工程數學(一) - 2B 2003.9~2004.1

If $y = x$ **is one complementary solution of** $(1-x^2)y'' - 2xy' + 2y = 0$**, find the other one. (1)** $y_2 = y_1 u_1$ **(2)Wronskian (3)** $\left(\frac{y_2}{y_1}\right)'$

(1) $y_2 = y_1 u$

$$(1-x^2)xu'' + (2-4x^2)u' = 0$$

$$u' = v$$

$$(1-x^2)xv' + (2-4x^2)v = 0$$

$$v = \frac{k}{x^2(1-x^2)}$$

$$u = \int v dx = \frac{-1}{x} + \ln\sqrt{\frac{1+x}{1-x}} + c$$

$$y_2 = 1 + x\ln\sqrt{\frac{1+x}{1-x}}$$

(2) *Wronskain*

$$w' - \frac{2x}{1-x^2}w = 0 \rightarrow w(x) = \frac{1}{1-x^2}$$

$$w = y_2 y_1' - y_1 y_2' = \frac{1}{1-x^2}$$

$$-xy_2' + y_2 = \frac{1}{1-x^2}$$

$$y_2 = 1 + x\ln\sqrt{\frac{1+x}{1-x}}$$

(3) $(\frac{y_2}{y_1})' = \frac{-y_2 y_1' + y_1 y_2'}{y_1^2} = \frac{-w}{y_1^2}$

$$y_2 = y_1 \int \frac{1}{y_1^2} w dx$$

$$y_2 = 1 + x\ln\sqrt{\frac{1+x}{1-x}}$$