

# 振動三部曲-激發、拍擊與共振(using Laplace transform)

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Two sources: external excitation or free vibration with two near frequencies

External excitation

$$\ddot{x}(t) + \omega^2 x(t) = \cos(\omega t)$$

Initial conditions:

$$x(0) = 0, \dot{x}(0) = 0$$

By taking Laplace transform, we have

$$(s^2 + \omega^2)X(s) = \frac{s}{s^2 + \omega^2}$$

Method 1: using  $\mathcal{L}\{tf(t)\} = -F'(s)$

$$X(s) = \frac{s}{(s^2 + \omega^2)^2}$$

$$X(s) = \frac{-1}{2} \frac{-2s}{(s^2 + \omega^2)^2}$$

$$X(s) = \frac{1}{2\omega} (-1) \frac{d}{ds} \left\{ \frac{\omega}{(s^2 + \omega^2)} \right\}$$

Method 2: using convolution

$$X(s) = \frac{1}{(s^2 + \omega^2)} \frac{s}{(s^2 + \omega^2)}$$

$$x(t) = \int_0^t \frac{\sin(\omega(t - \tau))}{\omega} \cos(\omega\tau) d\tau$$

Total solution:

$$x(t) = \frac{t}{2\omega} \sin(\omega t) \rightarrow \text{resonance}$$