二階微分方程式-含複數根

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1. Given a second order ODE

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = 0$$

2. General solution

$$x(t) = e^{st}$$

3. s must satisfy

$$s^2 + 2\xi\omega s + \omega^2 = 0$$

4. Two roots are

$$s_1 = (-\xi + \sqrt{\xi^2 - 1})\omega$$

$$s_2 = (-\xi - \sqrt{\xi^2 - 1})\omega$$

5. If $0 < \xi < 1$, two solutions are

$$x_1(t) = x_{1r}(t) + i x_{1i}(t) = e^{-\xi\omega t} cos(\sqrt{1-\xi^2}t) + i e^{-\xi\omega t} sin(\sqrt{1-\xi^2}t)$$

$$x_2(t) = x_{2r}(t) + i x_{2i}(t) = e^{-\xi\omega t} cos(\sqrt{1-\xi^2}t) - i e^{-\xi\omega t} sin(\sqrt{1-\xi^2}t)$$

6. Substituting the two solutions into the ODE, we have

$$\ddot{x}_{1r}(t) + 2\xi\omega\dot{x}_{1r}(t) + \omega^2 x_{1r}(t) + i\{\ddot{x}_{1i}(t) + 2\xi\omega\dot{x}_{1i}(t) + \omega^2 x_{1i}(t)\} = 0 + 0i$$

$$\ddot{x}_{2r}(t) + 2\xi\omega\dot{x}_{2r}(t) + \omega^2 x_{2r}(t) + i\{\ddot{x}_{2i}(t) + 2\xi\omega\dot{x}_{2i}(t) + \omega^2 x_{2i}(t)\} = 0 + 0i$$

7. Two complementary solutions are

$$x_{1c}(t) = e^{-\xi\omega t} cos(\sqrt{1-\xi^2}t)$$

$$x_{2c}(t) = e^{-\xi\omega t} sin(\sqrt{1-\xi^2}t)$$