

If the function $f(t) = \sin t$, then try to draw that

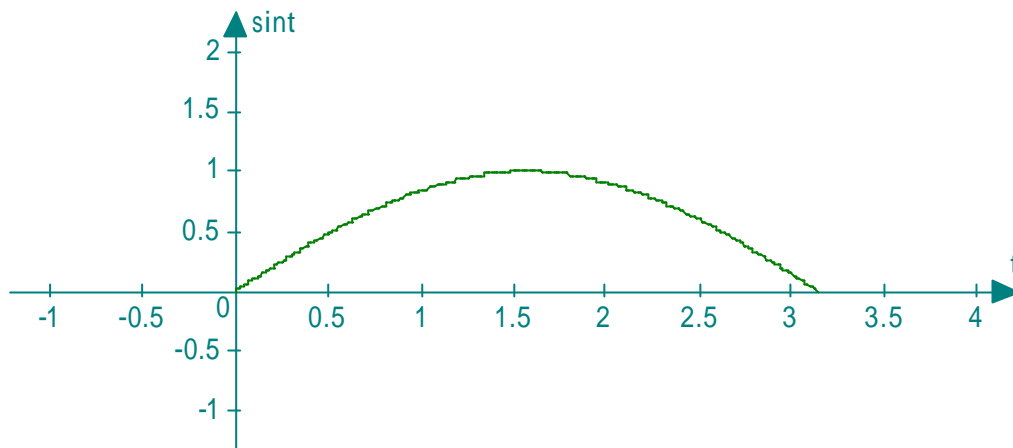
$$f(t)[u(t-0) - u(t-p)] * f(t)[u(t-0) - u(t-p)]$$

$$\begin{aligned} \int \sin(t-u)\sin u du &= \int (\sin t \cos u - \cos t \sin u)\sin u du \\ &= \frac{1}{2} \int [\sin t \sin(2u) - \cos t (1 - \cos(2u))] du \\ &= \frac{1}{2} \int [(\sin t \sin(2u) + \cos t \cos(2u)) - \cos t] du \\ &= \frac{1}{2} \int (\cos(t-2u) - \cos t) du \\ &= \frac{1}{2} \left(-\frac{1}{2} \sin(t-2u) - u \cos t \right) \\ &= -\frac{1}{4} \sin(t-2u) - \frac{1}{2} u \cos t \end{aligned}$$

$$\begin{aligned} \text{If } 0 \leq t \leq p, \text{ then } \int \sin(t-u)\sin u du &= \left(-\frac{1}{4} \sin(t-2u) - \frac{1}{2} u \cos t \right) \Big|_0^t \\ &= \frac{1}{2} \sin t - \frac{1}{2} t \cos t \end{aligned}$$

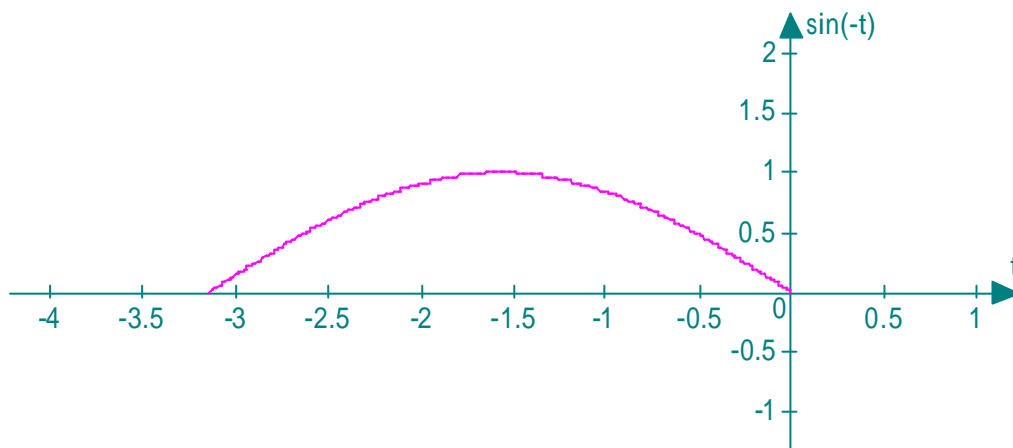
$$\begin{aligned} \text{If } p \leq t \leq 2p, \text{ then } \int \sin(t-u)\sin u du &= \left(-\frac{1}{4} \sin(t-2u) - \frac{1}{2} u \cos t \right) \Big|_{t-p}^p \\ &= -\frac{1}{2} \sin t - \frac{1}{2} (2p-t) \cos t \end{aligned}$$

$\sin t$



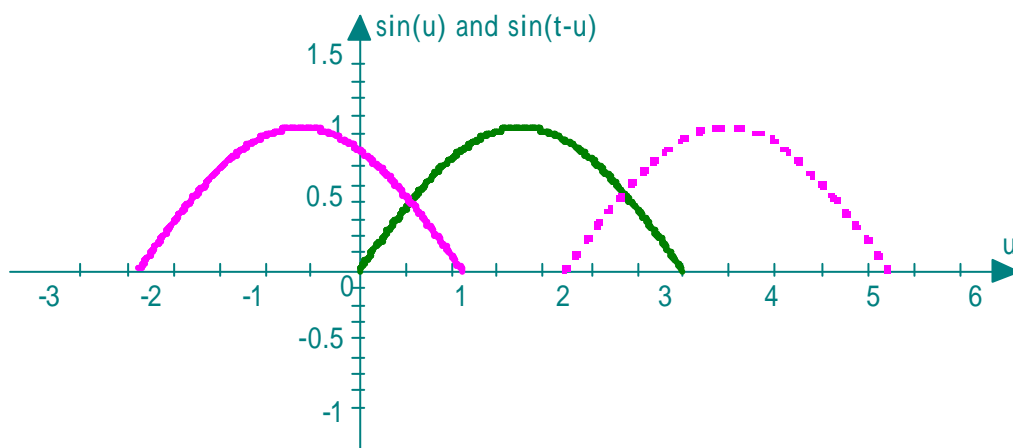
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$\sin(-t)$



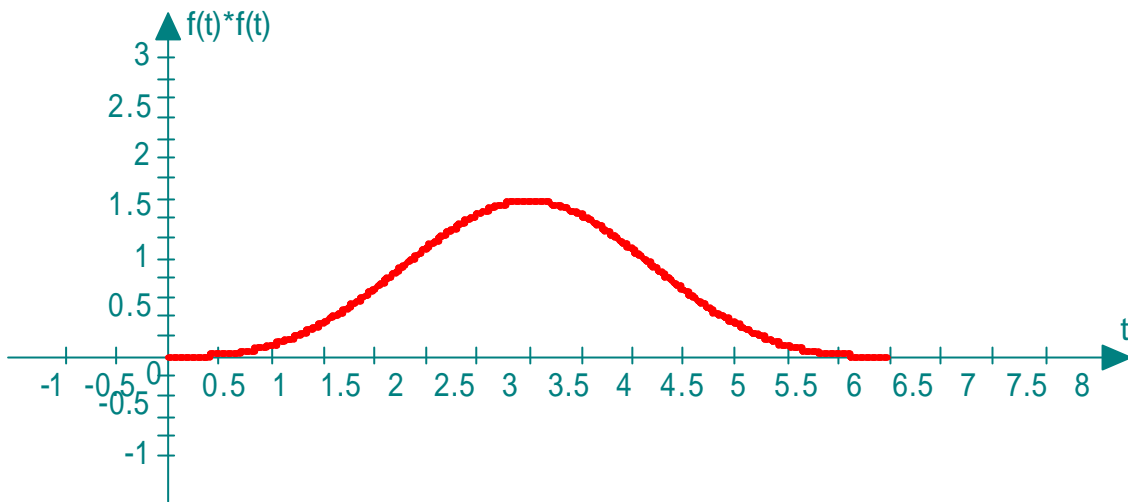
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$\sin(u)$ and $\sin(t-u)$



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$$f(t)[u(t-0) - u(t-p)] * f(t)[u(t-0) - u(t-p)]$$



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