Two sources: external excitation or free vibration with two near frequencies

case 2: free vibration with two near natural frequencies (pendulum)

$$\omega_1 = \sqrt{\frac{g}{L}}$$

$$\omega_2 = \sqrt{\frac{g}{L} + 2\frac{k}{m}\frac{a^2}{L^2}}$$

Initial conditions to determine coefficients

$$\theta_1(0) = \theta_0, \dot{\theta}_1(0) = 0$$

$$\theta_2(0) = 0, \dot{\theta}_2(0) = 0$$

General solution

$$\theta_1(t) = \frac{1}{2}\theta_0 cos(\omega_1 t) + \frac{1}{2}\theta_0 cos(\omega_2 t)$$

$$\theta_2(t) = \frac{1}{2}\theta_0 cos(\omega_1 t) - \frac{1}{2}\theta_0 cos(\omega_2 t)$$

General solution for beating as $\frac{ka^2}{mL^2}$ is very small

$$\theta_1(t) = \theta_0 cos(\frac{\omega_2 - \omega_1}{2})t \; cos(\frac{\omega_2 + \omega_1}{2})t$$

$$\theta_2(t) = \theta_0 sin(\frac{\omega_2 - \omega_1}{2})t \; sin(\frac{\omega_2 + \omega_1}{2})t$$

As $\frac{ka^2}{mL^2} = 0$, reduce to simple pendulum.

$$\theta_1(t) = \theta_0 cos(\omega_1 t)$$

$$\theta_2(t) = 0$$

Excitation case by support motion instead of external excitation