彈性力學 1999 第一次作業

$$F = RU = VR$$

$$F = \begin{bmatrix} 1 & 2/\sqrt{3} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \sqrt{F^T F} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & \frac{5}{2\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = FU^{-1} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad V = \begin{bmatrix} 5\sqrt{3}/6 & 1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SVD decomposition

$$[F] = \Phi \Sigma \Psi^T$$

$$[F] = \begin{bmatrix} 0.866026 & 0 & -0.5 \\ 0.50 & 0 & 0.866 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1.7321 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 0.5774 \end{bmatrix} \begin{bmatrix} 0.50 & 0 & -0.866 \\ 0.8660 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix} = \Phi \Sigma \Psi^T$$

Prove that

$$[R] = \Phi \Psi^T, \; U = \Psi \Sigma \Psi^T, \; V = \Phi \Sigma \Phi^T$$

$$F \psi_i = \sigma_i \phi_i$$

$$F^T \phi_i = \sigma_i \psi_i$$

and discuss its geometric meaning.

Physical explanation:

$$F[\psi_1 \; \psi_2 \; \psi_3] \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = RU[\psi_1 \; \psi_2 \; \psi_3] \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$F \Psi \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = R \Psi \Sigma \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$F = R \Psi \Sigma \Psi^T = \Phi \Sigma \Psi^T, \; F = RU = R \Psi \Sigma \Psi^T$$