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Response of suspended beams due to moving loads and vertical seismic ground excitations

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Abstract

In this paper, the vibration of a suspension bridge due to moving loads of equidistant, identical forces and shaken by vertical support motions caused by earthquake is studied. The suspension bridge is modelled as a single-span suspended beam. To conduct the beam vibration with time-dependent boundary conditions, the total response of the suspended beam is decomposed into two parts: the *quasistatic* component and the *dynamic* part based on the decomposition method. Since the quasi-static component of the suspended beam under the *static* action of *multiple* support motions has been obtained analytically, the remaining dynamic part can be solved using Galerkin's method. The numerical results indicate that the contribution of higher modes on the maximum acceleration of the suspended beams to moving loads will become significant as the propagation effect and multiple support motions of seismic waves in the subsoil of bridge supports has been taken into account. (c) 2007 Elsevier Ltd. All rights reserved.

Keywords: Suspension bridges; Moving loads; Support motions due to earthquakes

1. Introduction

Structural engineers often encounter a dynamic problem of multiple support motions when dealing with the analysis of long-span structures shaken by earthquake excitations [1-5,12]. For instance, the earthquake-induced response of a suspension bridge is a typical multipoint support vibration problem due to the propagation effect of seismic waves at construction site. Along with the rapid development of modern transportation networks, suspension bridges are often employed to span wide rivers or deep valleys in the infrastructure of a country. In recent years, numerous researchers have studied the dynamic behaviour of suspension bridges caused by moving loads [1,6,9, 10,16,17,20]. An important conclusion in these works revealed that the cable tensions of *short-span* suspension bridges induced by moving loads would be amplified significantly.

In addition, the Honshu-Shikoku Bridge project in Japan, which includes a series of long-span suspension bridges such as the Kita Bisan-Seto Bridge with a main span of 990 m and the

* Corresponding author. *E-mail address:* Fryba@ITAM.CAS.CZ (L. Frýba). Minami Bisan-Seto Bridge with a main span of 1100 m, enables the bridges to carry high-speed trains in addition to vehicular loads [27]. This represents a technology promotion in modern bridge construction. Recently, Diana et al. [28,29] adopted a train-track interaction model to study the railway runnability of long-span suspension bridges. Obviously, the riding comfort of passengers and the manipulation of a train running over a suspension bridge are of importance in studying the dynamic response of a vehicle–bridge system.

However, comparatively few studies have been conducted on the train-induced vibration for suspension bridges shaken by earthquake support excitations. Using an analytical approach, Fryba [7], Yang et al. [18], and Xia et al. [25] presented a resonant condition for the train-induced response of simply supported bridges. Such a condition provides a useful criterion for predicting the resonant speeds of a high speed train travelling on railway bridges. Concerning the stability problem of a train moving over a bridge shaken by earthquakes, Yang et al.'s book [19] pointed out that the presence of vertical ground excitations would affect drastically the stability of a moving train, especially, near the resonant excitations prescribed above. Xia et al. [26] revealed that for a train travelling over a

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Fig. 1. Suspended beam under moving loads and vertical support motions.

continuous seven-span viaduct shaken by earthquakes, lack of considering seismic travelling wave effect might lead the train manipulation to an unsafe conclusion.

The resonant vertical vibration of bridges has appeared recently that alarmed the civil engineers and initiated the papers [7,8]. This motivation suppressed the other important excitations and preferred the acceleration calculations. We are aware of the fact that the horizontal seismic forces are more important than the vertical one. Thus, the present study should be grasped as an introduction to a spatial problem of the interaction vehicle–bridge. The substantial simplifications in the paper emphasize the effect of vertical axle forces of vehicles and show the phenomenon of resonance for this particular case.

The conclusions of the paper might be served as a design basis for the selection of span length of a suspension bridge located at construction site in seismic regions. The method was also applied to the calculation of the world's largest designed suspension bridge, Messina Bridge of span 3.3 km, see [24].

2. Formulation of the problem

Due to the *near-fault* effect, the presence of vertical ground excitations plays an important role on the vertical vibration of bridge structures. For this reason, only the vertical support motions acting on suspension bridges are concerned in this study. To investigate the train-induced vibration of suspension bridges shaken by vertical support motions, as shown in Fig. 1, the suspension bridge is modelled as a single-span suspended beam with two hinged ends, and appreciable simplifications for the simulation of suspension bridge and train loads are outlined as follows:

- (1) The bridge is modelled as a linear elastic Bernoulli–Euler beam.
- (2) The *linearized* deflection theory of suspension bridges [6, 15] is adopted to formulate the equation of vertical motion for a single-span suspension bridge.
- (3) The bridge pylons are assumed to be undeformable during vibration so that the cable and the suspended beam provide the time-dependent boundary conditions as the earthquake shakes the beam.
- (4) Assuming the force travelling along the suspended beam, the load has a good isolation device so that it can be modelled as a sequence of equidistant and identical moving forces.



Fig. 2. Cable subjected to a uniformly distributed load w.

2.1. Equation of motion

When a parabolic cable in Fig. 2 is under the action of a uniform dead load w, the horizontal component H in the tensile cable is [11,15]:

$$H = \frac{w}{y''},\tag{1}$$

where y = the sag function of the cable,

$$y(x) = 4y_0[x/L - (x/L)^2],$$
(2)

and y_0 = cable sag at mid-span, L = span length. Based on the *linearized deflection theory* of suspension bridges [6,15], the vertical equation of motion for a single-span beam suspended by a parabolic cable is given by, [10]:

$$m\ddot{u} + c\dot{u} + EIu'''' - Hu'' - \Delta Hy'' = p(x, t),$$
(3)

where a superscript prime denotes partial derivative with respect to the coordinate x, the dot over a letter the partial derivative with respect to time t, m = mass of the beam and cable per unit length along x-axis, c = damping coefficient, u(x, t) = total displacement of the beam, EI = flexural rigidity of the beam, and p(x, t) = load function of moving loads passing the beam. The increase of horizontal component ΔH in the cable under the action of live loads and vertical support movements (u(0, t), u(L, t)) can be written as, [11]:

$$\Delta H = \frac{E_c A_c}{L_c} \int_0^L y' u' dx = \frac{E_c A_c}{L_c} \left[y' u |_0^L - \int_0^L y'' u dx \right]$$
$$= \frac{E_c A_c}{L_c} \left[-\frac{4y_0}{L} \left(u(0, t) + u(L, t) \right) + \frac{8y_0}{L^2} \int_0^L u dx \right], (4)$$

where $E_c A_c$ = axial rigidity of the cable, and the effective length L_c of the cable curve is given by, [11]:

$$L_{c} = \int_{0}^{L} \left(\frac{\mathrm{d}s}{\mathrm{d}x}\right)^{3} \mathrm{d}x = \int_{0}^{L} \left(\sqrt{1 + {y'}^{2}}\right)^{3} \mathrm{d}x.$$
 (5)

Substituting Eqs. (1), (2) and (4) into Eq. (3) yields the following equation for a suspended beam under the action of external loads and vertical support motions

$$m\ddot{u} + c\dot{u} + EIu'''' - Hu'' + A \int_0^L u dx$$

= $p(x, t) + \frac{AL}{2} [u(L, t) + u(0, t)],$ (6)

where

A

$$\mathbf{A} = \left(\frac{8y_0}{L^2}\right)^2 \frac{E_c A_c}{L_c}.\tag{7}$$

As shown in Fig. 1, a row of moving forces with identical weights P and equal intervals d is crossing a single-span suspended beam at constant speed v. The load function p(x, t) describing the action of train movements on the beam is expressed as, [7,8,21,22]:

$$p(x,t) = P \sum_{k=1}^{N} \delta \left[x - v(t - t_g - t_k) \right] \\ \times \left[H(t - t_g - t_k) - H(t - t_g - t_k - L/v) \right], \quad (8)$$

in which δ = Dirac's delta function, H(t) = Heaviside unit step function, k = 1, 2, 3, ..., N — the number of moving load on the beam, t_g = time lag for the moving loads entering the suspended beam after earthquake support motion has shaken the beam, and $t_k = (k - 1)d/v$ = arriving time of the k-th load at the beam.

The boundary conditions for the suspended beam with two hinged ends shaken by vertical support movements are:

$$u(0, t) = a(t),$$
 (9a)
(L_1) = b(t) (9b)

$$u(L, t) = b(t),$$
 (9b)

EIu''(0,t) = EIu''(L,t) = 0, (9c)

where a(t) and b(t) represent the vertical displacements at the two bridge supports, as depicted in Fig. 1.

The initial conditions are supposed to be zero when the first moving force enters the bridge:

$$u(x, 0) = \dot{u}(x, 0) = 0.$$

An observation of Eqs. Eq. (6) and (9a)–(9c) indicates that we face a beam-vibration problem with time-dependent boundary conditions. To obtain the total response of the suspended beam, a *quasistatic* decomposition method [2–4, 12] will be employed to solve this problem in the following sections.

2.2. Quasistatic decomposition method

For the time-dependent boundary value problem in beam vibration [12], the total deflection u(x, t) of the beam can be decomposed into two parts: the quasistatic displacement component U(x, t) and the dynamic displacement component $u_d(x, t)$, [2–4]:

$$u(x,t) = U(x,t) + u_d(x,t).$$
 (10)

Here, the *quasistatic* part U(x, t) represents the beam displacement induced by the static effect of support movements, and the remaining dynamic part $u_d(x, t)$ by the dynamic effects of the beam vibration. Substituting Eq. (10) into Eq. (6) and removing the terms with quasistatic components to the right hand side yields

$$m\ddot{u}_{d} + c\dot{u}_{d} + EIu_{d}''' - Hu_{d}'' + A \int_{0}^{L} u_{d} dx$$

= $p(x, t) + \frac{AL}{2} [u(L, t) + u(0, t)] - (m\ddot{U} + c\dot{U}) - f_{quasi},$
(11)

with the quasistatic force as

$$f_{\text{quasi}} = EI \frac{\partial^4 U(x,t)}{\partial x^4} - H \frac{\partial^2 U(x,t)}{\partial x^2} + A \int_0^L U(x,t) dx.$$
(12)

Since the quasistatic displacement U(x, t) of the suspended beam is only excited by the static action of support motions [5], the summation for the quasistatic force of f_{quasi} and the support excitations of $\frac{AL}{2}[u(L, t) + u(0, t)]$ in Eq. (11) will vanish, i.e.

$$\frac{AL}{2} \left[u(L,t) + u(0,t) \right] - \left[EI \frac{\partial^4 U(x,t)}{\partial x^4} - H \frac{\partial^2 U(x,t)}{\partial x^2} + A \int_0^L U(x,t) dx \right] = 0,$$
(13)

and the corresponding boundary conditions of the quasistatic displacement U(x, t) are, [12]:

$$U(0, t) = a(t),$$
 $U(L, t) = b(t),$ (14a)

$$EIU''(0,t) = EIU''(L,t) = 0.$$
 (14b)

Therefore, Eq. (11) is reduced to

$$m\ddot{u}_{d} + c\dot{u}_{d} + EIu_{d}^{''''} - Hu_{d}^{''} + A\int_{0}^{L} u_{d} dx$$

= $p(x, t) - (m\ddot{U} + c\dot{U}).$ (15)

Then, substituting Eqs. (10) and (14) into Eqs. (9a)–(9c) yields the following homogeneous boundary conditions for the dynamic displacement component $u_d(x, t)$ as:

$$u_d(0,t) = u_d(L,t) = 0,$$
 (16a)

$$EIu_d''(0,t) = EIu_d''(L,t) = 0.$$
 (16b)

Observing Eqs. (15) and (16) indicates that the differential equation in terms of the dynamic deflection component $u_d(x, t)$ with homogeneous boundary conditions can be transformed into a set of coupled generalized equations by Galerkin's method, whenever the quasistatic displacement U(x, t) has been obtained analytically.

3. Solutions

In this section, the analytical solution of quasistatic deflection U(x, t) for a suspended beam *statically* excited by vertical support movements will first be presented. Then, one can deal with the remaining dynamic part $u_d(x, t)$ in Eqs. (15) and (16) by Galerkin's method.

3.1. Solution of quasistatic displacement

To solve the partial integro-differential equation (13), one can transform this equation into the following nonhomogenous partial differential equation:

$$\frac{\partial^4 U(x,t)}{\partial x^4} - \lambda^2 \frac{\partial^2 U(x,t)}{\partial x^2} = -\frac{A}{EI} \int_0^L U(x,t) dx + \frac{AL}{2EI} [b(t) + a(t)], \quad (17)$$

with $\lambda = \sqrt{T/EI}$. Then, solving the differential equation in term of U(x, t) with respect to x leads to the following general solution:

$$U(x,t) = c_0(t) + c_1(t)\frac{x}{L} + d_0(t)\cosh\lambda x + d_1(t)\sinh\lambda x + \frac{Ax^2}{2H} \left[\int_0^L U dx - \frac{b(t) + a(t)}{2} \right].$$
 (18)

Integrating Eq. (18) from 0 to L, one can express the integration $\int_0^L U(x, t) dx$ in terms of the undetermined coefficients: $c_0(t)$, $c_1(t)$, $d_0(t)$, and $d_1(t)$. Then, substituting Eq. (18) into the boundary conditions in Eq. (14) yields the following solution

$$U(x,t) = \left[a(t) + (b(t) - a(t))\frac{x}{L}\right].$$
(19)

The quasistatic displacement shown in Eq. (19) represents the rigid body displacements due to the *quasistatic* effect of *multiple* support movements [a(t), b(t)] in different phases. Then, applying the quasistatic displacement in Eq. (19) to Eq. (15) leads to the following differential equation of motion in terms of $u_d(x, t)$:

$$m\ddot{u}_{d} + c\dot{u}_{d} + EIu_{d}^{'''} - Tu_{d}^{''} + A\int_{0}^{L} u_{d} dx$$

= $p(x, t) - m\ddot{U} - c\dot{U}.$ (20)

While the quasistatic deflection U(x, t) in Eq. (19) has been derived analytically, the solution of the dynamic deflection component $u_d(x, t)$ in Eq. (20) can be carried out by Galerkin's method in the following section.

3.2. Solution of dynamic response component

The linear partial differential equations of motion in Eq. (20) for the suspended beam can be transformed into a set of generalized equations of motion using Galerkin's method. First, multiplying both sides of Eq. (20) with respect to the variation of the dynamic deflection component δu_d , and then integrating the equation over the beam length *L*, one can obtain the following virtual work equation:

$$\int_0^L \left(m\ddot{u}_d + c\dot{u}_d + EIu_d^{\prime\prime\prime\prime} - Tu_d^{\prime\prime} \right) \delta u_d dx$$
$$+ \left(A \int_0^L u_d dx \right) \int_0^L \delta u_d dx$$
$$= \int_0^L p(x,t) \delta u_d dx - \int_0^L [m\ddot{U} + c\dot{U}] \delta u_d dx.$$
(21)

Taking the homogeneous boundary conditions (16a) and (16b) into account, the remaining dynamic deflection $u_d(x, t)$ of the total displacement of the suspended beam can be represented as, [6]:

$$u_d(x,t) = \sum_{n=1}^{\infty} q_n(t) \sin \frac{n\pi x}{L},$$
(22)

where $q_n(t)$ means the generalized coordinate associated with the *n*-th assumed mode of the suspended beam. Substituting Eqs. (8), (19) and (22) into (21), one can obtain the *n*-th generalized equation of motion in terms of the generalized coordinate q_n as follows:

$$m\ddot{q}_{n} + c\dot{q}_{n} + k_{n}q_{n} + \psi_{n} = \left[\sum_{k=1}^{N} F_{k}(\varpi_{n}, v, t)\right] + \frac{m(\ddot{a} - \ddot{b}\cos n\pi) + c(\dot{a} - \dot{b}\cos n\pi)}{n\pi/2},$$
(23)

with

$$k_n = \left(\frac{n\pi}{L}\right)^4 EI + \left(\frac{n\pi}{L}\right)^2 H,\tag{24}$$

$$\psi_n = \frac{2AL}{n\pi^2} (1 - \cos n\pi) \left[\sum_{k=1}^{\infty} \frac{1}{k} (1 - \cos k\pi) q_k \right],$$
(25)

and the generalized force $F_k(\varpi_n, v, t)$ of the k-th moving load is expressed as

$$F_k(\varpi_n, v, t) = \frac{2P}{L} \sin \varpi_n (t - t_g - t_k) \left[H(t - t_g - t_k) - H(t - t_g - t_k - L/v) \right],$$
(26)

where $\varpi_n = n\pi v/L$ represents the driving frequency of the *k*-th moving load to the *n*-th assumed mode of the suspended beam.

With the respective consideration of symmetrical/antisymmetrical modes, Eq. (23) can be rewritten as the following differential equations:

(1a) Symmetrical modes:
$$\sin(n\pi x/L)$$
 for $n = 1, 3, 5, 7...$
 $m\ddot{q}_n + c\dot{q}_n + k_n q_n + \Psi_n = \left[\sum_{k=1}^N F_k(\varpi_n, v, t)\right]$
 $-\frac{m(\ddot{a} + \ddot{b}) + c(\dot{a} + \dot{b})}{n\pi/2}.$ (27)

(1b) Antisymmetrical modes: $sin(n\pi x/L)$ for n = 2, 4, 6, 8...

$$m\ddot{q}_n + c\dot{q}_n + k_n q_n = \left[\sum_{k=1}^N F_k(\varpi_n, v, t)\right] - \frac{m(\ddot{a} - \ddot{b}) + c(\dot{a} - \dot{b})}{n\pi/2},$$
(28)

where

$$\Psi_n = \frac{8AL}{n\pi^2} \left[\sum_{k=1,3,5...}^{\infty} \frac{q_k}{k} \right]_{n=1,3,5...}.$$
(29)

An observation of Eqs. (27) and (28) indicates that the stiffening effect of the cable tensions is only effective on the symmetrical modes of the suspended beam, as the stiffness term $\Psi_n|_{n=1,3,5...}$ shown in Eqs. (27) and (29). Therefore, the first symmetrical vibration frequency of suspension bridges is generally higher than the first antisymmetrical frequency.

For the special case of *uniform* support motion, i.e. a(t) = b(t), Eqs. (27) and (28) can further be reduced to

Table 1 Properties of the suspended beam

<i>L</i> (m)	EI (kN m ²)	$E_c A_c$ (kN)	<i>m</i> (t/m)	<i>c</i> (kN s/m)	H (kN)	<i>y</i> ₀ (m)	Ω_1 (Hz)	Ω_2 (Hz)
150	3.3×10^8	6×10^7	16	1.92	29 400	15	1.31 (anti-symm.)	1.47 (symm.)

Table 2

n

Properties of moving loads and resonant speeds

N	<i>d</i> (m)	<i>P</i> (kN)	$v_{\rm res,1}~({\rm km/h})$	$v_{\rm res,2}~({\rm km/h})$
16	27.5	350	130	146

(2a) Symmetrical modes (n = odd)

$$n\ddot{q}_{n} + c\dot{q}_{n} + k_{n}q_{n} + \Psi_{n} = \left[\sum_{k=1}^{N} F_{k}(\varpi_{n}, v, t)\right] - \frac{4(m\ddot{a} + c\dot{a})}{n\pi}, \quad n = 1, 3, 5, 7....$$
 (30)

(2b) Antisymmetrical modes (n = even)

$$m\ddot{q}_{n} + c\dot{q}_{n} + k_{n}q_{n} = \left[\sum_{k=1}^{N} F_{k}(\varpi_{n}, v, t)\right],$$

$$n = 2, 4, 6, 8....$$
(31)

An observation of Eqs. (30) and (31) indicates that the *uniform* support motion only excites the symmetrical modes of dynamic components. Similarly, considering the *antiphased* support motions, i.e. b(t) = -a(t), Eqs. (27) and (28) can be reduced to:

(3a) Symmetrical modes (n = odd)

$$m\ddot{q}_{n} + c\dot{q}_{n} + k_{n}q_{n} + \Psi_{n} = \left[\sum_{k=1}^{N} F_{k}(\varpi_{n}, v, t)\right],$$

$$n = 1, 3, 5, 7....$$
(32)

(3b) Antisymmetrical modes (n = even)

$$m\ddot{q}_{n} + c\dot{q}_{n} + k_{n}q_{n} = \left[\sum_{k=1}^{N} F_{k}(\varpi_{n}, v, t)\right] - \frac{4(m\ddot{a} + c\dot{a})}{n\pi}, \quad n = 2, 4, 6, 8....$$
(33)

An observation of Eqs. (32) and (33) indicates that the *antiphased* support motions only excites the antisymmetrical modes of dynamic components. The amplification effect due to the input vertical support excitations on the train-induced vibration of the suspended beam will be studied in the following numerical investigations.

4. Resonance

Due to the regular arrangement of intervals d of wheel loads, as the train passes along a bridge at speed v, the bridge may experience a *quasi-periodic* action of successive moving forces with an *exciting passage frequency* v/d. Once the *exciting passage frequency* matches any of natural frequencies Ω of the bridge, the resonant response will be developed on the bridge [7,8,18,21,22]. Then, the dynamic response of the bridge may continuously build up as there are more vehicular loads passing through the bridge at the resonant speed of $v_{\rm res} = \Omega d$, [7,8,18].

On the other hand, the resonant response of a railway bridge causes not only excessive vibrations of the bridge but also increases the risk of derailment for the train [7, 8,13,21,22]. Moreover, the resonant response may result in the ballast destabilization and diminishing of running safety of trains on the track structure, [7,8]. Because of this, the maximum acceleration will be employed to evaluate the dynamic behaviour of suspension bridges under the action of moving loads and vertical support motions.

5. Numerical investigations

As shown in Fig. 1, a single-span suspended beam is under the simultaneous action of moving forces and vertical support motions. The properties of the suspended beam and moving forces are listed in Tables 1 and 2, respectively. The symbols of Ω_i in Table 1 represent the *i*-th modal frequency of the suspended beam. On the other hand, due to the stiffening effect of cable tensions on the first symmetric mode of the suspended beam, the frequency Ω_2 of the first symmetric vibration mode is generally higher than Ω_1 of the first antisymmetrical mode. For this reason, the first antisymmetrical vibration mode will first be excited by the moving loads travelling at the corresponding resonant speed $v_{res,1}$, as indicated in Table 2.

To compute the train-induced acceleration response for the suspended beam shaken by support motions, the first 20 shape functions expressed in Eq. (25) will be taken into account in the following examples, and the generalized equations of motion in Eq. (26) are discretized by Newmark's β method [14] in the time domain and then solved by the step-by-step *direct integration method*.

5.1. Resonance of acceleration response

To illustrate the train-induced resonance of a suspended beam, the train loads moving over the suspended beam at the first or second resonant speed, i.e. $v_{\text{res},1} = \Omega_1 d = 36$ m/s (=130 km/h) or $v_{\text{res},2} = \Omega_2 d = 40.4$ m/s (=146 km/h), will be employed to excite the first antisymmetrical or symmetrical vibration mode, respectively. The response curves in Fig. 3 depicts the time history responses of acceleration computed at the mid-point and third quarter-point sections of the beam. As can be seen, both the acceleration responses continuously build up while the moving loads pass the beam. But the resonant amplitude at the third quarter-point section induced by the moving loads travelling at the first resonant speed $v_{\text{res},1} = 130$ km/h is significantly larger than that at the



Fig. 3. Time history responses of acceleration of the suspended beam.



Fig. 4. $a_{\text{max}} - v - x/L$ three-phase-plot of the beam due to moving loads.

mid-point section due to the moving loads travelling at the second resonant speed $v_{\text{res},2} = 146$ km/h, even though the second resonant speed $v_{\text{res},2}$ is higher than the first resonant speed $v_{\text{res},1}$. The reason: when the moving loads travel over the suspension bridge, in which the bridge span (L = 150 m) is far larger than the interval (d = 27.5 m) of the moving loads, the presence of *multiple* loads passing over the bridge can apply a suppressing action to the first symmetrical mode (i.e. the second bending mode). Such an effect results in the acceleration amplitude at mid-span section of the bridge less severe compared with that at the third quarter-point section, [13,23].

On the other hand, a three-dimensional (3D) plot for the maximum acceleration a_{max} along the beam length (x/L) against various moving speeds (v) has been drawn in Fig. 4. Such a 3D plot is called $a_{\text{max}} - v - x/L$ three-phaseplot in the following examples. As expected, the number of acceleration amplitudes appearing along the beam length depends on the vibration shape that has been excited. The maximum acceleration computed of the suspended beam is $a_{\text{max}} = 0.12g$, which corresponds to the first antisymmetrical mode with the first resonant speed $v_{\text{res},1} = 130$ km/h. Under such a resonant condition for the first antisymmetrical mode that has been excited, two peak accelerations on the beam can be observed. Especially, as the train speed exceeds 150 km/h, higher vibration modes of the suspended beam will be excited as well.

5.2. Effect of uniform support motion

To conduct the effect of vertical ground motions on the maximum acceleration of a suspension bridge to the train



Fig. 5. Records of vertical ground surface acceleration (Kobe earthquake).

Fig. 6. Effect of time lags on $a_{\text{max}} - x/L$ plot of the beam shaken by uniform support motion.

loads travelling at the first resonant speed $v_{res,1} = 130 \text{ km/h}$, let us consider that the different cases of the input support motions have shaken the bridge before the train loads enter the bridge. Fig. 5 plots the input vertical support motion records of acceleration from Kobe earthquake in 1995. With the consideration of *uniform* support motion, i.e., a(t) = b(t)and the reduction to receive the maximum value of ground acceleration 0.05g (g = 0.98 m s⁻²), the $a_{\text{max}} - x/L$ plot has been given in Fig. 6. As expected, the inclusion of vertical earthquake support motions can totally amplify the acceleration amplitudes of the suspended beam, especially for the symmetrical modes. Noteworthy is the fact that the uniform support motion has resulted in a significant amplification on the mid-span acceleration of the beam due to the fundamental and higher symmetrical modes that have been excited even though the effect of different time lags for the train loads arriving at the bridge on the mid-span acceleration amplitudes is quite limited in this example.

5.3. Effect of antiphased support motion

Consider the same problem as the example in the Section 5.2 but the ground inputs of support motions are in an antiphased

Fig. 7. Effect of time lags on $a_{\text{max}} - x/L$ plot of the beam shaken by antiphased support motions.

action, i.e. b(t) = -a(t), the $a_{max} - x/L$ plot has been given in Fig. 7 as well. As indicated in Eq. (33), the antisymmetrical modes will dominate the maximum acceleration of the suspended beam under the action of the antiphased support excitations and the train loads travelling with various time lags. This interesting result is attributed to the fact that, as the antiphased support motions have shaken the suspended beam subjected to the train loads crossing the beam at the resonant speed $v_{res,1}$, the ground excitations may further result in the increase of peak acceleration responses around the first quarterpoint and third quarter-point sections on the beam. On the other hand, some secondary peaks on the acceleration response curves in Fig. 7 can be observed since the resonance associated with higher modes has been excited as well.

5.4. Effect of multiple support motions

Due to the propagation effect of seismic waves along the longitudinal direction of the beam axis, the input earthquake records acting at one supported end of the bridge may have a time delay compared to those at the other supported end. For this reason, the input support excitation records at the right support of the suspended beam in Fig. 1 will be assumed to lag behind those at the left support with a time delay t_d , i.e.

$$b(t) = \begin{cases} 0 & t \le t_d, \\ a(t - t_d) & t > t_d. \end{cases}$$

Here, the time delay t_d can be defined as L/v_w , v_w = travelling speed of seismic waves in subsoil along the longitudinal direction of the bridge. Moreover, the train loads entering the suspended beam with the resonant speed $v_{\text{res},1} = 130$ km/h are set to have a critical time lag of $0.5L/v_{\text{res},1}$ after the input ground excitation records have shaken the right support of the beam. Fig. 8 depicts the $a_{\text{max}} - x/L$ plot for the maximum acceleration along the beam length against various t_d .

The results indicate that with the consideration of seismic wave propagation effect, the mid-span of the suspended beam is no longer at the critical position of the maximum acceleration response. The contribution of higher modes to the maximum acceleration amplitudes due to the effect of seismic wave

Fig. 8. $a_{\text{max}} - x/L$ plot of the beam under multiple support motions with various time delays at supports.

passage needs to be taken into account for a suspended bridge with long span length.

6. Conclusions

In this study, the acceleration response of a suspended beam under the simultaneous action of moving loads and vertical support motions has been investigated. With the consideration of time-dependent boundary value problem, the quasistatic component of the total response of the suspended beam under the static action of support motions has been obtained analytically. Then, the remaining dynamic part of the total response can be solved using Galerkin's method and step-bystep *direct integration* based on Newmark's β method.

The numerical results in train-induced vibration of the suspended beam indicate that once the *exciting passage frequency* (v/d) of a row of moving forces matches any of the natural vibration frequencies of the bridge, resonance will be developed in the bridge. By counting the number of acceleration amplitudes appearing on the beam, one can identify the vibration mode that has been excited.

In the numerical simulation, a parametric study of uniform support motion, antiphased support motions, and seismic wave propagation effect on the acceleration response of the suspended beam subjected to the moving loads at the first resonant speed has been investigated. The following conclusions may be drawn:

- (1) The *uniform* support motion has resulted in a significant amplification on the mid-span acceleration of the beam due to the fundamental and higher symmetrical modes that have been excited.
- (2) As the anti-phased support motion has shaken the suspended beam subjected to the train loads crossing the beam at the resonant speed $v_{res,1}$, the antisymmetrical modes will be excited and the increase of peak acceleration responses will occur around the first quarter-point and third quarter-point sections on the beam.

(3) Considering the propagation effect of seismic waves along the longitudinal direction of the span length, the mid-span of the suspended beam is no longer at the critical position of the maximum acceleration response. Such a contribution of higher modes to the maximum acceleration amplitudes needs to be taken into account for a suspended bridge with long span length.

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