線性獨立、線性相關與 Wronskian

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1. Vector space $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4} \cdots$
2. function space $f_1, f_2, f_3, \cdots$
3. Linear independence for vector space

   $$\mathbf{v_1} = (1,0), \mathbf{v_2} = (0,1)$$

4. Linear dependence for vector space

   $$\mathbf{v_1} = (1,1), \mathbf{v_2} = (2,2)$$

5. Linear independence for function space

   $$f_1(x) = e^x, f_2(x) = e^{-x}$$

6. Linear dependence for function space

   $$f_1(x) = e^x, f_2(x) = e^{-x}, f_3(x) = cosh(x)$$

7. Linear independence for function space and vector space

   $$c_1 f_1(x) + c_2 f_2(x) + \cdots + c_n f_n(x) = 0 \ \ implies \ \ c_1 = c_2 = c_3 = \cdots = c_n = 0$$

   $$c_1 \mathbf{v}_1(x) + c_2 \mathbf{v}_2(x) + \cdots + c_n \mathbf{v}_n = 0 \ \ implies \ \ c_1 = c_2 = c_3 = \cdots = c_n = 0$$

8. Linear dependence for function space and vector space:

   Functions $f_1, f_2, \cdots f_n$ are linearly dependent on an interval I if and only if there exists $c_1, c_2, \cdots, c_n$ at least one of which is not zero, such that every x in I,

   $$c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + c_4 f_4(x) + \cdots + c_n f_n(x) = 0$$

9. inner product of vectors, determinant of vectors and Wronskian of function space

   $$\begin{vmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{vmatrix}, \quad \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix}$$