Test Sample of Calculus–95

—Created by **Fu-Sen Lin**

1. Find the limit $\lim_{x\to 2}(x^2 - 2x - 1) =?$
(A) -2 (B) 0 (C) 1 (D) -1

2. For the limit $\lim_{x\to 1} \sqrt{2}\, x = \sqrt{2}$, give an appropriate δ when $\varepsilon = 2h$.
(A) $\delta = \sqrt{3}h$ (B) $\delta = 3h/2$ (C) $\delta = \sqrt{2}\, h$ (D) $\delta = 2h$

3. Find the left-hand limit or show that it does not exist: $\lim_{x\to 2^-} (x - [x]) =?$
(A) 3 (B) 0 (C) 1 (D) does not exist

4. Evaluate the limit $\lim_{x\to 0} \dfrac{\cos^2 t}{1 + \sin t} =?$
(A) 0 (B) 1 (C) -1 (D) $\dfrac{3}{2}$

5. Find the limit $\lim_{\theta\to\infty} \dfrac{\sin^2\theta}{\theta^2 - 5} =?$
(A) 0 (B) $\dfrac{1}{2}$ (C) ∞ (D) -1

6. Find the values of a and b so that the following function is continuous everywhere.

$$f(x) = \begin{cases} x+1, & \text{if } x < 1 \\ ax + b, & \text{if } 1 \le x < 2 \\ 3x, & \text{if } x \ge 2 \end{cases}$$

(A) $(a,\, b) = (4, -1)$ (B) $(a,\, b) = (2,\, 4)$
(C) $(a,\, b) = (4, -2)$ (D) $(a,\, b) = (-2,\, 1)$

7. Find the slope of the tangent line to the curve $y = x^2 - 1$ at the point $x = -2$.
(A) 3 (B) 4 (C) -5 (D) -4

8. If $f(x) = x^3 + 2x^2 + 1$ find the derivative at x.
(A) $3x^2 + 2x$ (B) $3x^2 + 4x$ (C) $3x^2 + 4$ (D) $3x^2 + 1$

9. The height s in feet of a ball above the ground at t seconds is given by $s = -16t^2 + 40t + 100$. What is its instantaneous velocity at $t = 2$?
(A) 21 feets/sec. (B) -21 feets/sec. (C) 24 feets/sec. (D) -24 feets/sec.

10. Let $y = x^2 \cos x$. Find $D_x y =$?
(A)$-2x \sin x$ (B) $2x \sin x + x^2 \cos x$
(C)$-x^2 \sin x + 2x \cos x$ (D)$\sin x - 2x \cos x$

11. Let $y = \dfrac{1}{(3x^2 + x - 3)^9}$ Find $D_x y =$?
(A) $\dfrac{9}{(3x^2 + x - 3)^8}$ (B) $\dfrac{-9}{(3x^2 + x - 3)^{10}}$ (C) $\dfrac{-9(6x + 1)}{(3x^2 + x - 3)^{10}}$ (D) $\dfrac{-9(6x + 1)}{(3x^2 + x - 3)^8}$

12. Suppose that $g(t) = at^2 + bt + c$ and $g(1) = 5$, $g'(1) = 3$, and $g''(1) = -4$. Find $(a, b, c) =$?
(A) $(-2, 6, 0)$ (B) $(2, 7, 0)$ (C) $(-2, 7, 0)$ (D) $(2, -7, 0)$

13. Find the point on the curve $x^2y - xy^2 = 2$ where the tangent line is vertical, that is, where $dx/dy = 0$.
(A) $(0, 0)$ (B) $(2, 1)$ (C) $(2, 0)$ (D) $(-1, 1)$

14. Assuming that a soap bubble retains its spherical shape as it expands, how fast is its radius increasing when its radius is 3 inches if air is blown into it at a rate of 3 cubic inches per second?
(A) $\dfrac{1}{2\pi}$ in./s (B) $\dfrac{1}{3\pi}$ in./s (C) $\dfrac{1}{6\pi}$ in./s (D) $\dfrac{1}{12\pi}$ in./s

15. Let $y = f(x) = x^3$. Find the value of dy when $x = 0.5, dx = 1$.
(A) 0.25 (B) 0.5 (C) 0.75 (D) 1

16. Find the point on the curve $y = \dfrac{x^2}{4}$, $0 < x \leq 2\sqrt{3}$, that is closest to the point $(0, 4)$.
(A)$(0, 0)$ (B)$(2\sqrt{2}, 2)$ (C)$(2\sqrt{3}, 3)$ (D)$(1, \frac{1}{4})$

17. Let $g(x) = (x + 1)(x - 2)$. Use the Monotonicity Theorem to find where g is decreasing?
(A) $(-\infty, -1]$ (B) $(-\infty, -\frac{1}{2}]$ (C) $(-\infty, \frac{1}{2}]$ (D) $(-\infty, 2]$

18. Find the maximum values of $F(x) = 6\sqrt{x} - 4x$ on the interval $[0, 4]$.
(A) 0 (B) $\dfrac{9}{16}$ (C) $\dfrac{9}{4}$ (D) 4

19. A small island is 2 miles from the nearest point P on the straight shoreline of a large lake. If a woman on the island can row a boat 3 miles per hour and can walk 4 miles per hour, where should the boat be landed in order to arrive at a town 10 miles down the shore from P in the least time?

(A) $\dfrac{4}{\sqrt{7}}$ miles (B) $\dfrac{5}{\sqrt{7}}$ miles (C) $\dfrac{6}{\sqrt{7}}$ miles (D) $\dfrac{7}{\sqrt{7}}$ miles

20. Let $f(x) = x^3 - 3x + 5$. For the following statements which one is correct?
(A) f is increasing on $(-1, 1)$ (B) f has exactly one point of inflection
(C) f has one maximum value but no minimum value (D) f has a horizontal asymptote. 21. If f is the quadratic function defined by $f(x) = \alpha x^2 + \beta x + \gamma,\ \alpha \neq 0$. Where is the number c of the Mean Value Theorem in $[a, b]$.
(A) $\dfrac{2a+b}{3}$ (B) $\dfrac{a+b}{2}$ (C) $\dfrac{a+2b}{3}$ (D) $\dfrac{a+3b}{4}$

22.Find $\int \sin^2 x\, dx$.
(A) $\dfrac{\sin^3 x}{3} + c$ (B) $2\sin x \cos x + c$ (C) $x - \dfrac{\sin x}{4} + c$ (D)$\dfrac{x}{2} - \dfrac{\sin 2x}{4} + c$

23. If $dy/dx = x^2 + 1$, find the particular solution that satisfies $y = 1$ at $x = 1$.
(A) $y = \dfrac{x^3}{3} + x - \dfrac{1}{3}$ (B) $y = x^3 - x + 1$ (C) $y = -\dfrac{x^3}{2} + x + \dfrac{3}{2}$ (D) $y = 2x^3 - 1$

24. Find $\sum_{i=1}^{n} (2i^2 - 3i + 1) =?$
(A) $2^n - 3n + 1$ (B) $\dfrac{4n^3 - 3n^2 - n}{6}$ (C) $\dfrac{4n^3 - 3n^2 + 11n}{6}$ (D) $\dfrac{4n^3 + 2n^2 - n}{6}$

25. Suppose that an object is traveling along the t-axis in such a way that its velocity at time t seconds is $v = t + 2$ feet per second. How far did it travel between $t = 0$ and $t = 1$?
(A) $\dfrac{1}{2}\ ft$ (B) $\dfrac{3}{2}\ ft$ (C) $\dfrac{5}{2}\ ft$ (D) $\dfrac{7}{2}\ ft$

26. Let $f(x) = \dfrac{x^2}{2} + x$ on $[-2, 2]$. Calculate the Riemann sum $\sum_{i=1}^{n} f(\bar{x}_i)\, \Delta x_i$ for the partition obtained by dividing $[-2, 2]$ into eight equal subintervals and $\bar{x}_i$ is the midpoint of each subinterval.
(A) 2.625 (B) 2.615 (C) 2.605 (D) 2.595

27. Let $f(x) = 3 + |x - 3|$. Use the Interval Additive Property and linearity to evaluate $\int_0^4 f(x)dx =?$
(A) 15 (B) 16 (C) 17 (D) 18

28. Evaluate $\int_{-2}^{4} (2[x] - 3|x|)dx =?$
(A) -24 (B) -21 (C) 21 (D) 24

29. Let $\int \sqrt{3x+2}\, dx$. Use the method of substitution to find the indefinite integrals.

(A) $\frac{2}{9}(3x+2)^{\frac{1}{2}}+c$ (B) $\frac{2}{9}(3x+2)^{\frac{3}{2}}+c$ (C) $\frac{1}{3}(3x+2)^{\frac{1}{2}}+c$ (D) $\frac{1}{3}(3x+2)^{\frac{3}{2}}+c$

30. Find the area of the region between the curve $y=x^2-2x$ and $y=-x^2$.
(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

31. The base of a solid is bounded by one arch of $y=\sqrt{\cos x}\,, -\frac{\pi}{2}\le x\le\frac{\pi}{2}$, and the x-axis. Each cross section perpendicular to the x-axis is a square sitting on this base. Find the volume of the solid.
(A) -2 (B) -1 (C) 1 (D) 2

32. Find the volume of the solid generated by revolving the region R, bounded by the curves $y=1/x$, $x=1$, $x=4$, $y=0$, about the y-axis.
(A) 4 (B) 4π (C) 6 (D) 6π

33. Find the length of curve $y=\int_1^x \sqrt{u^3-1}\,du,\ 1\le x\le 2$.
(A) $\frac{8\sqrt{2}-2}{5}$ (B) $\frac{8\sqrt{2}-2}{3}$ (C) $\frac{4\sqrt{2}-1}{5}$ (D) $\frac{4\sqrt{2}-1}{3}$

34. Find the length of the curve $y=\frac{x^2}{4}-\ln\sqrt{x}\,, 1\le x\le 2$.
(A) $\frac{3}{4}+\frac{1}{2}\ln 2$ (B) $1+\frac{1}{2}\ln 2$ (C) $\frac{3}{4}+\frac{1}{4}\ln 2$ (D) $1+\frac{1}{4}\ln 2$

35. Let $f(x)=\frac{x-1}{x+1}$. Find the inverse $f^{-1}(x)=?$
(A) $\frac{1-x}{1+x}$ (B) $\frac{x-1}{x+1}$ (C) $\frac{x+1}{x-1}$ (D) $\frac{1+x}{1-x}$

36. Find the integral $\int_1^2 \frac{e^{3/x}}{x^2}\,dx$.
(A) $-\frac{1}{3}e^{3/2}+\frac{1}{3}e^3$ (B) $\frac{1}{3}e^{3/2}+\frac{1}{3}e^3$ (C) $\frac{1}{3}e^{3/2}-\frac{1}{3}e^3$ (D) $-\frac{1}{3}e^{3/2}-\frac{1}{3}e^3$

37. Consider $f(x)=\frac{a^x-1}{a^x+1}$ for fixed a, $a\ne 1$. Find a formula for $f^{-1}(x)=?$
(A) $\log_a\frac{1+x}{1-x}$ (B) $\log_a\frac{1-x}{1+x}$ (C) $\log_a\frac{x+1}{x-1}$ (D) $\log_a\frac{x-1}{x+1}$

38. The population of a certain country is growing at 3.2% per year; that is, if it is A at the beginning of a year, it is $1.032A$ at the end of that year. Assuming that it is 4.5 million now, what will it be at the end of 10 years?
(A) 4.64 million (B) 4.79 million (C) 6.17 million (D) 105 million

39. Solve differential equation $xy'+(1+x)y=e^{-x}$, with $y=0$ when $x=1$.
(A) $y=e^x\left(1-\frac{1}{x}\right)$ (B) $y=e^x(1-x)$
(C) $y=e^{-x}(1-x)$ (D) $y=e^{-x}\left(1-\frac{1}{x}\right)$

40. The lower edge of a wall hanging, 10 feet in height, is 2 feet above the observer's eye level. Find the ideal distance b to stand from the wall for viewing the hanging; that is, find b that maximizes the angle subtended at the viewer's eye.
(A) $2\sqrt{6}$ (B) $2\sqrt{5}$ (C) $2\sqrt{3}$ (D) $2\sqrt{7}$

41. Find $D_x y$ if $y = \sinh x \cosh 4x$.
(A) $4\sinh x \sinh 4x + \cosh x \cosh 4x$ (B) $\sinh x \sinh 4x + 4\cosh x \cosh 4x$
(C) $4\cosh x \sinh 4x + \sinh x \cosh 4x$ (D) $\cosh x \sinh 4x + 4\sinh x \cosh 4x$

42. Evaluate $\displaystyle\int \cosh 3x \, dx = ?$
(A) $3\sinh 3x + c$ (B) $-3\sinh 3x + c$ (C) $\frac{1}{3}\sinh 3x + c$ (D) $-\frac{1}{3}\sinh 3x + c$

43. The region bounded by $y = x + \sin x$, $y = 0$, and $x = \pi$ is revolved about the x-axis. Find the volume of the resulting solid.
(A) $\dfrac{1}{3}\pi^4 + \dfrac{5}{2}\pi^2$ (B) $\dfrac{5}{2}\pi^4 + \dfrac{1}{3}\pi^2$ (C) $\dfrac{1}{3}\pi^3 + \dfrac{5}{2}\pi$ (D) $\dfrac{5}{2}\pi^3 + \dfrac{1}{3}\pi$

44. Evaluate $\displaystyle\int_0^{\pi} \frac{\pi x - 1}{\sqrt{x^2 + \pi^2}} \, dx = ?$
(A) $(\sqrt{2} - 1)\pi^2 + \ln(\sqrt{2} + 1)$ (B) $(\sqrt{2} - 1)\pi^2 - \ln(\sqrt{2} + 1)$
(C) $(\sqrt{2} + 1)\pi^2 + \ln(\sqrt{2} - 1)$ (D) $(\sqrt{2} + 1)\pi^2 - \ln(\sqrt{2} - 1)$

45. Use integration by parts to evaluate $\displaystyle\int_{\pi/6}^{\pi/4} x \sec^2 x \, dx = ?$
(A) $\dfrac{\pi}{4} - \dfrac{\pi}{6\sqrt{3}} + \dfrac{1}{2}\ln\dfrac{2}{3}$ (B) $\dfrac{\pi}{3} - \dfrac{\pi}{2\sqrt{3}} + \dfrac{1}{2}\ln\dfrac{2}{3}$
(C) $\dfrac{\pi}{4} + \dfrac{\pi}{6\sqrt{3}} - \dfrac{1}{2}\ln\dfrac{2}{3}$ (D) $\dfrac{\pi}{3} + \dfrac{\pi}{2\sqrt{3}} - \dfrac{1}{2}\ln\dfrac{2}{3}$

46. Evaluate $\displaystyle\int_{\pi/6}^{\pi/4} \frac{\cos x}{\sin x(\sin^2 x + 1)^2} dx = ?$
(A) $\dfrac{1}{2}\ln\dfrac{5}{2} - \dfrac{3}{20}$ (B) $\ln\dfrac{5}{2} - \dfrac{3}{20}$ (C) $\dfrac{1}{2}\ln\dfrac{5}{2} + \dfrac{3}{20}$ (D) $\ln\dfrac{5}{2} + \dfrac{3}{20}$

47. Evaluate $\displaystyle\lim_{x \to 0} \frac{e^x - \ln(1+x) - 1}{x^2} = ?$
(A) -1 (B) 0 (C) 1 (D) does not exist.

48. Evaluate $\displaystyle\lim_{x \to 0} \left(\csc^2 x - \frac{1}{x^2}\right)^2 = ?$
(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{9}$ (D) does not exist

49. Find the area of the region under the curve $y = \dfrac{2}{4x^2 - 1}$ and above x-axis for $x \neq 1$.
(A) $\dfrac{\ln 3}{2}$ (B) $\ln 3$ (C) $\dfrac{1}{2}$ (D) $\ln 4$

50. Evaluate the improper integral $\int_0^1 \frac{x}{\sqrt[3]{1-x^2}}\,dx$ or show that it diverge.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) diverge

51. Find $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\sin \frac{k}{n}\right) \frac{1}{n} = ?$

(A) $\sin 1$ (B) $1-\sin 1$ (C) $\cos 1$ (D) $1-\cos 1$

52.A ball is dropped from a height of 100 feet. Each time it hits the floor, it rebounds to $\frac{2}{3}$ its previous height. Find the total distance it travels before coming to rest.

(A) 250 feet (B) 500 feet (C) 750 feet (D) 1000 feet

53. For what values of p does $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converge?

(A) $-\frac{3}{2}$ (B) 0 (C) 1 (D) $\frac{3}{2}$

54. In the following series which one is convergent?

(A) $\sum_{k=1}^{\infty} \left[\left(\frac{1}{2}\right)^k + \frac{k-1}{2k+1}\right]$ (B) $\sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{3}\right)$ (C) $\sum_{k=1}^{\infty} k^2 e^{-k^3}$ (D) $\sum_{k=100}^{\infty} \frac{k^{1/3}}{\sqrt{k+3}}$

55. Let $1 + x + \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} + \frac{x^4}{\sqrt{4}} + \frac{x^5}{\sqrt{5}} + \cdots$. Find the convergence set of the given power series.

(A) $-2 \le x \le -1$ (B) $-1 < x < 1$ (C) $-1 \le x < 1$ (D) $-1 < x \le 1$

56. If the power series $\sum_{n=1}^{\infty} nx^n$ converges, find the sum function.

(A) $\frac{1}{1-x}$ (B) $\frac{1}{(1-x)^2}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{(1-x)^2}$

57. Find the Maclaurin series for $f(x) = \int_0^x \frac{e^{t^2}-1}{t^2}\,dt$ and use it to calculate $f^{(4)}(0)$.

(A) -1 (B) 0 (C) 1 (D) $\frac{1}{30}$

58. Find the focus of the parabola $x^2 - 6x + 4y + 3 = 0$.

(A) $(3, \frac{3}{2})$ (B) $(3, \frac{1}{2})$ (C) $(3, -\frac{1}{2})$ (D) $(3, 0)$

59. Find the point of $x^2 + 14xy + 49y^2 = 100$ that are closest to the origin when $x > 0$ and $y > 0$.

(A) $(0, 0)$ (B) $(\frac{1}{5}, \frac{7}{5})$ (C) $(\frac{3}{2}, \frac{7}{3})$ (D) $(8, 1)$

60. Earth's orbit around the sun is an ellipse of eccentricity 0.0167 and major diameter 185.8 million miles. Find its perihelion.
(A) 92.9 (B) 91.3 (C) 93.1 (D) 92.3

61. Where is the symmetry of the polar equation $r = 4 - 3\cos\theta$?
(A) x-axis (B) y-axis (C) original (D) both axes

62. One leaf of the four-leaved rose $r = 3\cos 2\theta$, and find the area of the region enclosed by it.
(A) $\frac{9\pi}{4}$ (B) $\frac{9\pi}{5}$ (C) $\frac{9\pi}{7}$ (D) $\frac{9\pi}{8}$

Table 1: Answer Key

1	2	3	4	5	6	7	8	9	10
D	C	C	B	A	C	D	B	D	C
11	12	13	14	15	16	17	18	19	20
C	C	B	D	C	B	C	C	C	B
21	22	23	24	25	26	27	28	29	30
B	D	A	B	C	A	C	A	B	A
31	32	33	34	35	36	37	38	39	40
D	D	A	A	D	D	A	C	D	A
41	42	43	44	45	46	47	48	49	50
A	C	A	B	A	A	C	C	A	C
51	52	53	54	55	56	57	58	59	60
D	B	D	C	C	D	B	B	B	B
61	62								
A	D								