Solving another comp. sol. using Wronskian

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Given y_1 is one complementary solution for ODE,

$$y_1''(x) + a(x)y_1'(x) + b(x)y_1(x) \quad = \quad 0 \tag{1}$$

solve another complementary solution y_2. By setting the Wronskian

$$W(x) = W(y_1, y_2) = y_1 y_2' - y_1' y_2,$$

$W(x)$ satisfies the following first ODE

$$W'(x) + a(x)W(x) = 0$$

The solution is

$$W(x) = ke^{-\int a(x)dx}$$

Therefore, we have first order ODE for y_2 as follows:

$$y_2' - \frac{y_1'}{y_1} y_2 = \frac{k}{y_1} e^{-\int a(x)dx} \tag{2}$$

Example:

$$y'' + 3y' = 2y = 0$$

Sol: $y_1 = e^{-x}$, $y_2(x)$ satisfies

$$y_2' - \frac{-e^{-x}}{e^{-x}} y_2 = \frac{k}{e^{-x}} e^{-\int 3dx}$$

$$y_2' + y_2 = ke^{-2x}$$

$$y_2 = ce^{-x} + Ke^{-2x}$$

Exercise:

$$x^2 y''(x) - 4xy' - 6y = -6, \tag{3}$$

(a) if $y_1(x) = \frac{1}{x}$ is one of the complementary solution, solve y_2 using Wronskian.