Review: The solution of linear equation using the decomposition matrices of K matrix.

$$\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}1&0&0\\.5&1&0\\-1&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} .5&-1&0\end{matrix}&1\end{matrix}\right]\left[\begin{matrix}\begin{matrix}12&0&0\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0 &0&0\end{matrix}&0\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right] K=LDR=R'DR$$

 $KU=P \rightarrow R^{'}DRU=P \rightarrow DRU=R^{'}^{-1}P=Z \rightarrow RU=D^{-1}R^{'}^{-1}P=Y \rightarrow U=R^{-1}D^{-1}R^{'}^{-1}P$

 Find displacement from loading:

 $\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}p\_{1}\\p\_{2}\end{matrix}\\\begin{matrix}p\_{3}\\p\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\12\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\18\end{matrix}\end{matrix}\right\} KU=P$ $ \left[\begin{matrix}\begin{matrix}1&0&0\\.5&1&0\\-1&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} .5&-1&0\end{matrix}&1\end{matrix}\right]\left[\begin{matrix}\begin{matrix}12&0&0\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0 &0&0\end{matrix}&0\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}p\_{1}\\p\_{2}\end{matrix}\\\begin{matrix}p\_{3}\\p\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\12\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\18\end{matrix}\end{matrix}\right\} R^{'}DRU=P$

 $\left[\begin{matrix}\begin{matrix}12&0&0\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0 &0&0\end{matrix}&0\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}Z\_{1}\\Z\_{2}\end{matrix}\\\begin{matrix}Z\_{3}\\Z\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\0\end{matrix}\\\begin{matrix}0\\6\end{matrix}\end{matrix}\right\} DRU=R^{'}^{-1}P=Z$

 $\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}y\_{1}\\y\_{2}\end{matrix}\\\begin{matrix}y\_{3}\\y\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\6\end{matrix}\end{matrix}\right\} RU=D^{-1}R^{'}^{-1}P=Y$

 $\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\6\end{matrix}\end{matrix}\right\} U=R^{-1}D^{-1}R^{'}^{-1}P$

 Find rigid mode:

 $\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}y\_{1}\\y\_{2}\end{matrix}\\\begin{matrix}y\_{3}\\y\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}0\\0\end{matrix}\\\begin{matrix}1\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0\\1\end{matrix}\end{matrix}\right\}$

 $\left[ \begin{matrix}\begin{matrix}1&.5&0\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0 &0& 0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}s\_{1}\\s\_{2}\end{matrix}\\\begin{matrix}s\_{3}\\s\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}1\\0\end{matrix}\\\begin{matrix}1\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}-.5\\1\end{matrix}\\\begin{matrix}0\\1\end{matrix}\end{matrix}\right\}$

 $\left[ \begin{matrix}\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0&0&0 \end{matrix}&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}1\\0\end{matrix}\\\begin{matrix}1\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}-1\\1\end{matrix}\\\begin{matrix}0\\1\end{matrix}\end{matrix}\right\}$

 Numerical example: A beam element

$$u\_{4}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{1} u\_{3}$$

$$K=\frac{EI}{L^{3}}\left[\begin{matrix}\begin{matrix}12&6L&-12\\6L&4L^{2}&-6L\\-12&-6L&12\end{matrix}&\begin{matrix}6L\\2L^{2}\\-6L\end{matrix}\\\begin{matrix}6L&2L^{2}&-6L\end{matrix}&4L^{2}\end{matrix}\right]$$

 Let $EI=1$ and $L=1$

 $K=\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]$ $\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1& 0\\0&0& 0\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\ \begin{matrix} 0&-1& 0 \end{matrix}&1\end{matrix}\right]$ $\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&0\end{matrix}\right]$

$$\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}12&0&0\\6&1&0\\-12&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 6&-1&0\end{matrix}&1\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&0\end{matrix}\right]$$

 Find displacement from loading:

$$\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}1&0&0\\.5&1&0\\-1&0&1\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} .5&-1&0\end{matrix}&1\end{matrix}\right]\left[\begin{matrix}\begin{matrix}12&0&0\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\\begin{matrix} 0 &0&0\end{matrix}&0\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&1\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&1\end{matrix}\right]$$

 $\left[\begin{matrix}\begin{matrix}12&6&-12\\6&4&-6\\-12&-6&12\end{matrix}&\begin{matrix}6\\2\\-6\end{matrix}\\\begin{matrix} 6 & 2& -6\end{matrix}&4\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}p\_{1}\\p\_{2}\end{matrix}\\\begin{matrix}p\_{3}\\p\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\12\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\12\end{matrix}\\\begin{matrix}-24\\18\end{matrix}\end{matrix}\right\}$

 $\left[ \begin{matrix}\begin{matrix}1&.5&-1\\0&1&0\\0&0&0\end{matrix}&\begin{matrix}.5\\-1\\0\end{matrix}\\\begin{matrix} 0 &0 & 0 \end{matrix}&0\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}\overbar{p}\_{1}\\\overbar{p}\_{2}\end{matrix}\\\begin{matrix}\overbar{p}\_{3}\\\overbar{p}\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\6\end{matrix}\end{matrix}\right\}$ $u\_{4}=\infty $

 Find rigid mode:

$ \left[\begin{matrix}\begin{matrix}1&.5\\0&1\end{matrix}&\begin{matrix}-1&.5\\0&-1\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}0\\0\end{matrix}\right\}$ $\left[\begin{matrix}1&.5\\0&1\end{matrix}\right]\left\{\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\right\}=-\left[\begin{matrix}-1&.5\\0&-1\end{matrix}\right]\left\{\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\right\}$

$ \left[\begin{matrix}\begin{matrix}1&0\\0&1\end{matrix}&\begin{matrix}-1&1\\0&-1\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}0\\0\end{matrix}\right\}$ $\left\{\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\right\}=-\left[\begin{matrix}-1&1\\0&-1\end{matrix}\right]\left\{\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=-\left[\begin{matrix}\begin{matrix}-1&1\\0&-1\end{matrix}\\\begin{matrix}-1&0\\0&-1\end{matrix}\end{matrix}\right]\left\{\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\right\}$

 Numerical example: A beam element with a dummy link

$$u\_{4}$$

$$u\_{2}$$

$$u\_{3}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{2}$$

$$u\_{1} u\_{2}$$

$$v\_{2}=u\_{2}$$

$$u\_{2}$$

$$K=\frac{EI}{L^{3}}\left[\begin{matrix}\begin{matrix}12&-12&6L\\-12&12&-6L\\6L&-6L&4L^{2}\end{matrix}&\begin{matrix}6L\\-6L\\2L^{2}\end{matrix}\\\begin{matrix}6L&-6L&2L^{2}\end{matrix}&4L^{2}\end{matrix}\right] $$

 Let $EI=1$ and $L=1$

 $K=\left[\begin{matrix}\begin{matrix}12&-12&6\\-12&12&-6\\6&-6&4\end{matrix}&\begin{matrix}6\\-6\\2\end{matrix}\\\begin{matrix} 6& -6& 2\end{matrix}&4\end{matrix}\right]$ $\left[\begin{matrix}\begin{matrix}1&-1&.5\\0&0&0\\0&0&1\end{matrix}&\begin{matrix}.5\\0\\-1\end{matrix}\\\begin{matrix} 0 &0&-1\end{matrix}&1\end{matrix}\right]$ $\left[ \begin{matrix}\begin{matrix}1&-1&.5\\0&+1& 0\\0& 0& 1\end{matrix}&\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\\\begin{matrix} 0&0 &-1\\ 0 &-1 &0 \end{matrix}&\begin{matrix} 1& 0 \\ 0& +1\end{matrix}\end{matrix}\right]$

$$\left[\begin{matrix}\begin{matrix}12&-12&6\\-12&13&-6\\6&-6&4\end{matrix}&\begin{matrix}6&0\\-6&-1\\0&0\end{matrix}\\\begin{matrix} 6& -6 & 4 \\ 0& -1 & 0 \end{matrix}& \begin{matrix}2& 0\\0& +1\end{matrix}\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}12&0& 0\\-12&1& 0\\6&0& 1\end{matrix}&\begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix} 6& 0 &-1\\ 0& -1&0\end{matrix}&\begin{matrix}1&0\\0&1\end{matrix}\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&-1&.5\\0&+1&0\\0&0&1\end{matrix}&\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\\\begin{matrix}0 &0 &0 \\0 &0 &0 \end{matrix}&\begin{matrix}0& 0\\0& 0\end{matrix}\end{matrix}\right]$$

 Find displacement from loading:

$$\left[\begin{matrix}\begin{matrix}12&-12&6\\-12&13&-6\\6&-6&4\end{matrix}&\begin{matrix}6&0\\-6&-1\\0&0\end{matrix}\\\begin{matrix} 6& -6 & 4 \\ 0& -1 & 0 \end{matrix}& \begin{matrix}2& 0\\0& +1\end{matrix}\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}1&0& 0\\-1&1& 0\\.5&0& 1\end{matrix}&\begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix} .5& 0 &-1\\ 0& -1&0\end{matrix}&\begin{matrix}1&0\\0&1\end{matrix}\end{matrix}\right]\left[\begin{matrix}\begin{matrix}12&0& 0\\0&1& 0\\0&0& 1\end{matrix}&\begin{matrix}0&0\\0&0\\0&0\end{matrix}\\\begin{matrix}0& 0 &0\\0&0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\end{matrix}\right]\left[ \begin{matrix}\begin{matrix}1&-1&.5\\0&+1&0\\0&0&1\end{matrix}&\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\\\begin{matrix}0 &0 &0 \\0 &0 &0 \end{matrix}&\begin{matrix}1& 0\\0& 1\end{matrix}\end{matrix}\right]$$

 $\left[\begin{matrix}\begin{matrix}12&-12&6\\-12&12&-6\\6&-6&4\end{matrix}&\begin{matrix}6\\-6\\2\end{matrix}\\\begin{matrix} 6& -6& 2\end{matrix}&4\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}p\_{1}\\p\_{2}\end{matrix}\\\begin{matrix}p\_{3}\\p\_{4}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\-24\end{matrix}\\\begin{matrix}12\\12\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}24\\-24\end{matrix}\\\begin{matrix}12\\18\end{matrix}\end{matrix}\right\}$

 $\left[ \begin{matrix}\begin{matrix}1&-1&.5\\0&+1&0\\0&0&1\end{matrix}&\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\\\begin{matrix}0 &0 &0 \\0 &0 &0 \end{matrix}&\begin{matrix}0& 0\\0& 0\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\\begin{matrix}u\_{4}\\u\_{2}\end{matrix}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}\overbar{p}\_{1}\\\overbar{p}\_{2}\end{matrix}\\\begin{matrix}\begin{matrix}\overbar{p}\_{3}\\\overbar{p}\_{4}\end{matrix}\\\overbar{\overbar{p}}\_{2}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\\begin{matrix}0\\0\end{matrix}\end{matrix}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}2\\0\end{matrix}\\\begin{matrix}0\\\begin{matrix}6\\0\end{matrix}\end{matrix}\end{matrix}\right\}$ $u\_{4}=\infty $

 Find rigid mode:

$ \left[\begin{matrix}\begin{matrix}1&-1&.5\\0&+1&0\\0&0&1\end{matrix}&\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\\begin{matrix}u\_{4}\\\overbar{u}\_{2}\end{matrix}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}0\\0\\0\end{matrix}\right\}$ $\left[\begin{matrix}1&-1&.5\\0&+1&0\\0&0&1\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\u\_{3}\end{matrix}\right\}=-\left[\begin{matrix}.5&0\\0&-1\\-1&0\end{matrix}\right]\left\{\begin{matrix}u\_{4}\\\overbar{u}\_{2}\end{matrix}\right\}$

$ \left[\begin{matrix}\begin{matrix}1&0&0\\0&+1&0\\0&0&1\end{matrix}&\begin{matrix}1&-1\\0&-1\\-1&0\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\\begin{matrix}u\_{4}\\\overbar{u}\_{2}\end{matrix}\end{matrix}\end{matrix}\right\}=\left\{\begin{matrix}0\\0\\0\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\u\_{3}\end{matrix}\right\}=-\left[\begin{matrix}1&-1\\0&-1\\-1&0\end{matrix}\right]\left\{\begin{matrix}u\_{4}\\\overbar{u}\_{2}\end{matrix}\right\}$ $\left\{\begin{matrix}\begin{matrix}\begin{matrix}u\_{1}\\u\_{2}\end{matrix}\\\begin{matrix}u\_{3}\\u\_{4}\end{matrix}\end{matrix}\\\overbar{u}\_{2}\end{matrix}\right\}=-\left[\begin{matrix}\begin{matrix}1&-1\\0&-1\\-1&0\end{matrix}\\\begin{matrix}-1&0\\0&-1\end{matrix}\end{matrix}\right]\left\{\begin{matrix}u\_{4}\\\overbar{u}\_{2}\end{matrix}\right\}$