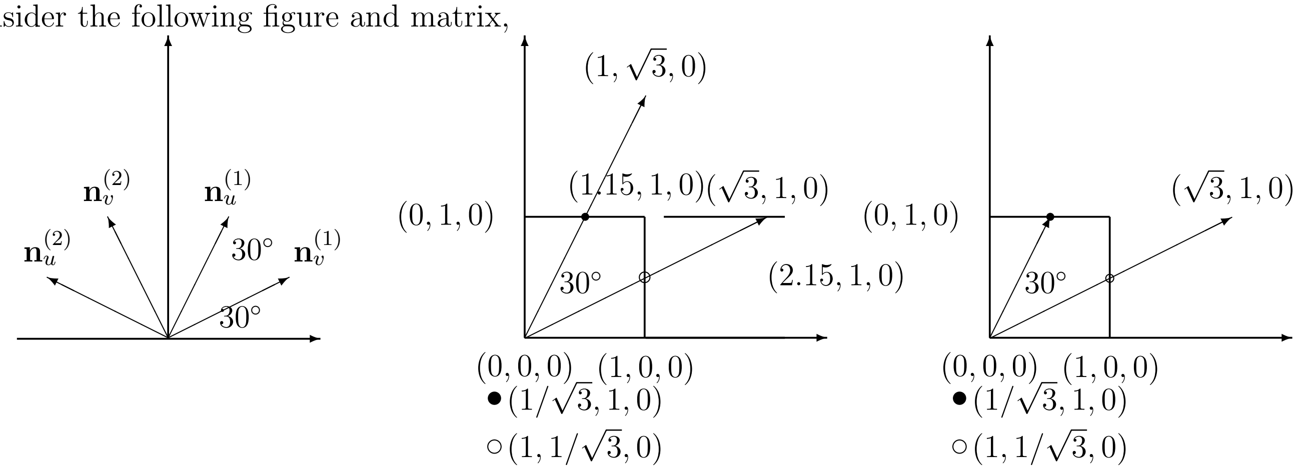
國立臺灣海洋大學河海工程學系 2000 工程數學 (一) 第一次大考解答

1. Given two lines,

$$\mathbf{u} = \lambda(\mathbf{i}+\mathbf{j}+\mathbf{k}), \;\; \mathbf{v} = \mathbf{k} + \mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$$

determine the minimum distance between the two lines.(5 %) Also, find the specific two points on those two lines make minimum distance. (5 %)
Ans: The two points are $(\lambda,\lambda,\lambda)$ and $(\mu, 2\mu, 3\mu+1)$ where $\lambda = -2/3, \mu = -1/2$. The minimum distance is $1/\sqrt{6}$.
2. Consider the following figure and matrix,



$$F = RU = VR = \begin{bmatrix} 1 & 2/\sqrt{3} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \sqrt{F^TF} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & \frac{5}{2\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \;\; V = \begin{bmatrix} 5\sqrt{3}/6 & 1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \;\; R = FU^{-1} = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Decompose the $\mathbf{P}$ vector into the combination of $\mathbf{n}_u^{(1)}$ and $\mathbf{n}_u^{(2)}$, i.e.,

$$\mathbf{P} = \alpha\mathbf{n}_u^{(1)} + \beta\mathbf{n}_u^{(2)}$$

determine α and β. (5 %) Also, calculate and plot the following vectors, (1). $U\alpha\mathbf{n}_u^{(1)}$ and $RU\alpha\mathbf{n}_u^{(1)}$ (5 %) (2). $U\beta\mathbf{n}_u^{(2)}$ and $RU\beta\mathbf{n}_u^{(2)}$ (5 %) (3). $R\alpha\mathbf{n}_u^{(1)}$ and $VR\alpha\mathbf{n}_u^{(1)}$ (5 %) (4). $R\beta\mathbf{n}_u^{(2)}$ and $VR\beta\mathbf{n}_u^{(2)}$. (5 %) (5). Sum the vectors of $RU\alpha\mathbf{n}_u^{(1)} + RU\beta\mathbf{n}_u^{(2)}$ (5 %) (6). Sum the vectors of $VR\alpha\mathbf{n}_u^{(1)} + VR\beta\mathbf{n}_u^{(2)}$. (5 %)
Ans: $\alpha = 1/2, \beta = -\sqrt{3}/2$.
(1). $\{\sqrt{3}/4, 3/4, 0\}$ (60^o), $\{3/4, \sqrt{3}/4, 0\}(30^o)$,
(2). $\{\sqrt{3}/4, -1/4, 0\}(-30^o)$, $\{1/4, -\sqrt{3}/4, 0\}(-60^o)$,
(3). $\{\sqrt{3}/4, 1/4, 0\}(30^o)$, $\{3/4, \sqrt{3}/4, 0\}(30^o)$,
(4). $\{\sqrt{3}/4, -3/4, 0\}(-60^o)$, $\{1/4, -\sqrt{3}/4, 0\}(-60^o)$,
(5). (1,0,0), (0^o), (6). (1,0,0) (0^o)

3. Fill in the direction of the following vectors in table by (1), (2) $\cdots$ (7).

Table 1: Direction of the following vectors. (10 %)

vector	$R^T\mathbf{n}_u^{(1)}$	$VR^3\mathbf{n}_v^{(2)}$	$\sqrt{U}\mathbf{n}_u^{(1)}$	$e^V\mathbf{n}_v^{(1)}$	$F^T\mathbf{n}_v^{(1)}$
direction	4	2	3	2	3

4. Given $x^2 - xy + y^2 = 1$, rewrite to the quadratic form $\mathbf{v}^T[A]\mathbf{v} = 1$, where $\mathbf{v} = \{x, y\}$ and $[A] = [A]^T$. Find $[A]$ (5 %) and determine its eigenvalues and eigenvectors. (5 %) Explain the geometric meaning for the eigenvalues and eigenvectors. (5 %)
Ans:

$$A = \begin{bmatrix} 1 & -1/2 \\ -1/2 & 1 \end{bmatrix}$$

Eigenvalues are $1/2$ and $3/2$. Eigenvectors are $\{1, 1\}$ and $\{1, -1\}$. The two axises are $1/\sqrt{\lambda_1}$ and $1/\sqrt{\lambda_2}$. The two eigenvectors are the principal axises.

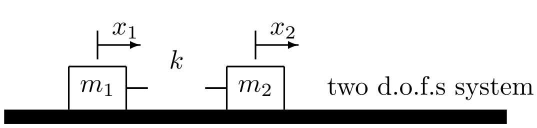
5. Fill in the following table.

Table 2: Direction of the following vectors. (10 %)

matrix	$[A] = [A]^T$	$[A][A]^\dagger = [A]^\dagger[A]$	$[A] = -[A]^T$	$[A] = [A]^\dagger$	$[A] = -[A]^\dagger$	$[A][A]^\dagger = [I]$
$[A]$	symmetric	normal	skew symmetric	Hermitian	skew Hermitian	unitary

6. Choose the right sentences. (10 %)
(1). Hermitian, anti-Hermitian and unitary matrices are all normal matrices.
(2). The absolute values of eigenvalues for any unitary matrix must be one.
(3). Real symmetric matrix must have real eigenvalues.
(4). The eigenvalues of anti-symmetric real matrix must be pure imaginary or zero.
(5). The eigenvectors of a Hermitian matrix w.r.t. different eigenvalues are orthogonal.
Ans: All are right.
7. Given a vibration system with two degrees of freedom with the following governing equations:

$$\begin{cases} m_1\ddot{x}_1(t) = k(-x_1(t) + x_2(t)) \\ m_2\ddot{x}_2(t) = k(x_1(t) - x_2(t)) \end{cases}$$



(1) Assume $x_1(t) = \alpha\, cos(\omega t)$, $x_2(t) = \beta\, cos(\omega t)$ are the solutions for the vibration, reduce the governing equations into eigen equations in the form of $\mathbf{Kp} = \omega^2\mathbf{p}$, where $\mathbf{p} = \{x_{10}, x_{20}\}^T$. Find the matrix of $\mathbf{K}$. (5%) (For simplicity, setting $m_1 = m_2 = k = 1$)

(2) Find the eigenvalues and eigenvectors of $\mathbf{K}$. (5%)

(3) Find the Rayleigh quotients of the eigenvectors. (5%)
Ans:

$$K = \left[\begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right]$$

The eigenvalues are 0 and 2. The eigenvectors are $\{1, 1\}$ and $\{1, -1\}$. The two Rayleigh quotients are 0 and 2.