Table 1: Comparsions of time domain and frequency domain approaches

| time domain | frequency domain |
|---|---|
| $x(t), p(t)$ | $\bar{X}(\bar{\omega}) = \mathcal{F}\{x(t)\}, \bar{P}(\bar{\omega}) = \mathcal{F}\{p(t)\}$ |
| $\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = p(t)$ | $(-\bar{\omega}^2 + 2\xi\omega\bar{\omega}i + \omega^2)\bar{X} = \bar{P}$ |
| $h(t), \delta(t)$ | $H(\bar{\omega},\omega) = \mathcal{F}\{h(t)\}, 1 = \mathcal{F}\{\delta(t)\}$ |
| $\ddot{h} + 2\xi\omega\dot{h} + \omega^2 h = \delta(t)$ (implulse function) | $(-\bar{\omega}^2 + 2\xi\omega\bar{\omega}i + \omega^2)\bar{X} = 1$ <br> $\bar{X} = H(\bar{\omega},\omega) = \frac{1}{(\omega^2 - \bar{\omega}^2 + i2\xi\omega\bar{\omega})}$ |
| $\ddot{x}_r + 2\xi\omega\dot{x}_r + \omega^2 x_r = cos(\alpha t)$ | $\bar{X}/H(\bar{\omega},\omega) = \frac{1}{2}(\delta(\bar{\omega}-\alpha) + \delta(\bar{\omega}+\alpha))$ <br> $x_r = \frac{1}{2}\{H(\alpha,\omega)e^{i\alpha t} + H(-\alpha,\omega)e^{-i\alpha t}\} = Re\{H(\alpha,\omega)e^{i\alpha t}\}$ |
| $\ddot{x}_i + 2\xi\omega\dot{x}_i + \omega^2 x_i = sin(\alpha t)$ | $\bar{x}/H(\bar{\omega},\omega) = \frac{1}{2}(\delta(\bar{\omega}-\alpha) - \delta(\bar{\omega}+\alpha))$ <br> $x_i = \frac{1}{2}\{H(\alpha,\omega)e^{i\alpha t} - H(-\alpha,\omega)e^{-i\alpha t}\} = Im\{H(\alpha,\omega)e^{i\alpha t}\}$ |

Fourier transform:

$$\bar{X}(\bar{\omega}) = \int_{-\infty}^{\infty} x(t)e^{-i\bar{\omega}t}dt$$

Inverse Fourier transform:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{X}(\bar{\omega})e^{i\bar{\omega}t}d\bar{\omega}$$

Find $h(t)$ by contour integration:

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{H}(\bar{\omega},\omega)e^{i\bar{\omega}t}d\bar{\omega}$$

case.1: $H(\bar{\omega},\omega) = \frac{1}{(\omega^2 - \bar{\omega}^2 + i2\xi\omega\bar{\omega})}$

case.2: $H(\bar{\omega},\omega) = \frac{1}{(\omega^2 - \bar{\omega}^2)}, \xi = 0$

case.3: $H(\bar{\omega},\omega) = \frac{1}{(-\bar{\omega}^2 + \omega^2(1 \pm i\eta))}, + \ if \ \bar{\omega} > 0, - \ if \ \bar{\omega} < 0$

case.4: $H(\bar{\omega},\omega) = \frac{1}{(-\bar{\omega}^2 + \omega^2)}, \eta = 0$

case.5: $H(\bar{\omega},\omega) = \frac{1}{i\bar{\omega}}$, or $\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{it}e^{i\bar{\omega}t}dt =?$

Can the solutions of (2) and (4) be derived by the limiting process ?