積分方程特論

Multiple-ellipses problem

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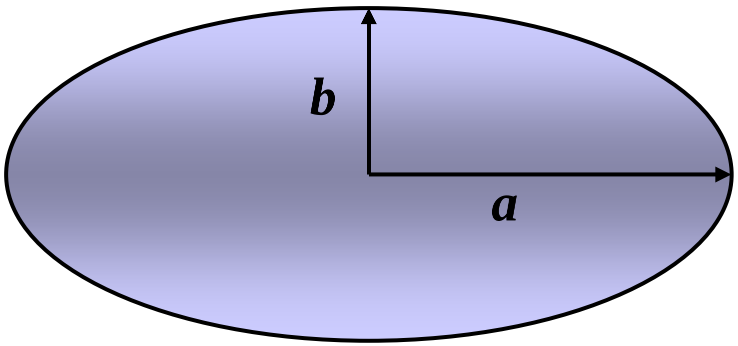
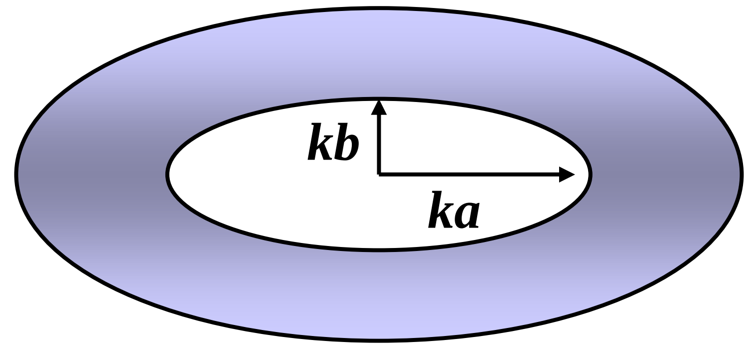
Ellipse and annular ellipse





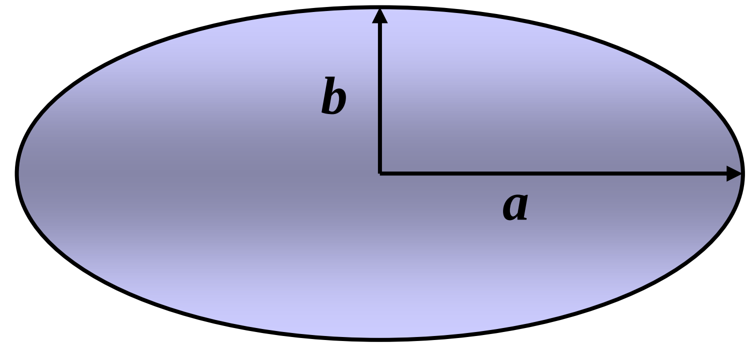
$$D = \frac{\pi a^3 b^3}{a^2 + b^2}$$

$$\phi = -\frac{a^2 b^2}{a^2 + b^2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)$$

$$D = \frac{\pi a^3 b^3}{a^2 + b^2}(1 - k^4)$$

$$\phi = -\frac{a^2 b^2}{a^2 + b^2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)$$

Prandlt solution



Governing equation: $\nabla^2\phi = -2$

Boundary condition: $\phi = 0$

Prandlt solution: $\phi = -\dfrac{a^2b^2}{a^2+b^2}\left(\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1\right)$

Present approach:

$$u = u' + \bar{u}$$

$$u' = -\frac{1}{2}(x^2+y^2) = -\frac{1}{2}c^2(\cosh^2\bar{\xi}\cos^2\bar{\eta}+\sinh^2\bar{\xi}\sin^2\bar{\eta})$$

$$\bar{u} = -u' = \frac{1}{4}c^2(\cosh 2\bar{\xi}) + \frac{1}{4}c^2\cos 2\bar{\eta}$$

$\Rightarrow$ $a_0 = \dfrac{1}{4}c^2\cosh 2\bar{\xi}$ $\quad a_2 = \dfrac{1}{4}c^2$ $\quad b_m = 0$

$$\int_0^{2\pi} T(s,x)u(s)dB(s) - \int_0^{2\pi} U(s,x)t(s)dB(s) = 0$$

$$\Rightarrow \quad p_0 = 0 \qquad p_2 = 2\frac{\sinh 2\bar{\xi}}{\cosh 2\bar{\xi}} a_2 = \frac{c^2}{2}\frac{\sinh 2\bar{\xi}}{\cosh 2\bar{\xi}} \qquad q_m = m\frac{\cosh m\bar{\xi}}{\sinh m\bar{\xi}} b_m = 0$$

$$2\pi u(x) = \int_0^{2\pi} T(s,x)u(s)dB(s) - \int_0^{2\pi} U(s,x)t(s)dB(s)$$

$$\Rightarrow \quad u(x) = \frac{1}{4}c^2\left[\cosh 2\bar{\xi} + \frac{1}{\cosh 2\bar{\xi}}\cosh 2\xi \cos 2\eta\right]$$

$$u = u' + \bar{u}$$

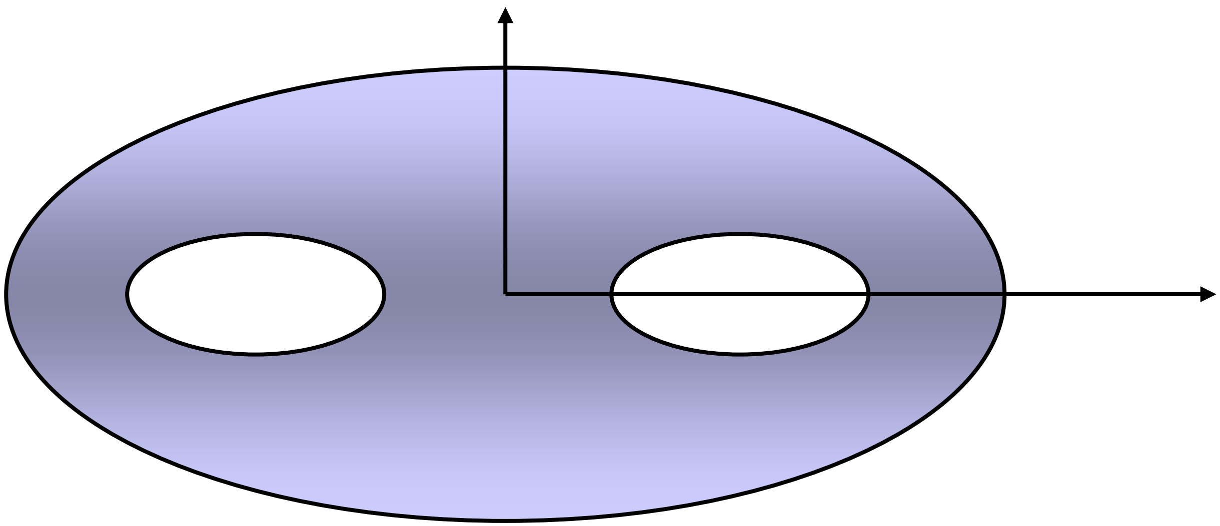
$$= -\frac{1}{2}c^2(\cosh^2\xi\cos^2\eta + \sinh^2\xi\sin^2\eta) + \frac{1}{4}c^2\left[\cosh 2\bar{\xi} + \frac{1}{\cosh 2\bar{\xi}}\cosh 2\xi\cos 2\eta\right]$$

$$= -\frac{c^2\cosh^2\bar{\xi}\sinh^2\bar{\xi}}{\cosh^2\bar{\xi} + \sinh^2\bar{\xi}}\left[\frac{\cosh^2\xi\cos^2\eta}{\cosh^2\bar{\xi}} + \frac{\sinh^2\xi\sin^2\eta}{\sinh^2\bar{\xi}} - 1\right]$$

$$= -\frac{a^2b^2}{a^2+b^2}\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right]$$

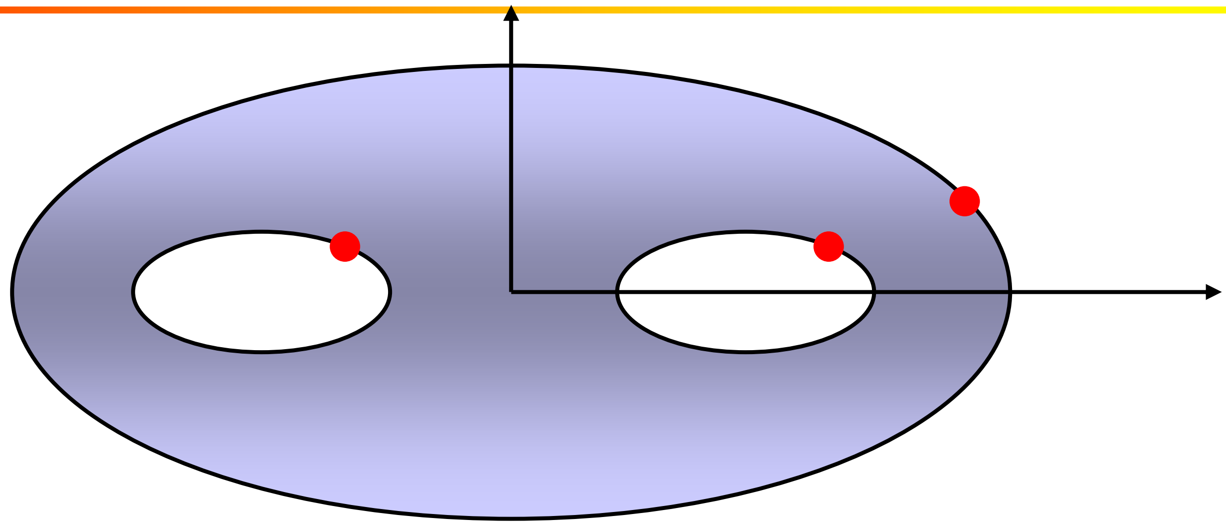
$$\phi = -\frac{a^2b^2}{a^2+b^2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)$$

Torsional rigidity



Present (2008)	Katsikadelis (1985)	Chou (1992)	Chou (1997)
0.2857	0.2934	0.2994	0.2842

Influence matrix



$$U(s,x)=\begin{cases}\xi+\ln\frac{c}{2}-\sum_{m=1}^{\infty}\frac{2}{m}e^{-m\xi}\cosh m\xi_0\cos m\eta\cos m\eta_0-\sum_{m=1}^{\infty}\frac{2}{m}e^{-m\xi}\sinh m\xi_0\sin m\eta\sin m\eta_0,\ \xi>\xi_0\\ \xi_0+\ln\frac{c}{2}-\sum_{m=1}^{\infty}\frac{2}{m}e^{-m\xi_0}\cosh m\xi\cos m\eta\cos m\eta_0-\sum_{m=1}^{\infty}\frac{2}{m}e^{-m\xi_0}\sinh m\xi\sin m\eta\sin m\eta_0,\ \xi<\xi_0\end{cases}$$

$$\begin{bmatrix}\xi+\frac{c}{2} & * & * & * & * & *\\ \xi+\frac{c}{2} & * & * & * & * & *\\ \xi_0+\frac{c}{2} & * & * & * & * & *\\ \xi_0+\frac{c}{2} & * & * & * & * & *\\ \xi_0+\frac{c}{2} & * & * & * & * & *\\ \xi_0+\frac{c}{2} & * & * & * & * & *\end{bmatrix}\quad \xi=\xi_0$$

Degenerate scale

$$\xi + \ln\frac{c}{2} = \tanh^{-1}(\frac{\alpha}{\beta}) + \ln(\frac{\sqrt{\alpha^2-\beta^2}}{2})$$

$x = c\cosh\xi\cos\eta$

$y = c\sinh\xi\sin\eta$

$\alpha = c\cosh\xi$

$\beta = c\sinh\xi$

$c^2 = \alpha^2 - \beta^2$

$$= \frac{1}{2}\ln(\frac{1+\frac{\beta}{\alpha}}{1-\frac{\beta}{\alpha}}) + \frac{1}{2}\ln(\alpha^2-\beta^2) - \ln 2$$

$$= \frac{1}{2}\ln(\frac{\alpha+\beta}{\alpha-\beta}) + \frac{1}{2}\ln(\alpha^2-\beta^2) - \ln 2$$

$$= \frac{1}{2}\ln[(\alpha^2-\beta^2)(\frac{\alpha+\beta}{\alpha-\beta})] - \ln 2$$

$$= \ln(\alpha+\beta) - \ln 2$$

$$= \ln(\frac{\alpha+\beta}{2})$$

The End

Thanks for your kind attention