

1 (a). Given a spiral curve, we can describe by time-like parameter as follows:

$$x(t) = \cos(t), y(t) = \sin(t), z(t) = t$$

Please describe the curve by using space-like parameter (arc length s). (5 %)

1 (b). Plot the curve from the starting point of $(1, 0, 0)$? (5 %)

1 (c). What is the arc length of the curve from $t = 0$ to $t = 2\pi$? (5 %)

1 (d). Please determine the radius of curvature for ρ and σ as shown below: (5 %)

$$\begin{Bmatrix} \dot{\hat{\mathbf{t}}} \\ \dot{\hat{\mathbf{n}}} \\ \dot{\hat{\mathbf{b}}} \end{Bmatrix} = \begin{bmatrix} 0 & \frac{1}{\rho} & 0 \\ -\frac{1}{\rho} & 0 & \frac{1}{\sigma} \\ 0 & -\frac{1}{\sigma} & 0 \end{bmatrix} \begin{Bmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{Bmatrix} \quad (1)$$

1 (e). Determine (5 %)

$$\left(\frac{d\mathbf{r}}{ds} \times \frac{d^2\mathbf{r}}{ds^2} \right) \cdot \frac{d^3\mathbf{r}}{ds^3} = ?$$

2. Given the radial position vector (\mathbf{r}) and radial basis function ($\phi(r)$)

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

$$r = \sqrt{x^2 + y^2 + z^2},$$

$$\mathbf{b} = \omega\mathbf{k},$$

find (20 %)

$$\nabla \cdot [\phi(r)\mathbf{r}] = ? \phi(r) + r \frac{d\phi(r)}{dr}$$

$$\nabla \times [\phi(r)\mathbf{r}] = ?$$

$$\nabla \cdot \mathbf{r} = ?$$

$$\nabla \times (\mathbf{b} \times \mathbf{r}) = ?$$

3. Given a cone with $z = 2\sqrt{x^2 + y^2}$, find the normal vector of the point $(1, 0, 2)$ on the cone. (5 %)

4. The temperature space field at the point (x, y, z) in space is inversely proportional to the square of the distance from (x, y, z) to the origin $(0, 0, 0)$, i.e., $T(x, y, z) = 1/(x^2 + y^2 + z^2)$. Find the rate of change of T at $(2, 3, 3)$ in the direction of $(3, 1, 1)$. (5 %) In which direction from $(2, 3, 3)$ does the temperature T increase most rapidly? (5 %) At the point $(2, 3, 3)$ what is the maximum rate of change of T ? (5 %)

5. For the cylindrical polar coordinates (ρ, ϕ, z) , determine (20 %) (1). $\partial\hat{\mathbf{e}}_\rho/\partial\rho$, (2). $\partial\hat{\mathbf{e}}_\phi/\partial\rho$, (3). $\partial\hat{\mathbf{e}}_\rho/\partial\phi$, (4). $\partial\hat{\mathbf{e}}_\phi/\partial\phi$.

6. Please fill in the following blanks.

Table 1: Three coordinate systems (15 %)

1	Curvilinear coordinate system	(x, y, z) system	(ρ, ϕ, z) system	(r, θ, ϕ) system
2	$x = x(u_1, u_2, u_3)$	$x = x$	$x = \rho \cos(\phi)$	$x = r \sin(\theta) \cos(\phi)$
3	$y = y(u_1, u_2, u_3)$	$y = y$	$y = \rho \sin(\phi)$	$y = r \sin(\theta) \sin(\phi)$
4	$z = z(u_1, u_2, u_3)$	$z = z$	$z = z$	$z = r \cos(\theta)$
5	$h_1 = \frac{\partial \mathbf{r}}{\partial u_1} $	1		
6	$h_2 = \frac{\partial \mathbf{r}}{\partial u_2} $	1		
7	$h_3 = \frac{\partial \mathbf{r}}{\partial u_3} $	1		
8	$(ds)^2$			
9	dV			
10	$\nabla \Phi$			

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $(ds)^2 = (\mathbf{dr}) \cdot (\mathbf{dr})$, $dV = h_1 h_2 h_3 du_1 du_2 du_3$ and $\nabla \Phi = \frac{1}{h_1} \frac{\partial \Phi}{\partial u_1} \hat{\mathbf{e}}_1 + \frac{1}{h_2} \frac{\partial \Phi}{\partial u_2} \hat{\mathbf{e}}_2 + \frac{1}{h_3} \frac{\partial \Phi}{\partial u_3} \hat{\mathbf{e}}_3$.

海大河工系 2000 第二次大考解答 by Chen for vector calculus

存檔:big02.ctx 建檔:Dec./20/2000 A3 (75 份)