment matrix $\mathbf{A}$ and the periodic structure of matrix $\mathbf{W}$ (circulant-like structure). Accordingly, only a unique

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subset of the repeated elements are chosen to represent the part of matrix **B** which contains such patterns. If Daubechies family of order 4 is adopted and the moment matrix size (M) is 64, only 64 elements input vector need to be constructed. Four elements from the first three level of details in the matrix **B** constitute the first 12 elements of the vector. Additionally, 4 elements from the fourth level details and 1 element from the summary level are added to the vector. The 17th element is the incident wave on the scatterer. All elements are complex numbers while each element is represented by its magnitude. The following algorithm describes the selection process:

$l_w = wavelet\ length$; $nn = 1$; $k = 1$; $j = 1$;
while $j < N$ **do**
  For $i = 1$ *to* $2^\wedge(nn-1) * l_w$ **step** $2^\wedge(nn-1)$;
    $X(k) = \mathbf{abs}(B(i, j))$;
    $k = k + 1$;
  end for;
  $j = j + N/(2^\wedge(nn))$;
  $nn = nn + 1$
end while;

This algorithm reduces the dimension of the input vector from the order $O(M^2)$ to the order of $O(l_w)$ while M is the moment matrix order and l_w is the length of the wavelet filter.

The learning process of the network is performed on a training set. The training set is constructed by varying the wave number within the non-characteristic range and calculating the input vector and its corresponding output $\boldsymbol{\phi}$.

Before feeding the network, all input data should be normalized. Data normalization is carried out in order to avoid large changes in the magnitude of the input vectors that may differ by several orders. These variations could represent an undesired domination and should be removed though a normalization process. The zero mean and unit standard deviation normalization are adopted as indicated below:

$$x_i' = \frac{(x_i - \overline{x})}{\sigma} \tag{11}$$

where x_i' is the normalized form of the ith input vector element, $\overline{x}$ is the mean, and σ is the standard deviation of input vector elements. Accordingly, undesired large changes in the signal magnitude are avoided, and differences between input vectors are amplified. This allows fast convergence.

5.2. Network architecture and training

The utilized NN is of a feed-forward type with one hidden layer of hyperbolic-tangent transfer function as depicted in Figure 1.

Training set is composed of input–output vector pairs of size 17 and 64, respectively. Each input–output pair is called a pattern. The input vector is selected from the sparsified moment matrix according to the algorithm presented in Section 5.1. The associated output vector is the magnitude of the complex scattered field on the surface calculated from the available analytic solutions for simple shapes of the scatterer. Many patterns are calculated at different non-characteristic wave numbers. The training process is conducted using backpropagation learning algorithm [25].

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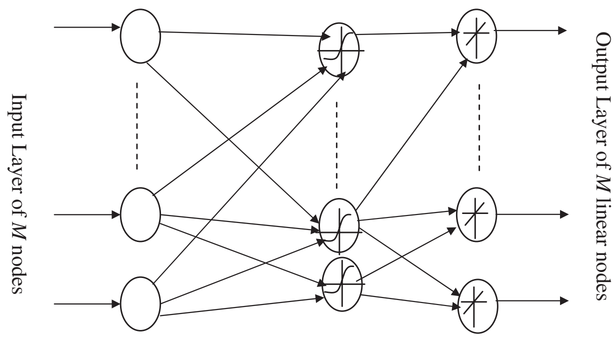

Figure 1. The architecture of the NN used in the mapping.

6. NUMERICAL RESULTS

The results are computed for the scattering of a plane wave from a hard sphere with different wave numbers. Due to the axisymmetric nature of the sphere, the one half circle is discretized into 64 points, expressed in integral equation (7). Moment matrix **A** is constructed as described in [8]. Operator **W** contains a Daubechies mother wavelet of order 4.

The training set consists of the inputs and outputs at different wave numbers in the range from $ka = 1 \rightarrow 10$ with a step of 0.45 (i.e. about 21 patterns). The network is trained with an accuracy of 10^{-6} and the maximum number of iterations is 2000. Convergence is reached after about 16 epochs as shown in Figure 2.

The scattering surface is taken as an acoustically hard sphere of a unit radius. The incoming unit plane wave travels towards the scatterer along the positive direction of the z-axis which is described as e^{-ikz} in the cylindrical co-ordinates. The training output results are calculated using the well-known analytical solution of an acoustic hard sphere which is given by

$$\varphi = \frac{-i}{(ka)^2} \sum_{n=0}^{\infty} (-i)^n (2n+1) \frac{P_n(\cos \vartheta)}{h_n^{(2)'}(ka)} \tag{12}$$

where φ is the total field on the surface of an acoustically hard sphere of radius a and ϑ is the co-latitude angle. The incidence angle is taken to be *zero* in this application. P_n is the Legendre polynomial of order n, and h_n is the spherical hankel function [26].

Tests are conducted at different wave numbers including some wave numbers at which non-uniqueness is encountered using traditional methods. The generalization ability of the trained NN is demonstrated in Figures 3 and 4. The inputs to the NN are calculated at different wave numbers, the corresponding output is denoted by $\boldsymbol{\phi}_{\text{nn}}$. These outputs are compared to the actual outputs $\boldsymbol{\phi}_{\text{ana}}$ which are obtained by Equation (12). The root-mean square error (RMSE) is taken as a performance measure which is defined as

$$\text{RMSE}_{\text{nn}} = \|\phi_{\text{nn}} - \phi_{\text{ana}}\| / \|\phi_{\text{ana}}\| \tag{13}$$

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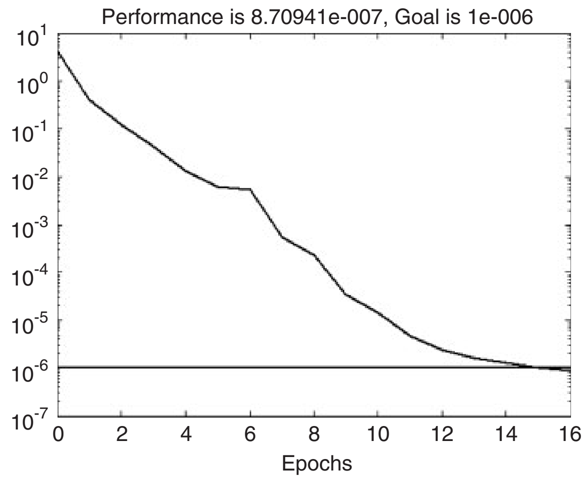

Figure 2. Training error *versus* number of iterations (epochs).

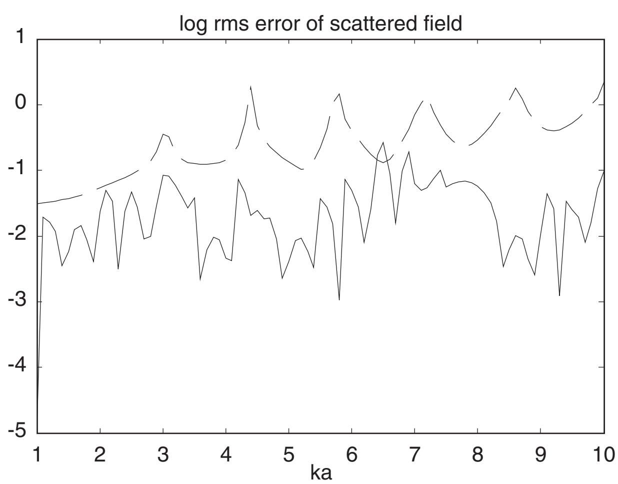

Figure 3. Log RMS error between the analytical solution with both NN-based solution and tradition numerical solution *versus* ka; (–) solid line represents the error with NN-based solution and (- -) dashed line represents the error with numerical solution.

and

$$\mathrm{RMSE}_{\mathrm{num}} = \|\phi_{\mathrm{num}} - \phi_{\mathrm{ana}}\| / \|\phi_{\mathrm{ana}}\| \tag{14}$$

where $\mathrm{RMSE}_{\mathrm{nn}}$ is the error incurred from NN outputs $\boldsymbol{\phi}_{\mathrm{nn}}$ while $\mathrm{RMSE}_{\mathrm{num}}$ is the error when the outputs of traditional numerical techniques $\boldsymbol{\phi}_{\mathrm{num}}$ is compared to the analytical solution $\boldsymbol{\phi}_{\mathrm{ana}}$. Figure 3 shows the variation of log(RMSE) for different wave numbers ranging from $ka = 1 \rightarrow 10$ with a step 0.35. The error is large at the characteristic wave numbers if the conventional methods

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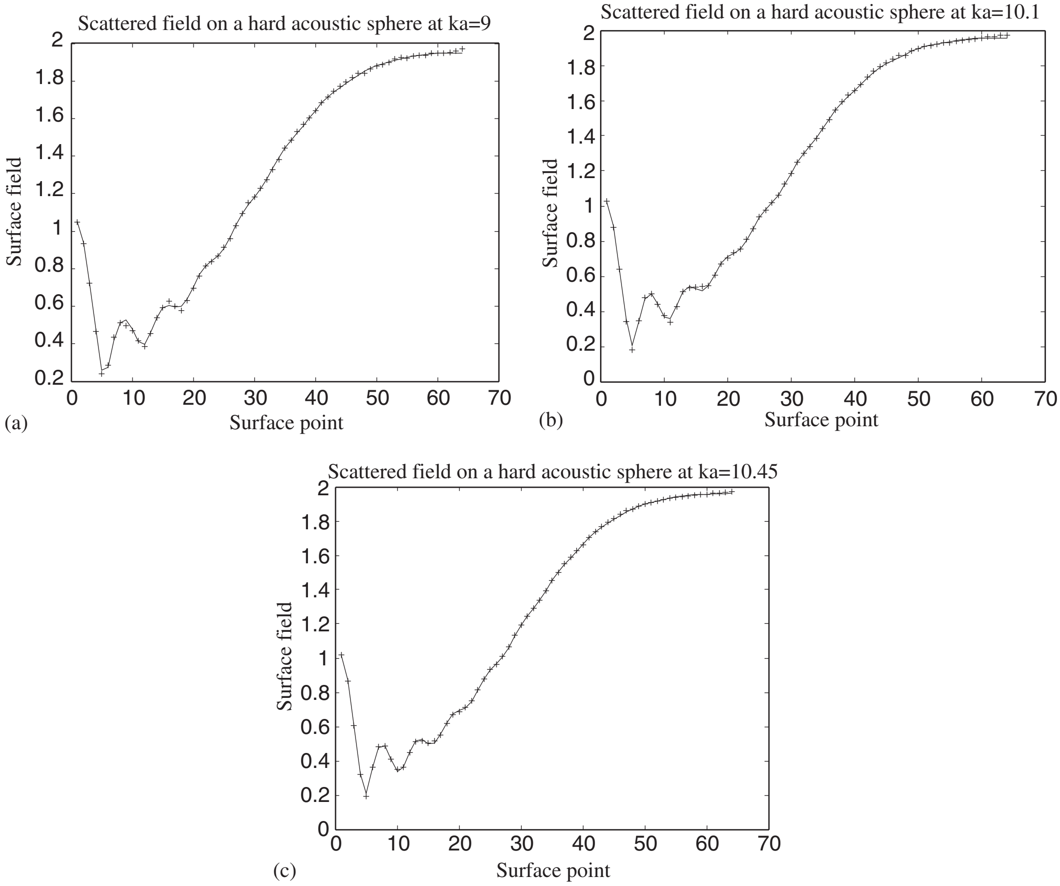

Figure 4. A comparison between the analytical solution and NN-based solution; (−) solid line represents the analytical solution and (+) the NN-based solution.

are used as it appears in Figure 3. These errors are suppressed using NN which is trained at non-characteristic wave numbers.

Comparisons between the analytical solution defined in Equation (12) and the corresponding NN-based solution for some high wave numbers are shown in Figure 4. It should be noted that two of these wave numbers are out of the training range. All results indicate excellent match between the NN-based solution and the exact one. This, clearly, reveals the generalization potential of the trained NN.

7. CONCLUSIONS

The moment method approach is a powerful tool for solving the integral equation of the acoustic scattering problems. However, the moment matrix suffers from non-uniqueness at the characteristic wave numbers of the corresponding acoustic scattering equation. Direct numerical methods cannot

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overcome the non-uniqueness problem. Moreover, the resulting matrix is always dense when the conventional expansion and testing functions are used. The wavelet expansion shows efficient sparsification of this matrix.

In this research, the potential of NNs is exploited in order to obtain a solution for the acoustic scattering problems through integral equation formulation. The moment matrix of the discretized integral equation is sparsified using wavelet decomposition. The sparsified matrix helps to reduce the neural network size and complexity and as a result fast learning is achieved. Appropriate patterns are chosen to represent the moment matrix. The NN is trained using the set of chosen patterns with wave numbers which are completely different from that which are used in testing. The trained NN is capable of obtaining a solution of the acoustic scattering problems at characteristic wave numbers and also other wave numbers with reduced computational load. Numerical results show excellent agreement between the neural network-based solution and the analytical solution for a spherical scatterer. Additionally, the trained NN is also tested at some wave numbers which are out of the training range.

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