# 海洋大學河海工程學系1999 彈性力學 期中考 (OPEN BOOK)

**1.** Given a circular bending deformation

$$x_1 = (\rho - X_2)sin(X_1/\rho) \quad (1)$$
$$x_2 = \rho - (\rho - X_2)cos(X_1/\rho) \quad (2)$$
$$x_3 = X_3 \quad (3)$$

where $\rho$ is a radius of curvature. Please answer the following questions.
(a). Is this plane deformation only ? (5 %)
(b). Find the deformation gradient $F$. (5 %)
(c). Find the right Cauchy-Green strain tensor $C$. (10 %)
(d). Find the displacement gradient tensor $H$. (10 %)
(e). Find the Lagrangian strain tensor $L$. (10 %)
(f). Find the infinitesimal strain tensor $\epsilon$. (10 %)
(g). Find the infinitesimal rotation tensor $\Omega$. (10 %)
(h). Find the rotational vector $\omega$. (10 %)

**2.** Conventionally, the deformation gradient $F$ was decomposed by

$$F = RU \text{ or } F = VR \quad (4)$$

where $\mathbf{dx} = F\mathbf{dX}$. In the course, we present new concept of singular value decomposition (SVD),

$$F = \Phi\Sigma\Psi^T \quad (5)$$

Please determine the relation of $R$ and $\Phi$ and $\Psi$. (10 %) Also, if we introduce two new vectors,

$$\mathbf{dy} = \Phi^T\mathbf{dx} \text{ and } \mathbf{dY} = \Psi^T\mathbf{dX} \quad (6)$$

please determine the formula between $\mathbf{dy}$ and $\mathbf{dY}$. (10 %) Please explain the physical, geometrical and numerical meanings for this transformation. (10 %)

**3.** Given the deformation gradient, $F$, please write down the procedures to derive $\Phi, \Psi$ and $\Sigma$, such that (10 %)

$$F = \Phi\Sigma\Psi^T \quad (7)$$