Paper 12 Hydrodynamic Forces on Multiple Cylinders in Waves

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SYNOPSIS

Some kinds of offshore mobile platforms are built up from several elements such as cylinders. In this paper two kinds of configurations, horizontal and vertical multiple cylinders, are taken up as simplified models of such platforms for a purpose to evaluate the hydrodynamic interaction effect between the elements.

First the methods to compute the hydrodynamic forces on multiple cylinders are given, in which the interaction effect is taken into consideration. Secondly with numerical examples of the wave force and the motion in waves of the multiple cylinders calculated by the methods, it is concluded that the effect is essential under certain circumstances for the theoretical prediction of their motion in waves.

1 INTRODUCTION

Some offshore mobile platforms have a common composite configuration, being constructed from many elemental sections such as cylinders and ellipsoids. In computing and discussing the wave exciting forces on the platforms and the internal forces on the bracings connecting the elements the hydrodynamic interaction effect between neighbouring elements has been usually neglected. The total wave force on the platforms, for instance, is usually provided by adding the forces on the elements. It seems to be that the neglect of the interaction effect has arisen not from its smallness, but because theoretical calculations taking the effect into account are very difficult to be performed.

In this paper two kinds of bodies (horizontal multiple cylinders and vertical multiple cylinders) composed of cylindrical elements will be adopted as the most simplified and typical models of the floating platforms with the composite configuration. Some theoretical and numerical examples will be given of studies describing the procedure to compute the hydrodynamic forces on the platforms. The interaction effect is essential under certain circumstances for the theoretical predictions of the response of the multiple cylinders to the sea.

It is a matter of importance what type of method is used to calculate the force on the composite body. A method with the following merit may be most desirable. For the purpose of calculating the hydrodynamic force including the interaction effect between the elements of the body, it is essential that only the hydrodynamic properties of each element is given. The method having such a merit will facilitate the calculation for a body having many elements and may be applied to the design arrangement of the elements. The procedure introduced in this paper is believed to have such a feature.

For the calculation performed in this paper, water is regarded as an ideal fluid and the wave motion is assumed to be infinitesimal.

1.1 Notation

- $\bar{A}_j$ Wave amplitude generated by jth oscillation of one two-dimensional cylinder.
- a Radius of circular cylinder.
- d Water depth.
- F_x Wave force on vertical cylinder in the x-direction.
- F_z Wave force on vertical cylinders in the z-direction.
- g Acceleration of gravity.
- k Wave number.
- m_j Added mass or moment of added mass of two-dimensional multiple cylinders.
- $2P$ Spacing between the centres of cylinders.
- T Draft of two-dimensional cylinder.
- ε_j Wave phase generated by jth oscillation of one two-dimensional cylinder.
- ξ_a Amplitude of incident waves.
- ρ Water density.
- ω Circular frequency of waves or oscillation of a body.

2 HYDRODYNAMIC FORCES UPON TWO-DIMENSIONAL MULTIPLE CYLINDERS

This Section introduces an approximate method of calculating the hydrodynamic forces and moment acting upon two-dimensional multiple cylinders. This simplified model of mobile platforms of multihulled vessel type may oscillate in or below the free surface of a fluid and is held at a fixed heading in regular waves. Many authors Ohkusu (**1**)†, Wang and Wahab (**2**), Nordenstrom *et al.* (**3**) and Takezawa *et al.* (**4**) have already reported on the procedures of computing exactly the added mass, moment of added mass and damping coefficients of half-immersed twin cylinders. The methods, however, are not convenient enough to be applied for the case of multiple cylinders having three or more element cylinders. Moreover they do not satisfy the requirement of being able to compute the hydrodynamic forces upon the multiple cylinders from only the hydrodynamic properties of one element cylinder. The method described below is an approximate one, but satisfies all the requirements.

Suppose that the left two-dimensional cylinder L of the configuration as shown in Fig. 1 is forced to perform an oscillation j ($j = 1$, swaying $x = \mathrm{Re}[e^{i\omega t}]$; $j = 2$, heaving $y = \mathrm{Re}[e^{i\omega t}]$; and $j = 3$, clockwise rolling about the point L, $\theta = \mathrm{Re}[e^{i\omega t}]$) about its mean position with the right cylinder R kept fixed. Here for simplicity we consider twin cylinders with identical cross-sections

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† *References are given in the Appendix.*

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although this is not an indispensable condition for applying the method described below. The distance between the point L and R is $2P$ and the cylinders' draft is T. Initially we enumerate two relations, which will be essential in deducing our approximate formula for hydrodynamic forces on multiple cylinders.

(1) Haskind relation (5)—if we know there exist diverging waves at $x_L = +\infty$, $\bar{A}_j e^{i\varepsilon_j} e^{i(\omega t - kx_L)}$, when cylinder L makes the oscillation j without the existence of cylinder R, then we can obtain the wave exciting force or moment acting upon the cylinder in the j-direction ($j = 1$: force in x-direction, $j = 2$: force in y-direction, and $j = 3$: rolling moment around L).

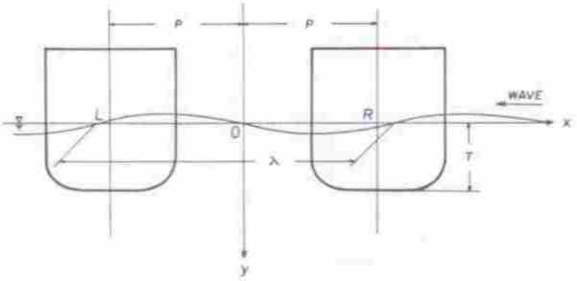

Fig. 1. Co-ordinate system

For a cylinder fixed in the incoming wave $\zeta_a e^{i(\omega t + kx_L)}$ the force or moment is given by,

$$\frac{i\rho g}{k}\zeta_a \bar{A}_j e^{i(\varepsilon_j + \omega t)} \qquad \ldots(1)$$

where x_L denote the x-co-ordinate with respect to the origin L, ρ density of a fluid, g acceleration of gravity and $k = \omega^2/g$.

(2) Bessho relation (6)—the transmitted wave ζ_- at $x_L = -\infty$ and the reflected wave ζ_+ at $x_L = +\infty$ generated by the cylinder fixed in the wave $\zeta_a e^{i(\omega t + kx_L)}$ are given by

$$\zeta_+ = iH^+(kT)\, e^{i(\omega t - kx_L)} \qquad \ldots(2)$$

$$\zeta_- = e^{i(\omega t + kx_L)} + iH^-(kT)\, e^{i(\omega t + kx_L)} \qquad \ldots(3)$$

where

$$H^{\pm}(kT) = i\, e^{i\varepsilon_2}\cos\varepsilon_2 \mp e^{i\varepsilon_1}\sin\varepsilon_1 \qquad \ldots(4)$$

The method adopted here to compute the forces on twin cylinders is based upon an approximation that there exists an interaction only between the two cylinders with respect to the progressing waves generated by the oscillation or the reflection of one cylinder. However, the standing waves being mainly only in the immediate vicinity of one cylinder have no influence upon the other cylinder.

The wave motion generated by the motion j of cylinder L is propagated towards cylinder R and becomes an incident wave $\bar{A}_j e^{i(\varepsilon_j + \omega t - kx_L)}$ upon cylinder R according to the approximation.

Using the Bessho relation (2) a reflected wave from cylinder R induced by this incident wave can be expressed in the neighbourhood of cylinder L as follows,

$$iH^+(kT)\, e^{-i4kP} \bar{A}_j e^{i\varepsilon_j} e^{i(\omega t + kx_L)}. \qquad \ldots(5)$$

By repeating this process we can obtain the total incoming wave acting upon cylinder R, which results from the exchange of reflected waves between the two cylinders. That is,

$$\bar{A}_j e^{i\varepsilon_j} \frac{e^{-i2kP}}{1 + H^+(kT)^2 e^{-i4kP}} e^{i(\omega t - kx_R)} \qquad \ldots(6)$$

The total incoming wave acting upon cylinder L is also given by

$$\bar{A}_j e^{i\varepsilon_j} \frac{iH^+(kT)\, e^{-i4kP}}{1 + H^+(kT)^2 e^{-i4kP}} e^{i(\omega t + kx_L)} \qquad \ldots(7)$$

where x_R is the x-co-ordinate with the origin R.

The force or moment f_{jl} which the incident wave (6) induces on cylinder R in the l-direction is obtained at once by substituting the wave amplitude and phase expressions into the Haskind relation (1) above, which gives,

$$f_{jl} = (-1)^l \frac{i\rho g}{k} \frac{e^{-i2kP}\bar{A}_j \bar{A}_l}{1 + H^+(kT)^2 e^{i4kP}} e^{i(\varepsilon_j + \varepsilon_l)} e^{i\omega t} \ldots(8)$$

We can get the force g_{jl} upon cylinder L induced by incident wave (7) by a similar manner and is given by,

$$g_{jl} = \frac{i\rho g}{k} \frac{iH^+(kT)\, e^{-i4kP}\bar{A}_j \bar{A}_l}{1 + H^+(kT)^2 e^{-i4kP}} e^{i(\varepsilon_j + \varepsilon_l)} e^{i\omega t} \qquad \ldots(9)$$

Repeating the same process we can also obtain the formula for the force on each cylinder when cylinder R oscillates but cylinder L is fixed.

With the application of the results obtained so far to the motion of each cylinder we can easily derive the hydrodynamic force on the twin cylinders as a whole making the oscillation j about the origin 0. The amplitude and phase of the wave at $x = +\infty$ generated by the oscillation of the twin cylinders are, for instance, given by the absolute value and phase angle of the following complex numbers.

$$\bar{A}_1 e^{i(\varepsilon_1 + kP)} \frac{2[1 - e^{i(2\varepsilon_2 - 2kP)}]}{2 - [e^{i(2\varepsilon_1 - 2kP)} + e^{i(2\varepsilon_2 - 2kP)}]}, (j = 1) \quad \ldots(10)$$

$$\bar{A}_2 e^{i(\varepsilon_2 + kP)} \frac{2[1 + e^{i(2\varepsilon_1 - 2kP)}]}{2 + [e^{i(2\varepsilon_1 - 2kP)} + e^{i(2\varepsilon_2 - 2kP)}]}, (j = 2) \quad \ldots(11)$$

$$2e^{ikP} \frac{P\bar{A}_2 e^{i\varepsilon_2}[1 - e^{i(2\varepsilon_1 - 2kP)}] + \bar{A}_3 e^{i\varepsilon_3}[1 - e^{i(2\varepsilon_2 - 2kP)}]}{2 - [e^{i(2\varepsilon_1 - 2kP)} + e^{i(2\varepsilon_2 - 2kP)}]},$$

$$(j = 3) \quad \ldots(12)$$

The added mass m_j and moment m_3 of added mass of the twin cylinders are expressed by the following equations.

$$m_j = 2m_j^0 + \mathrm{Re}\left[\frac{4\,i\rho}{k^2} \times \frac{(-1)^j \bar{A}_j^2 e^{i(2\varepsilon_j - 2kP)}}{2 + (-1)^j [e^{i(2\varepsilon_1 - 2kP)} + e^{i(2\varepsilon_2 - 2kP)}]}\right], (j = 1, 2) \quad \ldots(13)$$

$$m_3 = 2m_3^0 + 2P^2 m_2^0 + \mathrm{Re}\left[-\frac{4\mathrm{i}\rho}{k^2}\frac{(P\bar{A}_2\,\mathrm{e}^{\mathrm{i}\varepsilon_2} - \bar{A}_3\,\mathrm{e}^{\mathrm{i}\varepsilon_3})^2}{2 - [\mathrm{e}^{\mathrm{i}(2\varepsilon_1 - 2kP)} + \mathrm{e}^{\mathrm{i}(2\varepsilon_2 - 2kP)}]}\right] \quad \ldots(14)$$

where m_j^0 is the added mass or the moment of added mass of one cylinder.

It is clear from the Haskind relation (1) that the amplitude and the phase of the wave exciting force in the j-direction which acts on the twin cylinders fixed in a wave $\zeta_a \mathrm{e}^{\mathrm{i}(\omega t + kx)}$ is given by multiplying the equations (10), (11) and (12) by $\mathrm{i}\rho g/k$ respectively. If we do not take the interaction effect between two cylinders into consideration the wave force for $j = 2$ (heaving force) is given by

$$2(\mathrm{i}\rho g/k)\zeta_a \bar{A}_j\,\mathrm{e}^{\mathrm{i}\varepsilon_j}\cos(kP)\,\mathrm{e}^{\mathrm{i}\omega t} \quad \ldots(15)$$

It is obvious from equation (11) that the force becomes zero at the frequency where $2(\varepsilon_1 - kP) = (2n + 1)\pi$ is satisfied. But equation (15) shows that the zero wave force occurs at $-2kP = (2n + 1)\pi$ when we neglect the interaction effect. Accordingly the conclusion is that since ε_1 is not so small except when $kT \ll 1$, we cannot accurately predict the frequency at which the heaving motion of twin cylinders in waves vanishes, unless the interaction effect is taken into account. On the other hand if we know ε_1 for one cylinder, we can select the spacing between the two cylinders in such a way that their motion vanishes at a given frequency of incident waves. It is also evident from equations (13) and (14) that the interaction effect on added mass is in proportion to $\bar{A}_j^2$.

As is evident from the description given so far there exists no requirement for the application of the method that the left and right hand cylinders must have identical cross-sections, but only that the hydrodynamic properties of each of them is previously known. Accordingly, when the properties of twin cylinders are known, we can then easily compute the hydrodynamic forces acting upon three cylinders by considering the left cylinder to be twin cylinders and the right one to be one cylinder and applying the method. In this way one can calculate the forces acting upon multiple cylinders with an arbitrary number of cylindrical elements.

To illustrate the approximate method previously described some numerical examples were undertaken. It is necessary for the application of the method to have computed previously the hydrodynamic properties of an elemental cylinder. This was done by considering the cylinder's section as a Lewis form and using the method by Tasai (7).

Fig. 2 shows $\bar{A}_2$ (the amplitude of waves at infinity/motion amplitude) of two half-immersed circular cylinders performing a heaving oscillation, where a is a radius of the cylinders and $2P$ is the spacing between their centres. The lines in this figure indicate the values obtained by the approximate method and the dots show the exact values previously presented by the author (1). We may conclude from this figure that the approximate method produces results with sufficient accuracy.

The added mass and wave exciting force of the two-dimensional twin cylinders computed by the method make it possible to calculate their motions in waves.

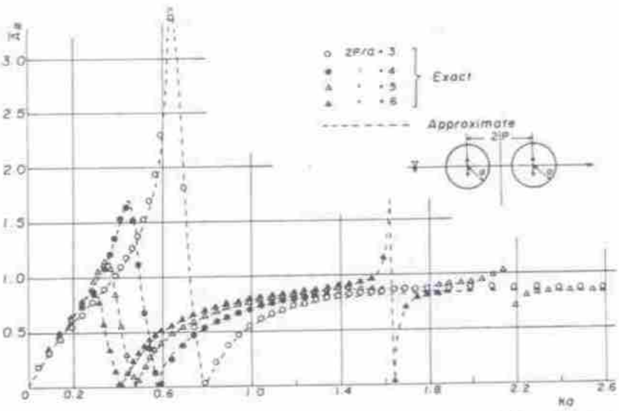

Fig. 2. Ratio of wave amplitude of heaving motion of two circular cylinders

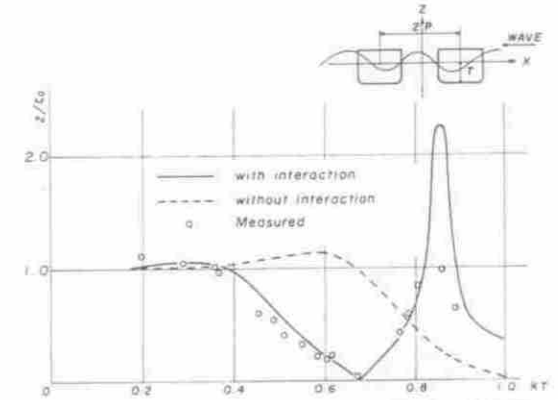

Fig. 3. Heaving amplitude of twin cylinders ($2P/T = 3$)

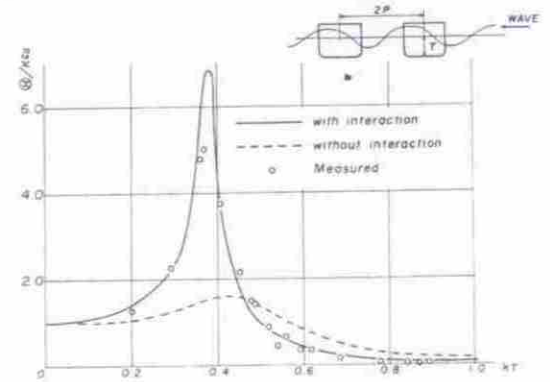

Fig. 4. Rolling amplitude of twin cylinders ($2P/T = 3$)

Figs 3 and 4 show the normalized amplitude of heave motion and roll motion of the twin cylinders with $2P/T = 3$ and with cross-sections as shown in Fig. 1. Full lines in these figures give the theoretical values computed by using the approximate method and accordingly take the interaction effect into consideration. In comparing them with the dotted lines which are the theoretical values obtained without taking the interaction into account and the dots which are experimental values, it is evident that if we neglect the interaction effect in the theoretical calculation of twin cylinders' motion, we will obtain the erroneous results especially in predicting the frequency of zero motion.

Figs 5 and 6 compare the heave and rolling motion of twin cylinders' and three cylinders' respectively. The spacing between the outside cylinders is identical ($2P/T = 1.6$) in both cases. We can see that if we add

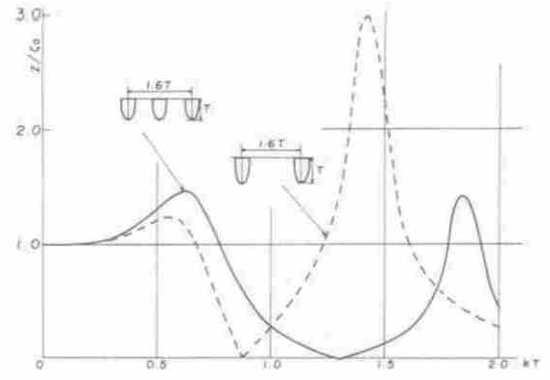

Fig. 5. Heaving amplitude of twin and three cylinders

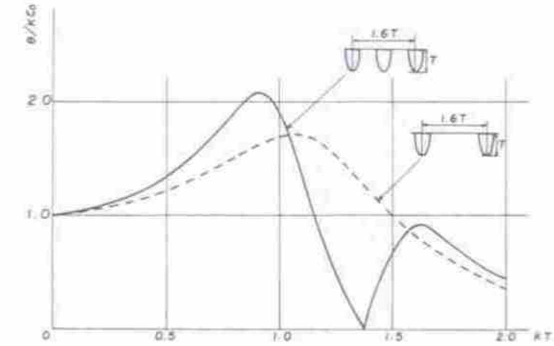

Fig. 6. Rolling amplitude of twin and three cylinders

only one more cylinder to the middle position between twin cylinders, their motion characteristics change remarkably.

3 HYDRODYNAMIC FORCES UPON VERTICAL MULTIPLE CYLINDERS

As a simplified model of some platforms of semi-submergible type consider the three cylinders configuration in Fig. 7. The interaction effect between the hydrodynamic forces acting upon them now needs to be investigated. Here we use the cylindrical co-ordinate systems (γ_1, θ_1, Z), (γ_2, θ_2, Z) and (γ_3, θ_3, Z) with their origins coinciding with the centres of cylinder 1, 2 and 3 respectively. The water depth is d, the distance between cylinders' bottom and water bottom h, the radii of the cylinders is a and the spacing between them $2P$.

When the wave, given by the following equation, is incident upon cylinder l,

$$\phi_I^1 = \sum_{m=-\infty}^{\infty} a_{m0} \frac{Z_0(z)}{dZ_0'(d)} \mathrm{Jm}(k\gamma_1)\, e^{im\theta_1} + \sum_{m=-\infty}^{\infty} \sum_{j=1}^{\infty} a_{mj} Z_j(z)\, \mathrm{Im}(\alpha_j \gamma_1)\, e^{im\theta_1} \quad \ldots(16)$$

or the cylinder is performing a periodical oscillation about its mean position, the wave

$$\phi_D^1 = \sum_{m=-\infty}^{\infty} A_{m0} \frac{Z_0(z)}{dZ_0'(d)} \mathrm{Hm}(k\gamma_1)\, e^{im\theta_1} + \sum_{m=-\infty}^{\infty} \sum_{j=1}^{\infty} A_{mj} Z_j(z)\, \mathrm{Km}(\alpha_j \gamma_1)\, e^{im\theta_1} \quad \ldots(17)$$

is radiated from the cylinder. The real roots $\alpha_j (j \neq 0)$ and imaginary root $\alpha_0 (= -ik)$ satisfy the equation

$$\alpha_j \tan \alpha_j d + \omega^2/g = 0 \quad \ldots(18)$$

where

$$Z_j(z) = \cos \alpha_j z / \sqrt{[\tfrac{1}{2}(1 + \sin 2\alpha_j d / 2\alpha_j d)]} \quad \ldots(19)$$

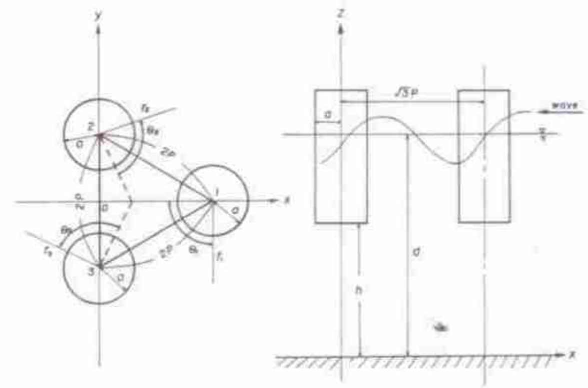

Fig. 7. Co-ordinate system

And Jm (x) is the first kind Bessel function, Hm (x) the second kind Hankel function, Im (x) and Km (x) the first and the second kind of modified Bessel function of mth order.

There exist the following relations between a_{mj} and A_{mj},

$$A_{m0} = \Big(\sum_{l=0}^{\infty} a_{ml} \mathscr{F}_{m0}^{l}\, dZ'(d) - a_{mo}\, \mathrm{Jm}(ka)\Big)/\mathrm{Hm}(ka) \quad \ldots(20)$$

$$A_{mj} = \sum_{l=0}^{\infty} (a_{ml} \mathscr{F}_{mj}^{l} - a_{ml} \delta_{jl}\, \mathrm{Im}(\alpha_l a))/\mathrm{Km}(ka) \quad \ldots(21)$$

Following the procedure given by Garret (**8**) the coefficients $\mathscr{F}_{mj}^{l}$ are determined by solving a boundary value problem in which a diffraction wave from cylinder 1 is produced by the wave described in equation (16). Knowing the coefficients $\mathscr{F}_{mj}^{l}$ we can easily calculate the wave exciting force acting upon cylinder 1 in the x and z-directions by using the following formulae,

$$F_x = \frac{\rho g \pi a^2}{i\omega} kd \tanh kd \sum_{j=0}^{\infty} \frac{\sin \alpha_j d - \sin \alpha_j h}{\sqrt{(N_j)}\alpha_j a} \times \Big[(a_{1j} - a_{-1j})\mathscr{F}_{1j}^{0} + \sum_{l=1}^{\infty} (a_{1l} + a_{-1l})\mathscr{F}_{1j}^{l}\Big] \quad \ldots(22)$$

$$F_z = \frac{\rho g \pi a^2}{i\omega} kd \tanh kd \sum_{l=0}^{\infty} a_{0l}\Big[F_{00}^{l} + 4 \sum_{n=1}^{\infty} F_{0n}^{l} \times (-1)^n \frac{h}{n\pi a} \cdot \frac{I_1(n\pi a/h)}{I_0(n\pi a/h)}\Big] \quad \ldots(23)$$

where

$$F_{mn}^{l} = \sum_{j=0}^{\infty} \frac{(-1)^n}{\alpha_j^2 h^2 - n^2 \pi^2} \mathscr{F}_{mj}^{l} N_j^{-\frac{1}{2}} \alpha_j h \sin \alpha_j h \ldots(24)$$

Using the additive theorem of Bessel functions the radiation wave potential ϕ_D^1 from cylinder 1 can be transformed into the form of an incident wave acting upon cylinder 2 given by,

$$\phi_I^2 = \sum_{m=-\infty}^{\infty} e^{-im(\pi/6)} \sum_{n=-\infty}^{\infty} (-1)^n a_{n0} H_{-n+m}(2kP)$$
$$\times\, e^{-in(\pi/6)} \frac{Z_0(z)}{dZ_0'(d)} \mathrm{Jm}\,(k\gamma_2)\, e^{im\theta_2}$$
$$+ \sum_{m=-\infty}^{\infty} \sum_{j=1}^{\infty} e^{-im(\pi/6)} \sum_{n=-\infty}^{\infty} a_{nj} K_{n+m}(2\alpha_j p)$$
$$\times\, e^{-in(\pi/6)} Z_j(z)\, \mathrm{Im}\,(\alpha_j \gamma_2)\, e^{im\theta_2}. \quad \ldots(25)$$

This incident wave on cylinder 2 has the same form as the wave described by equation (16). Accordingly, together with equation (17) we can determine again the diffraction from cylinder 2 induced by this incident wave. Thus we can calculate the effect which one cylinder has on the neighbouring cylinders and vice versa. An iterative procedure for calculating the interaction effect between the three cylinders is then possible and is described below.

If in the beginning, the incident waves acting upon each of the three cylinders are given in the form of equation (16) (to the case of the calculation of the wave forces on the cylinders fixed in waves), the diffracted waves from each cylinder caused by the first incident waves may be computed. The diffracted waves transform into new incident waves on the neighbouring cylinders which after the necessary computation results in a new diffracted wave induced by the new incident wave and so on. As a result of the repetition of this process we obtain three infinite series for the velocity potentials of the incident waves on each cylinder.

Assuming that the series converge then their sums have the form of equation (16). By using the coefficients $\mathscr{F}^l_{mj}$ and the equations (22) and (23), we can calculate the wave force on each cylinder induced by the series of incident waves. This is just the wave force on each of the three cylinders fixed in waves but also includes the interaction effects.

When each cylinder makes a periodic oscillation, the radiative waves from each cylinder described by equation (17) now occurs prior to the incident waves. In this case also infinite series of incident waves on each cylinder are obtained by the iteration procedure and the hydrodynamic force acting upon the oscillating three cylinder configuration may be determined by adding the force induced by the series of incident waves to the forces which act upon each cylinder performing the oscillation which is treated as a single cylinder.

Practically, it is difficult to prove the convergence of the infinite series of velocity potentials given by the iteration. But from some examples of the numerical calculations of their forms it appears that the terms decrease comparatively rapidly in magnitude. The velocity potential may be obtained by terminating the series after a certain number of terms provided that it satisfies the boundary condition on each cylinder with sufficient accuracy for practical application. This approximate velocity potential is satisfectory for the analysis.

A double series in the right hand side of the equation (17) may not be too important to the calculations, because it describes the potential of the standing wave and decreases rapidly away from the cylinder. Results of some numerical computation performed for various dimensions of cylinders show that it is not necessary to take into account the effect of the standing waves upon other cylinders when we compute the interaction effect in the hydrodynamic forces upon three cylinders satisfying the spacing condition $2P/a \geqslant 5$.

Some numerical examples were performed for the three cylinder configuration having the dimensions $2P/a = 5$ and (draft/a) = 3.65, where water depth d/a is 40.

Although semi-submergible drilling platforms like SEDCO135 have generally larger spacings between their legs, for instance of the order $2P/a \geqslant 10$, we adopt $2P/a = 5$ as a more severe case from the stand point of the interaction effect.

Figs 8 and 9 give the amplitudes of the wave exciting force in the x-direction (swaying force) and the force in

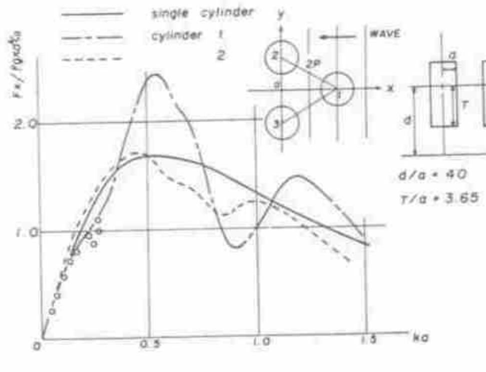

Fig. 8. Wave force in x-direction on three vertical cylinders ($2P/a = 5$)*

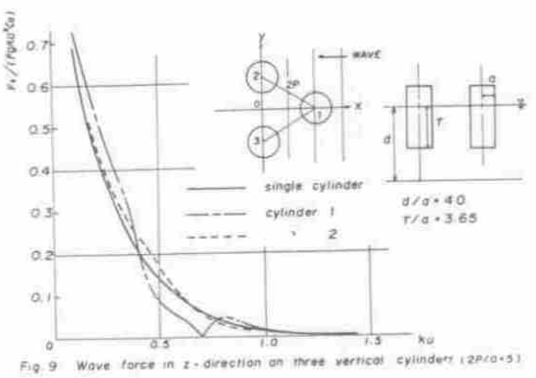

Fig. 9. Wave force in z-direction on three vertical cylinders ($2P/a = 5$)

z-direction upon each of the three cylinders fixed in a wave $\zeta_a e^{i(\omega k + kx)}$ propagating in the direction of the negative x direction. Solid lines represent the force on the single cylinder, broken lines the force on cylinder 1 and dotted lines the force on cylinder 2. The differences between either the solid lines, broken lines or dotted lines indicate the interaction effects and accordingly it may be concluded that the effect is negligible for the heave force but it is fairly large for the sway force. Here it should be noted that the magnitude of the interaction effect between vertical cylinders is roughly inversely proportional to the square root of the spacing between the cylinders and therefore the effect does not decrease much even if $2P/a = 10$. The results of Figs 8 and 9 include the effect of the standing wave term in equation (17).

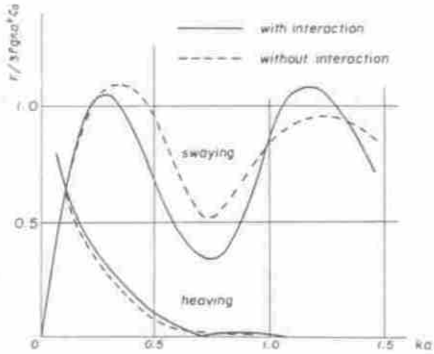

Fig. 10. Wave force on the whole three vertical cylinders ($2P/a = 5$)

Fig. 10 shows the sway force and the heave force on the three cylinder configuration. Comparison of the solid lines which include the interaction effect and the dotted lines which are the results without the effect but include the phase difference of the wave forces upon each cylinder show that the wave forces on the configuration are not so much influenced by the interaction effect.

Next the hydrodynamic force upon the three cylinder configuration making an oscillation in still water was determined. In this computation the standing wave terms in equation (17) were neglected. Figs 11 and 12

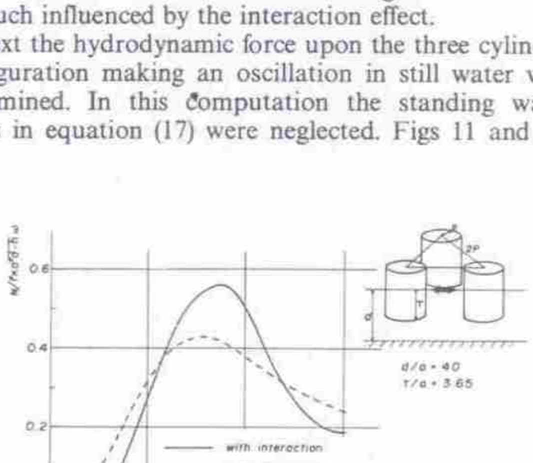

Fig. 11. Damping coefficient of swaying three vertical cylinders ($2P/a = 5$)

show the damping coefficient (damping/$\rho\omega\pi a^2(d - h)$) of the three cylinder configuration for the cases of heave and sway motion. Solid lines indicate the results with the interaction effect and dotted lines show the values for a single cylinder. That is, the values without the interaction effect. The interaction effect should be taken into account if one wants to perform exact calculation of the cylinders' sway motion in waves. For the heave motion the effect is also large, but one cannot be sure of its influence on the motion from these results because damping itself is very small.

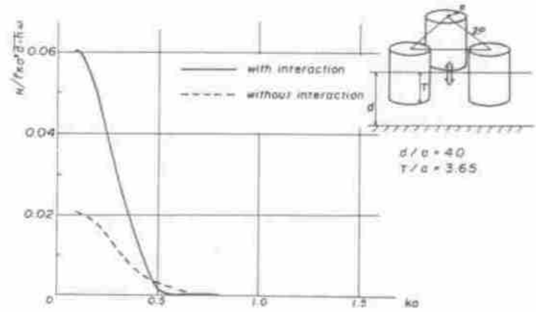

Fig. 12. Damping coefficient of heaving three vertical cylinders ($2P/a = 5$)

4 CONCLUSIONS

A method of calculating the hydrodynamic forces on horizontal and vertical multiple cylinders was developed and some numerical calculations were performed.

In the case of horizontal multiple cylinders which is taken as typical of platforms with multihulled vessel configurations, the wave exciting force on them and their motions in waves are much influenced by the interaction effect between elemental cylinders. This effect is also important in calculating the force on each element of the vertical multiple cylinders which are simplified configurations of some semi-submergible rigs with several legs. One may, however, neglect it for the force on the whole vertical multiple cylinders in most cases.

APPENDIX

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