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An improved form of the hypersingular boundary integral equation for exterior acoustic problems

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ABSTRACT

An improved form of the hypersingular boundary integral equation (BIE) for acoustic problems is developed in this paper. One popular method for overcoming non-unique problems that occur at characteristic frequencies is the well-known Burton and Miller (1971) method [7], which consists of a linear combination of the Helmholtz equation and its normal derivative equation. The crucial part in implementing this formulation is dealing with the hypersingular integrals. This paper proposes an improved reformulation of the Burton–Miller method and is used to regularize the hypersingular integrals using a new singularity subtraction technique and properties from the associated Laplace equations. It contains only weakly singular integrals and is directly valid for acoustic problems with arbitrary boundary conditions. This work is expected to lead to considerable progress in subsequent developments of the fast multipole boundary element method (FMBEM) for acoustic problems. Numerical examples of both radiation and scattering problems clearly demonstrate that the improved BIE can provide efficient, accurate, and reliable results for 3-D acoustics.

1. Introduction

The boundary integral equation (BIE) has been used for a long time in solving radiation and scattering problems in many scientific fields, such as potential theory, elastostatics, and acoustics. However, there is a defect; the BIE fails to yield unique solutions for exterior acoustic problems at characteristic frequencies of the associated interior Dirichlet problem. It should be noted that the non-uniqueness is purely a mathematical drawback, and those fictitious characteristic frequencies have no physical meaning. To remove the non-uniqueness difficulty from exterior acoustic problems is one of the major tasks in the research of BIEs. Two major methods, appropriate for practical applications, have been applied to overcome this difficulty.

Schenck [1] proposed a combined Helmholtz integral equation formulation (CHIEF) that successfully removed the non-uniqueness by adding some additional Helmholtz integral relations in the interior domain. This resulted in an over-determined system of equations, which was then solved using a least-squares technique. This method is perhaps widely used in engineering applications. However, selecting the optimum number and suitable positions of interior points may become difficult as the wave number increases. Some modified CHIEF methods were proposed by several other researchers [2–6]; however, they still do not solve the non-uniqueness problem completely, especially, in the high-frequency range.

Another well-known formulation to overcome the non-uniqueness problem at characteristic frequencies was the method by proposed Burton and Miller [7]. This method consisted of a linear combination of the Helmholtz equation and its normal derivative equation. It was proved in Ref. [7] that the linear combination of these two equations would yield a unique solution for all frequencies if the coupling constant of the derivative equation was appropriately chosen. However, the major difficulty in this formulation is that the normal derivative of the Helmholtz integral equation involves a hypersingular integral. Burton and Miller used a double surface integral method throughout the integral equation to reduce the order of hypersingularity. Although such a technique results in numerically tractable kernels, it is computationally expensive to evaluate a double surface integral. Other regularization techniques such as the work by Meyer et al. [8] and Terai [9] are valid for planar elements only. The concept of Hadamard finite-part integral [10] has also been used to evaluate the hypersingular integral. Based on Hadamard finite-part interpretation, Chien et al. [11] employed some identities in the integral equation related to an interior Laplace problem to reduce the order of kernel singularity. Liu and Rizzo [12] and Liu and Chen [13] presented a weakly singular form of the hypersingular integral equations by subtracting a two-term

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Taylor series from the density function. Certain integral identities of static Green's function were used to assess the added-back terms.

More recently, Yang [14,15] expressed the unknown functions as a truncated Fourier–Legendre series. Some weakly singular integrals and the hypersingular integral were analytically evaluated using some properties of Legendre functions. Harris and Chen [16] proposed a high-order Galerkin method in terms of the singularity subtraction approach to reduce the hypersingular operator to a weakly singular one. In Ref. [16], the numerical procedures included two particular iterative solvers: the conjugate gradient normal method (CGN) and the generalized minimal residual method (GMRES). Gray et al. [17] employed multiple polar coordinate transformations and analytic integration to evaluate directly Galerkin hypersingular integrals without recourse to the Hadamard's finite part. Yan et al. [18] considered the normal derivative of solid angles on the surface. Seydou et al. [19] proposed a method based on Green's theorem for calculating singular and hypersingular integrals.

In the present paper, an improved form of the hypersingular and strongly singular BIEs for exterior acoustic problems based on the Burton–Miller formulation is presented. This improved form is used to regularize the hypersingular integrals using a new singularity subtraction technique and properties from the associated Laplace equations. It contains only weakly singular integrals and is directly valid for acoustic problems. Discretization of this weakly singular form of the hypersingular and strongly singular BIEs is straightforward. No special numerical integration quadratures are required to compute all the integrals and hence the quadrature for conventional BIE can be applied directly.

The improved form in this paper is mainly solving 3-D exterior acoustic radiation and scattering problems with Neumann boundary conditions on the surface of the structure, which is quite common in practice. We note that the improved form of BIE presented here is intended for use in solving practical problems, such as acoustic radiation and scattering from spherical and cylindrical bodies. However, for Dirichlet or Robin boundary conditions, acoustic radiation and scattering problems are still effectively solved using the improved form.

This paper is organized as follows. The basic BIE formulation is reviewed in Section 2. The weakly singular regularization method is presented in Section 3. Numerical examples demonstrating the effectiveness and accuracy of the improved form for acoustic radiation and scattering from spherical and cylindrical bodies are presented in Section 4. Section 5 concludes the paper with further discussions.

2. BIE formulation of exterior acoustic problems

The problem under consideration in this paper is the solution of the Helmholtz equation in the domain E exterior to a closed bounded surface S. To be precise, we consider propagation of time-harmonic acoustic waves in a homogeneous isotropic acoustic medium described by the Helmholtz equation

$$\nabla^2\phi(x)+k^2\phi(x)=0,\quad x\in E. \tag{1}$$

For the exterior acoustic problems it is necessary to introduce a condition at infinity. This ensures the physical requirement that all scattered and radiated waves are outgoing. This is termed the Sommerfeld radiation condition:

$$\lim_{r\to\infty} r\left(\frac{\partial\phi}{\partial r}-ik\phi\right)=0, \tag{2}$$

where ϕ is the total acoustic wave (velocity potential or acoustic pressure), $k=\omega/c$ the wave number, ω the angular frequency, c the wave speed in the acoustic medium E, r the distance from a fixed origin to a general field point, and $i=\sqrt{-1}$.

The integral representation of the solution to the exterior Helmholtz equation is

$$C(x)\phi(x)=\int_S\left[-\frac{\partial G_k(x,y)}{\partial n_y}\phi(y)+G_k(x,y)\frac{\partial\phi(y)}{\partial n_y}\right]dS_y+\phi_{in}(x), \tag{3}$$

where x is the collocation point, y the field point, S_y denotes a subset of surface S, $\phi_{in}(x)$ is a prescribed incident wave (for scattering problems only), the coefficient $C(x)$=1, ½, or 0 when the collocation point x is in the exterior region E (acoustic medium), on the boundary S (if it is smooth), or in the interior region B (a body or scatterer), respectively, and n_y is the outward normal at y. The free space Green's function G_k for 3-D problems is given by

$$G_k(x,y)=\frac{e^{ikr}}{4\pi r}\quad with\ \ r=|x-y|. \tag{4}$$

Eq. (3) with $x\in S$ is the commonly used form of the conventional BIE for acoustic wave problems. This is a singular form of the conventional BIE, which can be converted into a weakly singular form readily using an integral expression for the coefficient $C(x)$. If the normal derivative is given on boundary S, the acoustic pressure can be computed at any point in E using Eq. (3). However, Eq. (3) does not possess a unique solution at certain characteristic frequencies with the corresponding interior problems. A number of different methods [1,7,13,16] have been proposed for overcoming this non-uniqueness problem. One of the most effective and robust method is the Burton–Miller method [7] and for this reason, this is the method we shall use here.

Taking the derivative of Eq. (3) with respect to the normal at the collocation point $x(n_x)$ and letting x approach S give the following boundary integral equation:

$$C(x)\frac{\partial\phi(x)}{\partial n_x}=\int_S\left[-\frac{\partial^2 G_k(x,y)}{\partial n_y\,\partial n_x}\phi(y)+\frac{\partial G_k(x,y)}{\partial n_x}\frac{\partial\phi(y)}{\partial n_y}\right]dS_y+\frac{\partial\phi_{in}(x)}{\partial n_x},\quad x\in S, \tag{5}$$

where $C(x)$=1/2 if S is smooth around x. In the right-hand side of Eq. (5), the two integrands become hypersingular [12,20] (integrand has a $1/r^3$ singularity) and strongly singular (integrand has a $1/r^2$ singularity). A well-known formulation to overcome the non-uniqueness problem is the method proposed by Burton and Miller [7]. This approach consists of a linear combination of the Helmholtz integral Eq. (3) and its normal derivative Eq. (5):

$$\begin{aligned}\alpha\int_S\frac{\partial^2 G_k(x,y)}{\partial n_y\,\partial n_x}\phi(y)dS_y-\alpha\int_S\frac{\partial G_k(x,y)}{\partial n_x}\frac{\partial\phi(y)}{\partial n_y}dS_y\\=\int_S\left[-\frac{\partial G_k(x,y)}{\partial n_y}\phi(y)+G_k(x,y)\frac{\partial\phi(y)}{\partial n_y}\right]dS_y\\-\frac{1}{2}\phi(x)-\frac{1}{2}\alpha\frac{\partial\phi(x)}{\partial n_x}+\phi_{in}(x)+\alpha\frac{\partial\phi_{in}(x)}{\partial n_x},\quad x\in S,\end{aligned} \tag{6}$$

where α is a non-zero coupling constant. It can be shown that the imaginary part of α is non-zero. Discussions on the choices of α, found to be rather non-restrictive, are given in Refs. [8,11,12]. Regarding Eq. (6) it was proved in Ref. [7] that the linear combination of the two equations would yield a unique solution for all frequencies. However, the major difficulty in this formulation is that the normal derivative of the Helmholtz integral equation involves a hypersingular integral. The question of how to effectively handle this hypersingular integral is still under investigation by many researchers [21–23]. Various regularization techniques to reduce the order of singularity can be found in the literature mentioned in the introduction (Section 1). Here, this paper proposes an improved reformulation of the Burton–Miller method and is used to regularize the hypersingular and strongly singular integrals by using a new singularity subtraction technique and properties from the associated Laplace equations in Section 3.

3. Improved weakly singular form of the hypersingular BIE

In the present study, an improved form is used to regularize the hypersingular and strongly singular integrals using a new singularity subtraction technique and properties from the associated Laplace equations. We need the following three integral identities for the static Green's function $G_0 = 1/(4\pi r)$ [24,25]:

$$\int_S \frac{\partial G_0(x,y)}{\partial n_y} dS_y = 0, \quad \text{the first identity,} \tag{7}$$

$$\int_S \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y = 0, \quad \text{the second identity,} \tag{8}$$

$$\int_S \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} (y - x) dS_y = \int_S \frac{\partial G_0(x,y)}{\partial n_x} n_y \, dS_y, \quad \text{the third identity,} \tag{9}$$

where the collocation point x is in domain E.

We now introduce Eq. (6) with a view to remove the hypersingular term. In order to obtain a weakly singular form, we need to apply a transformation to the hypersingular term

$$\int_S \frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} \phi(y) dS_y. \tag{10}$$

A special treatment, proposed by Chien et al. [11], is needed to take the differential operator into the integral. Therefore, the hypersingular term can be transformed into an integrable form by subtracting the second derivative of the Green's function as follows:

$$\int_S \phi(y) \frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} dS_y = \int_S \phi(y) \left[\frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} - \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} \right] dS_y + \int_S \phi(y) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y. \tag{11}$$

As point y approaches point x, kernel of the first integral in the right-hand side of Eq. (11) can be expanded by Taylor series:

$$\frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} - \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} = \frac{k^2}{8\pi r} + \frac{ik^3}{12\pi} + o(r). \tag{12}$$

Since the above kernel is weakly singular, clearly, the first integral in the right-hand side of Eq. (11) is weakly singular.

Now, we consider the second integral in the right-hand side of Eq. (11). It contains singularity $1/r^3$, which corresponds to an associated interior problem of the Laplace equation $\nabla^2\phi=0$, with the boundary condition $\phi_0=\phi$ on surface S. In order to calculate this integral, it is better to convert this integral into a different form, using potential theory and a singularity subtraction technique. However, the Laplace equation is just a special case of the Helmholtz equations, wave number $k=0$. Therefore, the simplest way to accomplish this transformation is to replace the function $G_k(x,y)$ by function $G_0(x,y)$ in Eq. (11).

The second integral in the right-hand side of Eq. (11) can be transformed into an integrable form, and then the differential operator can be applied to the new integrand, since the new integrand is weakly singular. In order to do this, we make use of the following identity [23,25]:

$$\int_S b(y-x) \frac{\partial^2 G_k(x,y)}{\partial n_y \partial n_x} dS_y = \int_S b n_y \frac{\partial G_k(x,y)}{\partial n_x} dS_y \int_S - k^2 b(y-x) \frac{\partial G_k(x,y)}{\partial n_x} dE_y - \frac{b n_x}{2}, \tag{13}$$

where b is a vector in $\mathbb{R}^3$ (a function of space). The second integral on the right-hand side is a volume integral. Volume E_y denotes the interior region enclosed by surface S_y in $\mathbb{R}^3$.

It is noticed that the second integral in the right-hand side of Eq. (11) can be written identically by subtracting and adding back terms, as follows:

$$\int_S \phi(y) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y = \int_S [\phi(y) - \phi(x) - \nabla\phi(x)(y-x)] \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y + \phi(x) \int_S \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y + \int_S \nabla\phi(x)(y-x) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y. \tag{14}$$

Substituting Eq. (8) (the second identity) into Eq. (14) yields

$$\int_S \phi(y) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y = \int_S [\phi(y) - \phi(x) - \nabla\phi(x)(y-x)] \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y + \int_S \nabla\phi(x) \cdot (y-x) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y, \tag{15}$$

where $\nabla\phi(x)$ denotes the domain gradient of ϕ at the point x. The first term in the right-hand side of Eq. (15) contains what is essentially the difference between $\phi(y)$ and the terms up to and including the first derivative terms of its Taylor's series about $y=x$. This is a general rule. The higher the order of singularity, the more the terms to be retained. Hence the remainder of terms in square brackets in Eq. (15) is of order r^2 and so the first integral behaves as r^{-1}, which means it is weakly singular. Clearly, the first term in the right-hand side of Eq. (15) can be evaluated using an appropriate quadrature rule.

The singularity in the second integral in the right-hand side of Eq. (15) remains to be addressed. In order to overcome this singularity, set $b=\nabla\phi(x)$ (remembering that $x \in S$ must be a point at which the normal is well defined) and use Eq. (13) for $k=0$ to obtain

$$\int_S \nabla\phi(x)(y-x) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y = \int_S \nabla\phi(x) n_y \frac{\partial G_0(x,y)}{\partial n_x} dS_y \int_S - k^2 \nabla\phi(x)(y-x) \frac{\partial G_0(x,y)}{\partial n_x} dE_y - \frac{1}{2} \nabla\phi(x) n_x. \tag{16}$$

Clearly, the Laplace equation is just a special case of the Helmholtz equations with wave number $k=0$. Therefore the volume integral disappears along with the second integral in the right-hand side of Eq. (16) and we obtain

$$\int_S \nabla\phi(x)(y-x) \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y} dS_y = \int_S \nabla\phi(x) n_y \frac{\partial G_0(x,y)}{\partial n_x} dS_y - \frac{1}{2} \nabla\phi(x) n_x. \tag{17}$$

Substituting Eq. (17) into Eq. (15) yields that

$$\int_S \phi(y)\frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}dS_y = \int_S [\phi(y)-\phi(x)-\nabla\phi(x)(y-x)]\frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}dS_y + \int_S \nabla\phi(x)n_y\frac{\partial G_0(x,y)}{\partial n_x}dS_y - \frac{1}{2}\nabla\phi(x)n_x, \tag{18}$$

where every integral in the right-hand side is now weakly singular.

Substituting the result in Eq. (18) into the second integral of Eq. (11), we obtain

$$\int_S \phi(y)\frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y}dS_y = \int_S \phi(y)\left[\frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} - \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}\right]dS_y + \int_S [\phi(y)-\phi(x)-\nabla\phi(x)(y-x)]\frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}dS_y + \int_S \nabla\phi(x)n_y\frac{\partial G_0(x,y)}{\partial n_x}dS_y - \frac{1}{2}\nabla\phi(x)n_x, \tag{19}$$

where all integrals in the right-hand side are weakly singular. Hence, we have reformulated the hypersingular integral into an improved form involving boundary integrals that are only weakly singular.

We have reformulated the hypersingular integral of Burton–Miller formulation (Eq. (6)) using a weakly singular form. Now we consider the strongly singular integral in Eq. (6), which can be regularized using the identity (Eq. (7)) shown previously to yield

$$\int_S \frac{\partial G_k(x,y)}{\partial n_x}\frac{\partial\phi(y)}{\partial n_y}dS_y = \int_S \left[\frac{\partial G_k(x,y)}{\partial n_x} + \frac{\partial G_0(x,y)}{\partial n_y}\right]\frac{\partial\phi(y)}{\partial n_y}dS_y - \int_S \frac{\partial G_0(x,y)}{\partial n_y}\left[\frac{\partial\phi(y)}{\partial n_y} - \frac{\partial\phi(x)}{\partial n_y}\right]dS_y, \tag{20}$$

which represents a weakly singular kernel as y approaches x when S is smooth near x.

Substituting Eqs. (19) and (20) into (6) yields

$$\begin{aligned}
&\alpha\int_S \frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y}\phi(y)dS_y - \alpha\int_S \frac{\partial G_k(x,y)}{\partial n_x}\frac{\partial\phi(y)}{\partial n_y}dS_y \\
&= \alpha\Bigg[\int_S \phi(y)\left[\frac{\partial^2 G_k(x,y)}{\partial n_x \partial n_y} - \frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}\right]dS_y \\
&\quad + \int_S [\phi(y)-\phi(x)-\nabla\phi(x)\cdot(y-x)]\frac{\partial^2 G_0(x,y)}{\partial n_x \partial n_y}dS_y \\
&\quad + \int_S \nabla\phi(x)\cdot n_y\frac{\partial G_0(x,y)}{\partial n_x}dS_y - \frac{1}{2}\nabla\phi(x)n_x \\
&\quad - \int_S \left[\frac{\partial G_k(x,y)}{\partial n_x} + \frac{\partial G_0(x,y)}{\partial n_y}\right]\frac{\partial\phi(y)}{\partial n_y}dS_y \\
&\quad + \int_S \frac{\partial G_0(x,y)}{\partial n_y}\left[\frac{\partial\phi(y)}{\partial n_y} - \frac{\partial\phi(x)}{\partial n_y}\right]dS_y\Bigg] \\
&= \int_S \left[-\frac{\partial G_k(x,y)}{\partial n_y}\phi(y) + G_k(x,y)\frac{\partial\phi(y)}{\partial n_y}\right]dS_y \\
&\quad - \frac{1}{2}\phi(x) - \frac{1}{2}\alpha\frac{\partial\phi(x)}{\partial n_x} + \phi_{in}(x) + \alpha\frac{\partial\phi_{in}(x)}{\partial n_x}, \quad x\in S.
\end{aligned} \tag{21}$$

Eq. (21) is the improved weakly singular form of the hypersingular and strongly singular BIEs for acoustic wave problems, which has not only an unique solution (as in Burton–Miller method) but also no hypersingular and strongly singular integrals (unlike Burton–Miller method). Therefore, subsequent solution of Eq. (21) can be directly solved by the standard quadrature formula.

Compared with the method in Ref. [23], in the present method, discretization is much easier because the domain gradient $\nabla\phi(x)$ can be evaluated readily using shape functions on an element. The method in Ref. [25] faces strongly singular integrals and is suited only for solving acoustic problems with Neumann boundary conditions on the surface of structures. The present method faces no hypersingular and strongly singular integrals and is valid for acoustic problems with arbitrary boundary conditions.

4. Numerical examples

The improved form of the hypersingular BIE has been implemented in a code using Fortran 90. Numerical studies on radiation and scattering from spherical and cylindrical bodies with smooth surfaces are conducted to verify the improved BIE with linear quadrilateral elements. The coupling constant is taken to be $\alpha = i/k$ for all the problems. It should be noted that this adheres to the aforementioned requirement that $\mathrm{Im}(\alpha) \neq 0$. Discussions on choices of α that have been found to be rather non-restrictive are given in Refs. [8,11,12].

The first numerical example is the radiation and scattering problems of a spherical body of radius a in an infinite acoustic domain (Fig. 1). The whole sphere is considered for modeling the problem with 96 linear quadrilateral elements.

For the radiation problem, the exact analytical solution of the acoustic pressure ϕ at a distance r from the center of a pulsating sphere, pulsating with uniform radial velocity υ_0 on the surface S and $\partial\phi/\partial n = ikz_0\upsilon_0$ is given by [11]

$$\frac{\phi(r)}{z_0\upsilon_0} = \frac{a}{r}\frac{ika}{1+ika}e^{-ik(r-a)}, \tag{22}$$

where $z_0 = \rho c$ is the characteristic impedance and k the wave number. As shown in Fig. 2, nondimensionalized surface acoustic pressure $|\phi/(z_0\upsilon_0)|$ is plotted versus nondimensionalized wave numbers ka=0–8 with 219 frequency steps. With these small frequency increments, fictitious frequencies for the conventional BIE can be identified clearly at $ka=\pi$ and 2π, near which the conventional BIE results deviate substantially from the analytical solution (a clear indication of the non-uniqueness of the conventional BIE solution). However, the improved BIE proposed by this paper provides very good agreement with the analytical solution throughout the range of frequencies. The improved BIE results can converge to the analytical solution with a finer mesh (results are not shown on the plot).

For the scattering problem, the rigid sphere where $\partial\phi/\partial n = 0$ on the surface S is impinged upon by an incident unit plane wave $\phi_{in} = e^{ikx}$ along the x-axis. The analytical solution for scattered pressure $\phi_s(r,\theta)$ at a distance r from the center of the sphere and

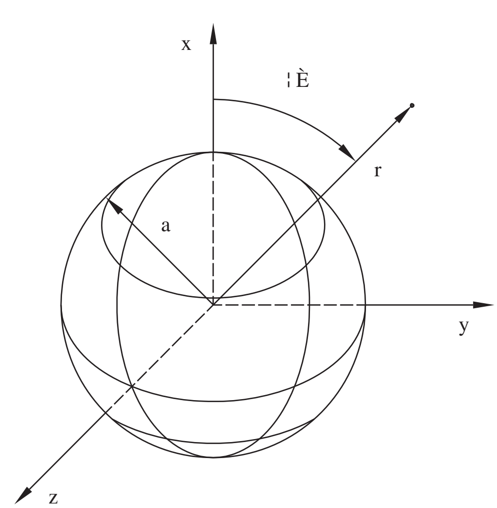

Fig. 1. A sphere body with radius a.

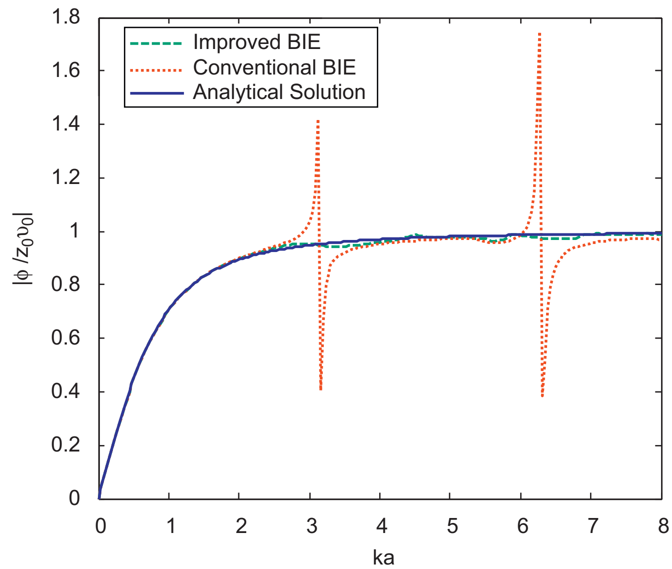

Fig. 2. Dimensionless pressure on the surface of a pulsating sphere.

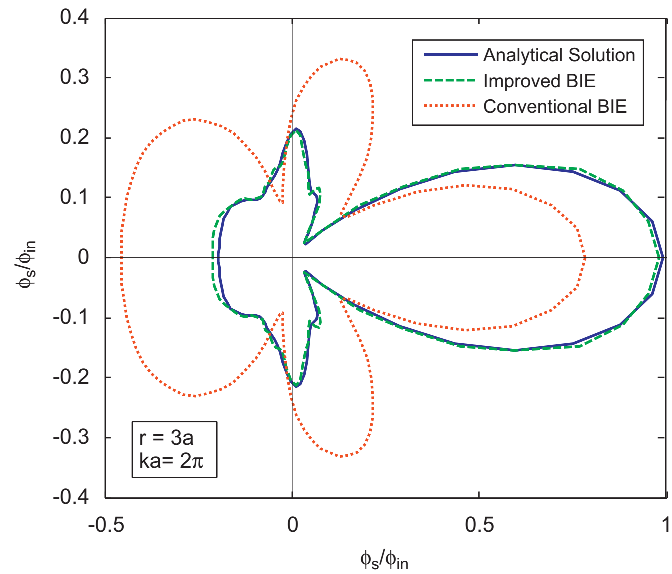

Fig. 4. Angular dependence of ϕ_S/ϕ_{in} for a rigid sphere when $ka=2\pi$ and $r=3a$.

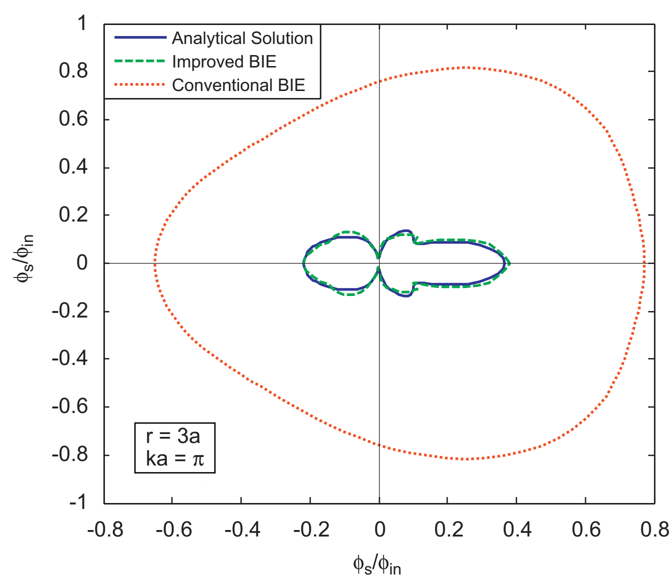

Fig. 3. Angular dependence of ϕ_S/ϕ_{in} for a rigid sphere when $ka=\pi$ and $r=3a$.

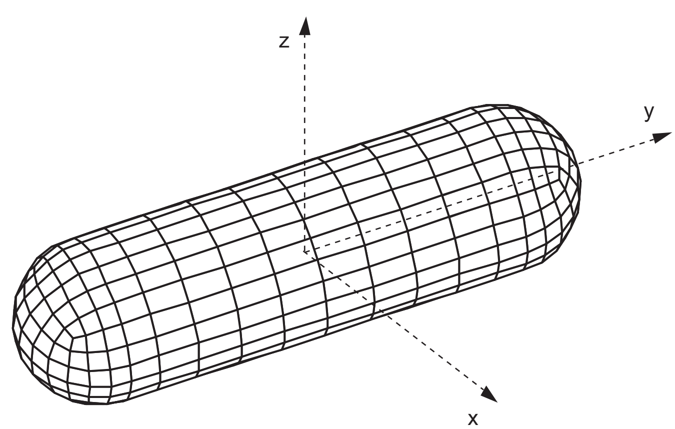

Fig. 5. A capsule-like cylindrical body with radius = 1.0 m and total length = 7.0 m.

angle θ from the x-axis is given by [26]

$$\phi_S(r,\theta)=\sum_{n=1}^{\infty}-\frac{i^n(2n+1)j_n'(ka)}{h_n'(ka)}P_n(\cos\theta)h_n(kr), \tag{23}$$

where P_n denotes the Legendre function of the first kind, h_n is the spherical Hankel function of the first kind, and j_n the spherical Bessel function of the first kind. As shown in Figs. 3 and 4, the variation of ϕ_S/ϕ_{in} at a distance $r=3a$ is plotted versus the polar angle θ (Fig. 1) for wave numbers, and compared with the analytical solution and the conventional BIE. Wave numbers $ka=\pi$ and 2π are the fictitious frequencies of both the conventional and the normal derivative BIEs, so conventional BIE cannot be applied successfully. However, Figs. 3 and 4 clearly demonstrate that the results using the improved BIE proposed by this paper are very stable and have excellent agreement with the analytical solution.

The second numerical example is the radiation and scattering problem of a capsule-like cylindrical body with radius = 1.0 m and total length = 7.0 m in an infinite acoustic domain. This is the same test problem used in Liu and Chen [13]. Since no analytical solutions are readily available for this problem, a comparison with the CHIEF method [1,2] verifies the accuracy and efficiency of the proposed method. In the present work, the whole cylinder is considered for modeling the problem with 456 linear quadrilateral elements (Fig. 5).

For the radiation problem (a pulsating capsule), a uniform normal velocity of unit magnitude is applied on the surface of the cylinder. As shown in Fig. 6, the magnitude of acoustic pressure at the point (0, 10, 0) in the main axis direction (y-axis) is plotted versus frequencies 0–250 Hz (with 251 frequency steps). The fictitious frequencies for conventional BIE can be identified clearly at 134, 153, 185, and 224 Hz. It can be seen that results using the conventional BIE deteriorate near the four fictitious frequencies. The CHIEF method [1,2], which can overcome this fictitious frequency difficulty, uses the Helmholtz integral representation at additional points inside the body (interior points) and solves an over-determined system of equations. It is well known that the success of the CHIEF method to overcome the fictitious frequency difficulty is largely determined by successful selections of the interior points, which are case dependent and often difficult for complicated structures or in high-frequency range. In this case,

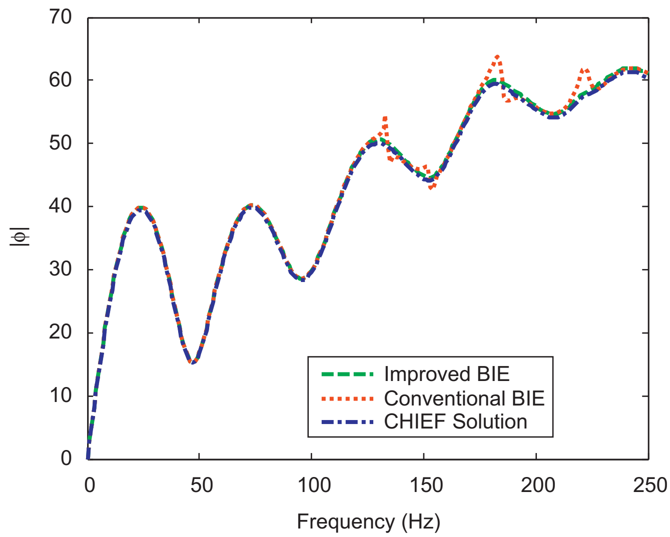

Fig. 6. Radiated wave from a cylinder in the main axis direction.

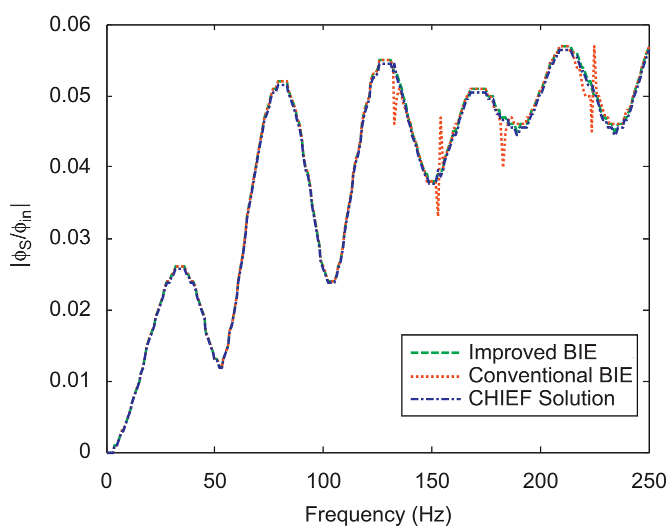

Fig. 7. Scattering from a cylinder in the main axis direction.

three interior points are placed along the main axis of the cylinder for the CHIEF solution. Fig. 6 shows that the improved BIE proposed by this paper provides stable and smooth results, and very good agreement with the CHIEF solution throughout the range of frequencies is obtained.

For the scattering problem, the rigid cylinder where $\partial\phi/\partial n = 0$ on the surface S is impinged upon by an incident unit plane wave $\phi_{in} = e^{ikx}$ along the x-axis. As shown in Fig. 7, the magnitude of scattered waves $|\phi_S/\phi_{in}|$ at the point (0,10,0) in the main axis direction, is plotted versus frequencies 0–250 Hz (with 251 frequency steps). The conventional BIE cannot be applied successfully near the four fictitious frequencies (134, 153, 185, and 224 Hz). However, results using the improved BIE proposed by this paper and the CHIEF solution stay closely along a smooth curve as expected.

5. Conclusion

Numerical results have demonstrated that the improved BIE proposed by this paper to regularize the hypersingular and strongly singular integrals into weakly singular form in the Helmholtz integral equation is very efficient and accurate. Discretization of this weakly singular form is quite straightforward. All the computations have been accomplished with the standard quadrature procedure without any special numerical integration schemes. The new method greatly improves computational efficiency and has tractable integral kernels. Another improvement in the numerical examples presented in this paper is the use of linear quadrilateral elements in place of the usual piecewise-constant elements to handle curved boundaries.

Here surface of the structure is constrained to be smooth enough. Further investigations will extend the new technique to problems with arbitrary shape structures. The improved form is evaluated analytically. Analytical integration has the benefit of accuracy and efficiency and is well suited for integration with the fast multipole boundary element method (FMBEM). This work is expected to lead to considerable progress in subsequent developments of the FMBEM for acoustic problems. The improved BIE is also suited for the high-frequency acoustic problems.

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References

[1] Schenck HA. Improved integral formulation for acoustic radiation problems. J Acoust Soc Am 1968;44:41–58.
[2] Seybert AF, Rengarajan TK. The use of CHIEF to obtain solutions for acoustic radiation using boundary integral equations. J Acoust Soc Am 1987 1299–306.
[3] Segalman DJ, Lobitz DW. A method to overcome computational difficulties in the exterior acoustic problems. J Acoust Soc Am 1992;91:1855–61.
[4] Wu TW, Seybert AF. A weighted residual formulation for the CHIEF method in acoustics. J Acoust Soc Am 1991;90:1608–14.
[5] Benthien W, Schenck HA. Nonexistence and nonuniqueness problems associated with integral equation methods in acoustics. Comput Struct 1997;65:295–305.
[6] Chen JT, Liu LW, Hong HK. Spurious and true eigensolutions of Helmholtz BIEs and BEMs for a multiply connected problem. Proc R Soc London A 2003;459:1891–924.
[7] Burton AJ, Miller GF. The application of integral equation methods to the numerical solution of some exterior boundary value problems. Proc R Soc London A 1971;323:201–10.
[8] Meyer WL, Bell WA, Stallybrass MP, Zinn BT. Prediction of the sound field radiated from axisymmetric surfaces. J Acoust Soc Am 1979;65:631–8.
[9] Terai T. On calculation of sound field around three-dimensional objects by integral equation methods. J Sound Vib 1980;69:71–100.
[10] Hadamard J. Lectures on Cauchy's problem in linear partial differential equations. New Haven: Yale University Press; 1923.
[11] Chien CC, Rajiyah H, Atluri SN. An effective method for solving the hypersingular integral equations in 3D acoustics. J Acoust Soc Am 1990;88:918–37.
[12] Liu YJ, Rizzo FJ. A weakly singular form of the hypersingular boundary integral equation applied to 3D acoustic wave problems. Comput Method Appl Mech Eng 1992;96:271–87.
[13] Liu YJ, Chen SH. A new form of the hypersingular boundary integral equation for 3-D acoustics and its implementation with C^0 boundary elements. Comput Method Appl Mech Eng 1999;173:375–86.
[14] Yang YA. A numerical method for scattering from acoustically soft and hard thin bodies in two dimensions. J Sound Vib 2002;250:773–93.
[15] Yang YA. Evaluation of 2D Green's boundary formula and its normal derivative using Legendre polynomials with an application to acoustic scattering problems. Int J Numer Methods Eng 2002;5:905–27.
[16] Harris PJ, Chen K. On efficient preconditioners for iterative solution of a Galerkin boundary element equations for the three dimensional exterior Helmholtz problem. J Comput Appl Math 2003;156:303–18.
[17] Gray LJ, Glaeser JM, Kaplan T. Direct evaluation of hypersingular Galerkin surface integrals. SIAM J Sci Comput 2004;25:1534–56.
[18] Yan ZY, Cui FS, Hung KC. Investigation on the normal derivative equation of Helmholtz integral equation in acoustics. Comput Model Eng Sci 2005;7: 97–106.

[19] Seydou F, Duraiswami R, Seppanen T, Gumerov NA. Computation of singular and hypersingular integrals by Green identity and application to boundary value problems. Eng Anal Boundary Elem 2009;33:1124–31.
[20] Hwang WS. Hypersingular boundary integral equations for exterior acoustic problems. J Acoust Soc Am 1997;101:3336–42.
[21] Yang SA. Evaluation of the Helmholtz boundary integral equation and its normal and tangential derivatives in two dimensions. J Sound Vib 2007;301:864–77.
[22] Engleder S, Steinbach O. Stabilized boundary element methods for exterior Helmholtz problems. Numer Math 2008;110:145–60.
[23] Chen K, Cheng J, Harris PL. A new study of the Burton and Miller method for the solution of a 3D Helmholtz problem. IMA J Appl Math 2009 163–77.
[24] Rudolphi TJ. The use of simple solutions in the regularization of hypersingular boundary integral equations. Math Comput Model 1991;15:269–78.
[25] Liu YJ, Rudolphi TJ. Some identities for fundamental solutions and their applications to weakly singular boundary element formulations. Eng Anal Boundary Elem 1991;8:301–11.
[26] Hickling R, Wang NM. Scattering of sound by a rigid movable sphere. J Acoust Soc Am 1966;39:276–9.