Given the following deformation

$\bar{x} = x + u = 1.05x\cos\alpha - 0.98y\sin\alpha$

$\bar{y} = y + v = 1.05x\sin\alpha + 0.98y\cos\alpha$

$\bar{z} = z + w = z$

where $\alpha = \dfrac{\pi}{3}$

(1) Using the examples, construct Table in 1.

(2) Find $F, c_{11}, c_{12}, c_{22}, \theta, \sqrt{c_{11}c_{22} - c_{12}^2}$ and J.

(3) Plot the undeformed and deformed states.

Sol :

(1)

	Tensor	Undeformated	Relation	Deformed
Point	0	x	$\bar{x} = 1.05x\cos\alpha - 0.98y\sin\alpha$ $\bar{y} = 1.05x\sin\alpha + 0.98y\cos\alpha$ $\bar{z} = z$	$\bar{x}$
Infinitesimal element	1	dx	$d\bar{x} = \begin{bmatrix} 0.525 & -0.84871 & 0 \\ 0.90933 & 0.49 & 0 \\ 0 & 0 & 1 \end{bmatrix} dx$	$d\bar{x}$
Line length	1	dL	$\sqrt{c_{11}} = 1.05, \sqrt{c_{12}} = 0, \sqrt{c_{22}} = 0.98$	$d\bar{L}$
Angle	0	dx, dy	$\cos\theta = 0, \theta = \dfrac{\pi}{2}$	$d\bar{x}, d\bar{y}$
Area	0	dx, dy	$d\bar{A} = 1.029dA$	$d\bar{x}, d\bar{y}$
Volume	0	dx, dy, dz	$d\bar{V} = 1.029dV$	$d\bar{x}, d\bar{y}, d\bar{z}$

(2)

$$F_{ij} = \frac{\partial \bar{x}_i}{\partial x_j}$$

$$F_{11} = \frac{\partial \bar{x}_1}{\partial x_1} = 1.05\cos\alpha, \quad F_{12} = \frac{\partial \bar{x}_1}{\partial x_2} = -0.98\sin\alpha, \quad F_{13} = \frac{\partial \bar{x}_1}{\partial x_3} = 0$$

$$F_{21} = \frac{\partial \bar{x}_2}{\partial x_1} = 1.05\sin\alpha, \quad F_{12} = \frac{\partial \bar{x}_2}{\partial x_2} = 0.98\cos\alpha, \quad F_{13} = \frac{\partial \bar{x}_2}{\partial x_3} = 0$$

$$F_{31} = \frac{\partial \bar{x}_3}{\partial x_1} = 0, \quad F_{12} = \frac{\partial \bar{x}_3}{\partial x_2} = 0, \quad F_{13} = \frac{\partial \bar{x}_3}{\partial x_3} = 1$$

$$[F] = \begin{bmatrix} 1.05\cos\alpha & -0.98\sin\alpha & 0 \\ 1.05\sin\alpha & 0.98\cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.525 & -0.84871 & 0 \\ 0.90933 & 0.49 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[c]=[F]^T[F]=\begin{bmatrix}1.05\cos\alpha & 1.05\sin\alpha & 0\\ -0.98\sin\alpha & 0.98\cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1.05\cos\alpha & -0.98\sin\alpha & 0\\ 1.05\sin\alpha & 0.98\cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1.1025 & 0 & 0\\ 0 & 0.9604 & 0\\ 0 & 0 & 1\end{bmatrix}$$

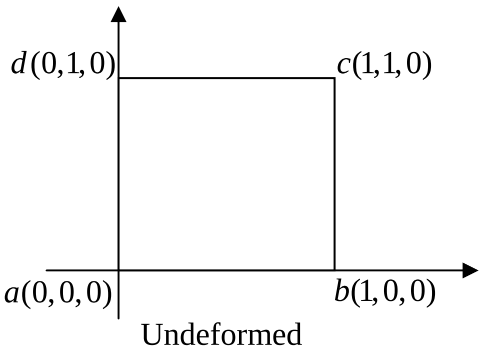
$$c_{11}=1.1025,\quad c_{12}=0,\quad c_{22}=0.9604$$

$$\cos\theta=\frac{c_{12}}{\sqrt{c_{11}c_{22}}}=0,\quad \theta=\frac{\pi}{2}$$

$$\sqrt{c_{11}c_{22}-c_{12}^2}=\sqrt{1.1025*0.9604}=1.029$$

$$J=\det\|F\|=1.029\cos^2\alpha+1.029\sin^2\alpha=1.029$$

(3)



$$\bar{a}_1=1.05\times0\times\cos\frac{\pi}{3}-0.98\times0\times\sin\frac{\pi}{3}=0$$

$$\bar{a}_2=1.05\times0\times\sin\frac{\pi}{3}+0.98\times0\times\cos\frac{\pi}{3}=0$$

$$\bar{a}_3=0$$

$$\bar{b}_1=1.05\times1\times\cos\frac{\pi}{3}-0.98\times0\times\sin\frac{\pi}{3}=0.525$$

$$\bar{b}_2=1.05\times1\times\sin\frac{\pi}{3}+0.98\times0\times\cos\frac{\pi}{3}=0.90933$$

$$\bar{b}_3=0$$

$$\bar{c}_1=1.05\times1\times\cos\frac{\pi}{3}-0.98\times1\times\sin\frac{\pi}{3}=-0.3237$$

$$\bar{c}_2=1.05\times1\times\sin\frac{\pi}{3}+0.98\times1\times\cos\frac{\pi}{3}=1.3993$$

$$\bar{c}_3=0$$

$$\bar{d}_1=1.05\times0\times\cos\frac{\pi}{3}-0.98\times1\times\sin\frac{\pi}{3}=-0.8487$$

$$\bar{d}_2=1.05\times0\times\sin\frac{\pi}{3}+0.98\times1\times\cos\frac{\pi}{3}=0.49$$

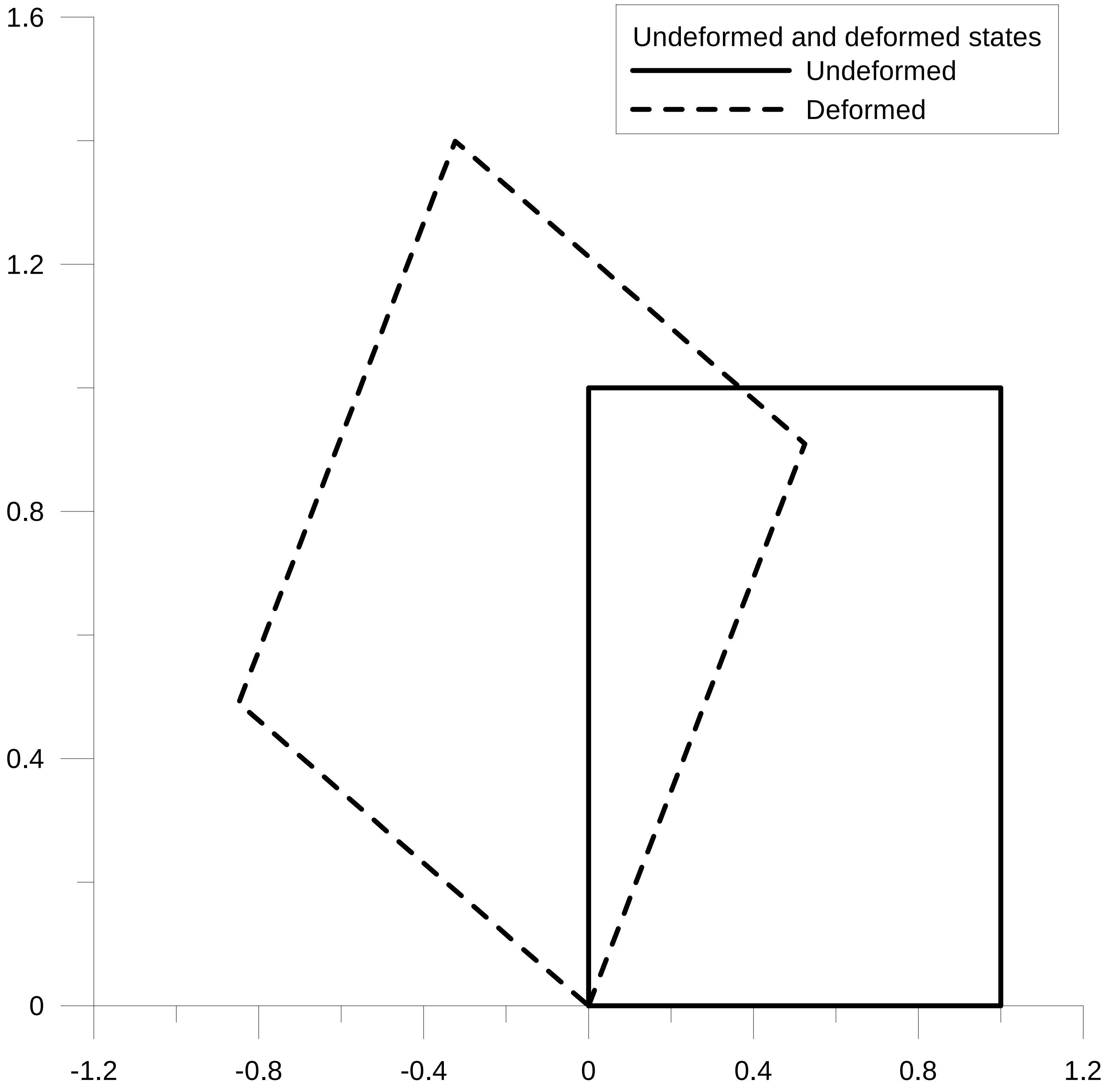
$$\bar{d}_3=0$$

$$\bar{a}=(0,0,0)$$

$$\bar{b}=(0.525,0.90933,0)$$

$$\bar{c}=(-0.3237,1.3993,0)$$

$$\bar{d}=(-0.8487,0.49,0)$$


Undeformed and deformed states
Undeformed
Deformed
1.6
1.2
0.8
0.4
0
-1.2
-0.8
-0.4
0
0.4
0.8
1.2