海洋大學河海工程學系2007 工程數學 (一) 二B 班積分因子

1. Solve the following ordinary differential equations.

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2} \tag{1}$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \tag{2}$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2} \tag{3}$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy} \tag{4}$$

Plot the solutions and indicate the orthogonal relationships.

ODE	Eq.(1)	Eq.(2)	Eq.(3)	Eq.(4)
Separable	No	No	No	No
Exact	No	No	Yes	Yes
Integrating factor	y^{-2}	x^{-2}	1	1
Homogeneous	Yes	Yes	Yes	Yes
Solution	$x^2 + y^2 = 2ky$	$x^2 + y^2 = 2kx$	$x^2y - \frac{y^3}{3} = c$	$\frac{x^3}{3} - xy^2 = c$
Orthogonality	Eq.(2)	Eq.(1)	Eq.(4)	Eq.(3)

Figures: