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Modal analysis of free vibration of liquid in rigid container by the method of fundamental solutions

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ABSTRACT

Modal analysis of free vibration of liquid in a rigid container having arbitrary shape requires a numerical method. The method of fundamental solutions has an advantage over the finite element method and the boundary element method in that it does not require either volume mesh generation or surface mesh generation. This paper presents the formulation of the method of fundamental solutions for determining natural frequencies and mode shapes of free vibration of liquid in rigid container. Modal analyses are performed for cylindrical container, cylindrical quadrant container, cylindrical equilateral triangle container, hemispherical container, and cylindrical container with baffle. It is shown that natural frequencies of certain modes obtained by the method of fundamental solutions agree with analytical results and other numerical results.

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1. Introduction

Sloshing is caused by disturbance to a partially filled liquid container. The dynamic stability of a moving container may be seriously compromised if the frequency of disturbance is close to a natural frequency of liquid free-surface motion. Although there are an infinite number of natural frequencies, only a few lowest ones are likely to be excited by disturbance. Determination of natural frequencies by an analytical method is possible if liquid is assumed to be incompressible and inviscid. In that case, the equation of motion is reduced to the Laplace equation with velocity potential as the dependent variable. The kinematic free-surface condition gives an eigenvalue problem, from which natural frequencies and corresponding mode shapes are determined.

Modal analyses of liquid free-surface motion in rigid containers having simple shapes by analytical methods are described by Ibrahim [1]. If the shape of the container is arbitrary, a numerical method such as the finite element method may be used [2]. However, this method requires domain mesh generation even for a Laplace problem. Therefore, a boundary-type method like the boundary element, which requires only boundary mesh generation, is more efficient for this type of analysis [3–5]. An alternative boundary-type method that, unlike the boundary element method, does not require boundary mesh generation is the method of fundamental solutions. Since this method has been shown to be capable of providing accurate solutions to the eigenvalue problems that have Laplace and biharmonic operators [6,7], it should be considered as a competitive method for modal analysis of liquid free-surface motion in a container having an arbitrary shape.

In this paper, the method of fundamental solutions is used to find natural frequencies and mode shapes of free vibration of liquid in rigid containers. The following sections present the governing equation and boundary conditions for liquid motion, the formulation of method of fundamental solutions for setting up an eigenvalue problem, and numerical results for cylindrical, cylindrical quadrant, cylindrical equilateral triangle, spherical containers, and cylindrical container with annular ring baffle. Results from the method of fundamental solutions are then compared with analytical results for cylindrical, cylindrical quadrant, and spherical containers and with other numerical results for cylindrical container with annular ring baffle.

2. Governing equation and boundary conditions

Fig. 1 illustrates a rigid container filled with an incompressible, inviscid, and irratational liquid. The governing equation for the liquid motion is the Laplace equation of velocity potential (Φ):

$$\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial^2 \Phi}{\partial y^2}+\frac{\partial^2 \Phi}{\partial z^2}=0. \quad (1)$$

The boundary condition at the rigid boundary is

$$\frac{\partial \Phi}{\partial n}=0, \quad (2)$$

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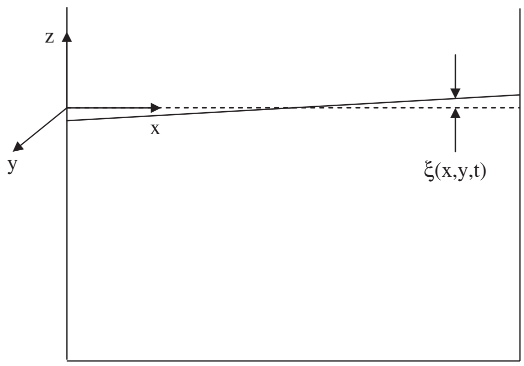

Fig. 1. Rigid container filled with liquid.

where n is the coordinate normal to the boundary. Assume that the undisturbed liquid level is at $z=0$. Disturbance will cause free surface elevation $\xi(x,y,t)$. For small-amplitude waves, the boundary conditions at the free surface for ξ and Φ are

$$\frac{\partial \xi}{\partial t} = \frac{\partial \Phi}{\partial z}, \tag{3}$$

$$\frac{\partial \Phi}{\partial t} = -g\xi. \tag{4}$$

Eliminating ξ from Eqs. (3) and (4) results in the kinematic boundary condition

$$\frac{\partial^2 \Phi}{\partial t^2} = -g\frac{\partial \Phi}{\partial z}. \tag{5}$$

A functional form of Φ that satisfies Eq. (5) is

$$\Phi(x,y,z,t) = \phi(x,y,z)e^{i\omega t} \tag{6}$$

provided that

$$\frac{\partial \phi}{\partial z} = \frac{\omega^2}{g}\phi. \tag{7}$$

Substituting Φ from Eq. (6) into Eqs. (1) and (2) yields, respectively, the Laplace equation of ϕ and the homogeneous Neumann boundary condition of ϕ with Eq. (7) being the kinematic boundary condition.

3. Method of fundamental solutions

Suppose the total number of boundary nodes (N) is divided into N_f nodes on the free surface and N_r nodes on the rigid boundary. Coordinates of free boundary nodes are (x_i,y_i,z_i) with $i=1, 2,\ldots,N_f$, whereas coordinates of rigid boundary nodes are $(x_{N_f+i},y_{N_f+i},z_{N_f+i})$ with $i=1, 2,\ldots,N_r$. The method of fundamental solutions approximates $\phi(x,y,z)$ as

$$\phi(x,y,z) = \sum_{j=1}^{N} a_j G(x,y,z,\bar{x}_j,\bar{y}_j,\bar{z}_j), \tag{8}$$

where the fundamental solution of the Laplace equation is

$$G(x,y,z,\bar{x}_j,\bar{y}_j,\bar{z}_j) = \frac{1}{\sqrt{(x-\bar{x}_j)^2+(y-\bar{y}_j)^2+(z-\bar{z}_j)^2}}, \tag{9}$$

and $(\bar{x}_j,\bar{y}_j,\bar{z}_j)$ are coordinates of source points located outside the domain. The number of source points is N. Each source point is situated at a suitable distance from the boundary in the normal direction.

$$\bar{x}_j = x_j + s\Delta\cos\alpha, \tag{10a}$$

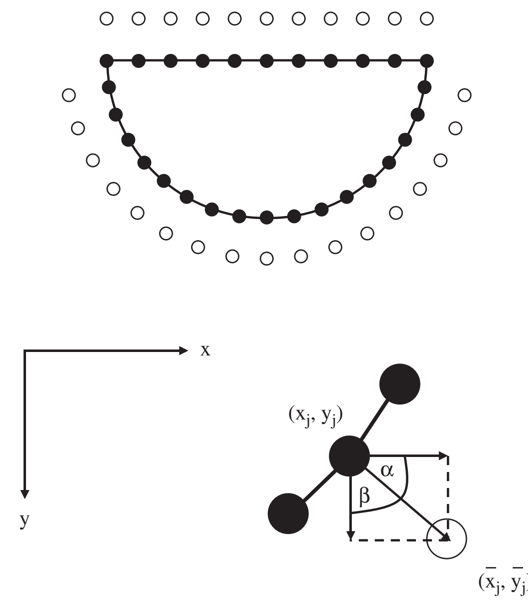

Fig. 2. Location of source point (white circle) relative to boundary point (black circle).

$$\bar{y}_j = y_j + s\Delta\cos\beta, \tag{10b}$$

$$\bar{z}_j = z_j + s\Delta\cos\gamma, \tag{10c}$$

where Δ is grid spacing, s is the source point parameter, and $(\cos\alpha, \cos\beta, \cos\gamma)$ is the vector of directional cosines. Fig. 2 illustrates the placement of a source point. Mitric and Rashed [8] suggested that s should be sufficiently large in order for the solution to be accurate. It is found in this study that s should be at least 1.5, and that solution accuracy is relatively insensitive to s if it is larger than this value.

The homogeneous Neumann boundary condition of ϕ gives the following matrix equation:

$$\mathbf{B}_f\vec{a}_f + \mathbf{B}_r\vec{a}_r = \vec{0}. \tag{11}$$

The dimensions of $\mathbf{B}_f$ and $\mathbf{B}_r$ are, respectively, $N_r \times N_f$ and $N_r \times N_r$. Their components are

$$(B_f)_{i,j} = \frac{\partial G}{\partial n}(x_{N_f+i}, y_{N_f+i}, z_{N_f+i}, \bar{x}_j, \bar{y}_j, \bar{z}_j), \tag{12}$$

$$(B_r)_{i,j} = \frac{\partial G}{\partial n}(x_{N_f+i}, y_{N_f+i}, z_{N_f+i}, \bar{x}_{N_f+j}, \bar{y}_{N_f+j}, \bar{z}_{N_f+j}). \tag{13}$$

Vectors $\vec{a}_f$ and $\vec{a}_r$ are defined as $\vec{a}_f = (a_1, a_2, \ldots, a_{N_f})$ and $\vec{a}_r = (a_{N_f+1}, a_{N_f+2}, \ldots, a_N)$. The kinematic boundary condition of ϕ gives the following matrix equation:

$$\mathbf{C}_f\vec{a}_f + \mathbf{C}_r\vec{a}_r = \frac{\omega^2}{g}(\mathbf{D}_f\vec{a}_f + \mathbf{D}_r\vec{a}_r). \tag{14}$$

The dimensions of $\mathbf{C}_f$ and $\mathbf{C}_r$ are, respectively, $N_f \times N_f$ and $N_f \times N_r$. Their components are

$$(C_f)_{i,j} = \frac{\partial G}{\partial z}(x_i, y_i, z_i, \bar{x}_j, \bar{y}_j, \bar{z}_j), \tag{15}$$

$$(C_r)_{i,j} = \frac{\partial G}{\partial z}(x_i, y_i, z_i, \bar{x}_{N_f+j}, \bar{y}_{N_f+j}, \bar{z}_{N_f+j}). \tag{16}$$

The dimensions of $\mathbf{D}_f$ and $\mathbf{D}_r$ are, respectively, $N_f \times N_f$ and $N_f \times N_r$. Their components are

$$(D_f)_{i,j} = G(x_i, y_i, z_i, \bar{x}_j, \bar{y}_j, \bar{z}_j), \tag{17}$$

$$(D_r)_{i,j} = G(x_i, y_i, z_i, \bar{x}_{N_f+j}, \bar{y}_{N_f+j}, \bar{z}_{N_f+j}). \tag{18}$$

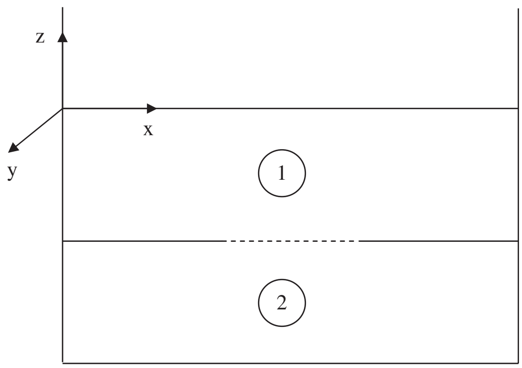

Fig. 3. Partitioning of container with baffle.

In order to formulate an eigenvalue problem, eliminate $\vec{a}_r$ from Eqs. (11) and (14), and rearrange the result.

$$(\mathbf{D}_f - \mathbf{D}_r\mathbf{B}_r^{-1}\mathbf{B}_f)^{-1}(\mathbf{C}_f - \mathbf{C}_r\mathbf{B}_r^{-1}\mathbf{B}_f)\vec{a}_f = \frac{\omega^2}{g}\vec{a}_f. \tag{19}$$

This equation can be solved for eigenvalue ω^2/g, and the corresponding eigenvector $\vec{a}_f$, from which $\vec{a}_r$ can be determined by using Eq. (11). The mode shape is determined by using Eq. (3) to find the expression of ξ.

$$\xi(x,y,t) = \frac{1}{\omega}\frac{\partial\phi}{\partial z}e^{(\mathrm{i}\omega t-\pi/2)}. \tag{20}$$

The sloshing motion in a container may be suppressed, and natural frequencies may be changed by installing baffle below the free surface. The method of fundamental solutions can be used to determine natural frequencies in this case. Consider a cylindrical tank with annular ring baffle as shown in Fig. 3. The domain is divided into two non-overlapping subdomains. Each subdomain has a distinct velocity potential function, which is governed by the Laplace equation. Boundary conditions for $\phi^{(1)}$ are

$$\frac{\partial\phi^{(1)}}{\partial z} = \frac{\omega^2}{g}\phi^{(1)} \text{ (free surface)}, \tag{21}$$

$$\frac{\partial\phi^{(1)}}{\partial n} = 0 \text{ (rigid boundary of subdomain 1)}. \tag{22}$$

Boundary condition for $\phi^{(2)}$ is

$$\frac{\partial\phi^{(2)}}{\partial n} = 0 \text{ (rigid boundary of subdomain 2)}. \tag{23}$$

In addition, ϕ and $\partial\phi/\partial z$ are required to be continuous at the interface between subdomains 1 and 2. Therefore,

$$\phi^{(1)} = \phi^{(2)} \text{ (interface)}, \tag{24}$$

$$\frac{\partial\phi^{(1)}}{\partial z} = \frac{\partial\phi^{(2)}}{\partial z} \text{ (interface)}. \tag{25}$$

Suppose the total number of boundary nodes (N_1) in domain 1 consists of N_f nodes on the free surface, N_{r1} nodes on the rigid boundary, and N_m nodes on the interface. Suppose that the number of boundary nodes (N_2) in domain 2 consists of N_{r2} nodes on the rigid boundary and N_m nodes on the interface. Let ($x_i^{(1)}$, $y_i^{(1)}$, $z_i^{(1)}$) with i = 1, 2,…,N_f denote coordinates of free boundary nodes in domain 1; $(x_{N_f+i}^{(1)}, y_{N_f+i}^{(1)}, z_{N_f+i}^{(1)})$ with i = 1, 2,…,N_{r1} denote coordinates of rigid boundary nodes in domain 1; and $(x_{N_f+N_{r1}+i}^{(1)}, y_{N_f+N_{r1}+i}^{(1)}, z_{N_f+N_{r1}+i}^{(1)})$ with i = 1, 2,…,N_m denote coordinates of interface nodes in domain 1. In addition, let ($x_i^{(2)}$, $y_i^{(2)}$, $z_i^{(2)}$) with i = 1, 2,…,N_{r2} denote coordinates of rigid boundary nodes in domain 2; and $(x_{N_{r2}+i}^{(2)}, y_{N_{r2}+i}^{(2)}, z_{N_{r2}+i}^{(2)})$ with i = 1, 2,…,N_m denote coordinates of interface nodes in domain 1. The expressions for $\phi^{(1)}$ and $\phi^{(2)}$ are analogous to Eq. (8)

$$\phi^{(1)}(x,y,z) = \sum_{j=1}^{N} a_j^{(1)} G(x,y,z,\bar{x}_j^{(1)},\bar{y}_j^{(1)},\bar{z}_j^{(1)}), \tag{26}$$

$$\phi^{(2)}(x,y,z) = \sum_{j=1}^{N} a_j^{(2)} G(x,y,z,\bar{x}_j^{(2)},\bar{y}_j^{(2)},\bar{z}_j^{(2)}). \tag{27}$$

Eq. (23) gives the following matrix equation:

$$\mathbf{B}^{(2)}\vec{a}^{(2)} = \vec{0}. \tag{28}$$

The dimension of $\mathbf{B}^{(2)}$ is $N_{r2} \times N_2$. Its components are

$$B_{i,j}^{(2)} = \frac{\partial G}{\partial n}(x_i^{(2)}, y_i^{(2)}, z_i^{(2)}, \bar{x}_j^{(2)}, \bar{y}_j^{(2)}, \bar{z}_j^{(2)}). \tag{29}$$

Eq. (24) gives the following matrix equation:

$$\mathbf{E}^{(2)}\vec{a}^{(2)} = \mathbf{E}^{(1)}\vec{a}^{(1)}. \tag{30}$$

The dimension of $\mathbf{E}^{(2)}$ is $N_m \times N_2$. Its components are

$$E_{i,j}^{(2)} = G(x_{N_{r2}+i}^{(2)}, y_{N_{r2}+i}^{(2)}, z_{N_{r2}+i}^{(2)}, \bar{x}_j^{(2)}, \bar{y}_j^{(2)}, \bar{z}_j^{(2)}). \tag{31}$$

The dimension of $\mathbf{E}^{(1)}$ is $N_m \times N_1$. Its components are

$$E_{i,j}^{(1)} = G(x_{N_f+N_{r1}+i}^{(1)}, y_{N_f+N_{r1}+i}^{(1)}, z_{N_f+N_{r1}+i}^{(1)}, \bar{x}_j^{(1)}, \bar{y}_j^{(1)}, \bar{z}_j^{(1)}). \tag{32}$$

Eqs. (28) and (30) can be solved for $\vec{a}^{(2)}$ in terms of $\vec{a}^{(1)}$.

$$\vec{a}^{(2)} = \begin{bmatrix} \mathbf{B}^{(2)} \\ \mathbf{E}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{E}^{(1)} \end{bmatrix} \vec{a}^{(1)}. \tag{33}$$

The next step is to formulate an eigenvalue problem from Eqs. (21), (22), (25), and (33). Eq. (25) gives the following matrix equation:

$$\mathbf{F}^{(1)}\vec{a}^{(1)} = \mathbf{F}^{(2)}\vec{a}^{(2)}. \tag{34}$$

The dimension of $\mathbf{F}^{(2)}$ is $N_m \times N_2$. Its components are

$$F_{i,j}^{(2)} = \frac{\partial G}{\partial z}(x_{N_{r2}+i}^{(2)}, y_{N_{r2}+i}^{(2)}, z_{N_{r2}+i}^{(2)}, \bar{x}_j^{(2)}, \bar{y}_j^{(2)}, \bar{z}_j^{(2)}). \tag{35}$$

The dimension of $\mathbf{F}^{(1)}$ is $N_m \times N_1$. Its components are

$$F_{i,j}^{(1)} = \frac{\partial G}{\partial z}(x_{N_f+N_{r1}+i}^{(1)}, y_{N_f+N_{r1}+i}^{(1)}, z_{N_f+N_{r1}+i}^{(1)}, \bar{x}_j^{(1)}, \bar{y}_j^{(1)}, \bar{z}_j^{(1)}). \tag{36}$$

Substitute $\vec{a}^{(2)}$ from Eq. (33) into Eq. (34). After being rearranged, the resulting equation becomes

$$\mathbf{H}\vec{a}^{(1)} = \vec{0}, \tag{37}$$

where

$$\mathbf{H} = \mathbf{F}^{(1)} - \mathbf{F}^{(2)} \begin{bmatrix} \mathbf{B}^{(2)} \\ \mathbf{E}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{E}^{(1)} \end{bmatrix}. \tag{38}$$

Eqs. (22) and (37) are then combined into

$$\mathbf{B}_f^{(1)}\vec{a}_f^{(1)} + \mathbf{B}_{rm}^{(1)}\vec{a}_{rm}^{(1)} = \vec{0}. \tag{39}$$

The dimensions of $\mathbf{B}_f^{(1)}$ and $\mathbf{B}_{rm}^{(1)}$ are, respectively, $(N_{r1}+N_m) \times N_f$ and $(N_{r1}+N_m) \times (N_{r1}+N_m)$. Their components are

$$(B_f^{(1)})_{i,j} = \frac{\partial G}{\partial n}(x_{N_f+i}^{(1)}, y_{N_f+i}^{(1)}, z_{N_f+i}^{(1)}, \bar{x}_j^{(1)}, \bar{y}_j^{(1)}, \bar{z}_j^{(1)}) \; (i \leqslant N_{r1}), \tag{40}$$

$$(B_{rm}^{(1)})_{i,j} = \frac{\partial G}{\partial n}(x_{N_f+i}^{(1)}, y_{N_f+i}^{(1)}, z_{N_f+i}^{(1)}, \bar{x}_{N_f+j}^{(1)}, \bar{y}_{N_f+j}^{(1)}, \bar{z}_{N_f+j}^{(1)}) \; (i \leqslant N_{r1}), \tag{41}$$

$$(B_f^{(1)})_{N_{r1}+i,j} = H_{i,j}, \tag{42}$$

$$(B_{rm}^{(1)})_{N_{r1}+i,j} = H_{i,N_{r1}+j}. \tag{43}$$

Vectors $\vec{a}_f^{(1)}$ and $\vec{a}_{rm}^{(1)}$ are defined as $\vec{a}_f^{(1)} = (a_1^{(1)}, a_2^{(1)}, \ldots, a_{N_f}^{(1)})$ and $\vec{a}_{rm}^{(1)} = (a_{N_f+1}^{(1)}, a_{N_f+2}^{(1)}, \ldots, a_{N_1}^{(1)})$. Eq. (21) gives the following matrix equation:

$$\mathbf{C}_f^{(1)}\vec{a}_f^{(1)} + \mathbf{C}_{rm}^{(1)}\vec{a}_{rm}^{(1)} = \frac{\omega^2}{g}(\mathbf{D}_f^{(1)}\vec{a}_f^{(1)} + \mathbf{D}_{rn}^{(1)}\vec{a}_{rm}^{(1)}). \tag{44}$$

The dimensions of $\mathbf{C}_f{}^{(1)}$ and $\mathbf{C}_{rm}{}^{(1)}$ are, respectively, $N_f \times N_f$ and $N_f \times (N_{r1}+N_m)$. Their components are

$$(C_f^{(1)})_{i,j} = \frac{\partial G}{\partial z}(x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, \bar{x}_j^{(1)}, \bar{y}_j^{(1)}, \bar{z}_j^{(1)}), \tag{45}$$

$$(C_{rm}^{(1)})_{i,j} = \frac{\partial G}{\partial z}(x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, \bar{x}_{N_f+i}^{(1)}, \bar{y}_{N_f+i}^{(1)}, \bar{z}_{N_f+i}^{(1)}). \tag{46}$$

The dimensions of $\mathbf{D}_f{}^{(1)}$ and $\boldsymbol{D}_{rm}^{(1)}$ are, respectively, $N_f \times N_f$ and $N_f \times (N_{r1}+N_m)$. Their components are

$$(D_f^{(1)})_{i,j} = G(x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, \bar{x}_j^{(1)}, \bar{y}_j^{(1)}, \bar{z}_j^{(1)}), \tag{47}$$

$$(D_{rm}^{(1)})_{i,j} = G(x_i^{(1)}, y_i^{(1)}, z_i^{(1)}, \bar{x}_{N_f+i}^{(1)}, \bar{y}_{N_f+i}^{(1)}, \bar{z}_{N_f+i}^{(1)}). \tag{48}$$

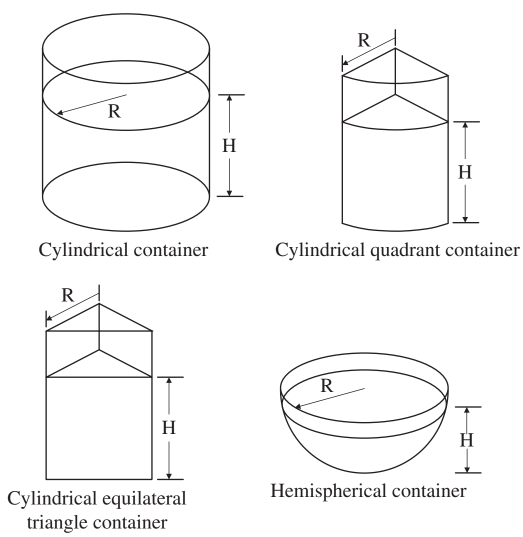

Fig. 4. Four partially filled liquid containers for which modal analyses by the method of fundamental solutions are performed.

After eliminating $\vec{a}_{rm}^{(1)}$ from Eqs. (39) and (44), and rearranging, the resulting equation is

$$[\mathbf{D}_f^{(1)} - \mathbf{D}_{rm}^{(1)}(\mathbf{B}_{rm}^{(1)})^{-1}\mathbf{B}_f^{(1)}]^{-1}[\mathbf{C}_f^{(1)} - \mathbf{C}_{rm}^{(1)}(\mathbf{B}_{rm}^{(1)})^{-1}\mathbf{B}_f^{(1)}]\vec{a}_f^{(1)} = \frac{\omega^2}{g}\vec{a}_f^{(1)}, \tag{49}$$

which is the required eigenvalue problem.

It should be noted that the coefficient matrix in Eqs. (19) or (49) is a dense matrix, and may have a large condition number if the number of free boundary nodes is large. Fortunately, it is found that the method of fundamental solutions is capable of giving a sufficiently accurate solution using only a few hundred free boundary nodes. The system of equations from Eqs. (19) or (49) can, therefore, be solved by a direct method.

4. Results and discussion

Four types of containers considered are illustrated in Fig. 4. The parameters for all 4 containers are $R = H = 1$. The first five non-degenerate natural frequencies of the four containers, as well as corresponding mode shapes, are determined by the method of fundamental solutions. The numbers of nodes used for cylindrical container, cylindrical quadrant container, cylindrical equilateral triangle container, and hemispherical container are, respectively, 1340, 1463, 1602, and 1267. It is found that fewer nodes (or larger grid spacing) may result in missing natural frequencies. Therefore, it is recommended that different solutions using different values of grid spacing should be compared in each case to ensure that there is no missing natural frequency. Mode shapes are shown in Fig. 5. Computed natural frequencies are shown in Table 1, and are compared with analytical results [1]. It can be seen that computed natural frequencies are very close to analytical values.

Next, the method of fundamental solutions is used to find natural frequencies of free-surface vibration in cylindrical container with annular ring baffle, as illustrated in Fig. 6. The parameters in this case are $R = H = 1$; $h = 0.1$, and 0.3; and $r = 0.2$, 0.4, 0.6, and 0.8. Natural frequencies (ω) are converted to sloshing frequency parameter (defined as $\omega(R/g)^{1/2}$). Since no analytical results are available, results from the method of fundamental solutions are compared with results obtained by Biswal et al. [2] and Gedikli and Erguven [3]. It can be seen from Table 2 that the present results agree with results by other researchers.

It is interesting to note that solutions of plate vibration, membrane vibration, and acoustic problems by the boundary element method [9] and the method of fundamental solutions

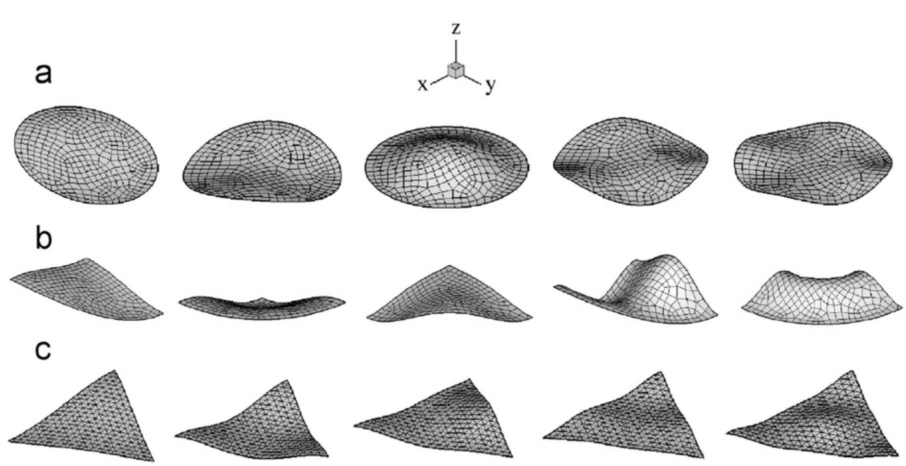

Fig. 5. Shapes of the first five non-degenerate modes of free vibration of liquid in (a) cylindrical container and hemispherical, (b) cylindrical quadrant, and (c) cylindrical equilateral triangle container.

Table 1
Natural frequencies of the first five non-degenerate modes of free-surface vibration in four types of partially-filled liquid container

Mode	Container			
	Cylindrical[a]	Cylindrical quadrant[a]	Cylindrical equilateral triangle[b]	Spherical[c]
1	4.144 (4.108)	5.465 (5.462)	6.414	3.908 (3.912)
2	5.461 (5.462)	6.132 (6.128)	8.440	5.251 (5.259)
3	6.128 (6.128)	7.225 (7.222)	9.066	6.056 (6.062)
4	6.418 (6.418)	8.112 (8.111)	10.424	6.248
5	7.222 (7.222)	8.297 (8.296)	11.092	7.077 (7.194)

Numbers without parenthesis indicate results from the method of fundamental solutions. Numbers in parentheses indicate analytical results.
[a] Analytical results in parentheses.
[b] Only results from the method of fundamental solutions shown.
[c] Analytical results in parentheses, except result for mode 4.

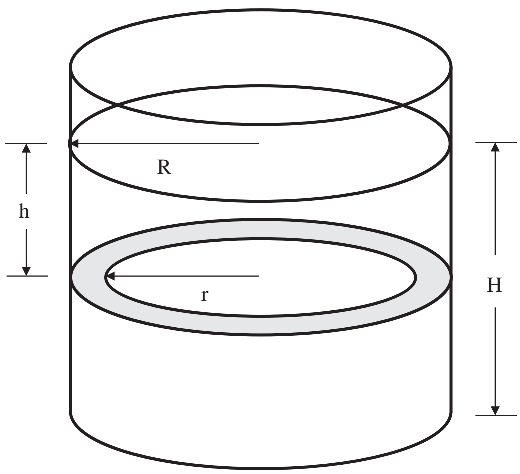

Fig. 6. Partially filled cylindrical containers with annular ring baffle for which modal analysis by the method of fundamental solutions is performed.

Table 2
Sloshing frequency parameters of the first mode of free-surface vibration in partially filled liquid cylindrical container with baffle as illustrated in Fig. 5

h/H	r/R			
	0.2	0.4	0.6	0.8
0.1	0.596	0.672	0.852	1.140
	(0.60)[a]	(0.69)	(0.89)	(1.19)
	(0.61)[b]	(0.70)	(0.88)	(1.15)
0.3	0.971	1.021	1.133	1.235
	(0.97)	(1.03)	(1.15)	(1.28)
	(0.99)	(1.06)	(1.15)	(1.25)

Numbers without parenthesis indicate results from the method of fundamental solutions. Numbers in parentheses indicate numerical results from Biswal et al. [2] and Gedikli and Erguven [3].
[a] Result from Biswal et al. [2] in the second row of each cell.
[b] Result from Gedikli and Erguven [3] in the third row of each cell.

[10,11] may produce spurious eigensolutions. Governing equations of such problems are the Helmholtz equation or the bi-Helmholtz equation. On the other hand, the governing equation of the free vibration of liquid in rigid container is the Laplace equation. No spurious eigensolutions have been observed in this study and previous studies of free vibration of liquid in rigid container by the finite element method [2] and the boundary element method [4].

5. Conclusions

Modal analysis of free vibration of liquid in a rigid container requires the solution of an eigenvalue problem derived from the Laplace problem having Neumann and kinematic boundary conditions. The method of fundamental solutions is suitable for this type of problem due to its meshless nature and its accuracy. This paper shows that the method of fundamental solutions can be effectively used to find natural frequencies and mode shapes of free vibration of liquid in cylindrical container, cylindrical quadrant container, cylindrical equilateral triangle container, hemispherical container, and cylindrical container with baffle. It is perceivable that this method is capable of computing natural frequencies for container of arbitrary shape. Results from this paper, therefore, confirm the superiority of the method of fundamental solutions in solving linear problems.

Acknowledgment

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