範圍:拉氏轉換　班級：_____學號：_____姓名：_____

國立台灣海洋大學河海工程學系工程數學(二) 2B 班第一次大考解答

1. Define zero order Bessel function $J_0(t)$ which is one of the solution for

$$t\ddot{y}(t) + \dot{y}(t) + ty(t) = 0, \quad (1)$$

Solve $\mathcal{L}\{J_0(t)\}$. (10%)　解 1. $1/\sqrt{s^2+1}$

2. $J_0(t)$ can be expressed by

$$J_0(t) = \sum_{m=0}^{\infty} \frac{(-1)^m}{2^m (m!)^2} t^{2m}, \quad (2)$$

解 2. $\sum_{m=0}^{\infty} \frac{(-1)^m (2m)!}{2^m (m!)^2 s^{2m+1}}$

Determine the series form for $\mathcal{L}\{J_0(t)\}$. (10%)

3. Compare the solutions 1. with 2., and prove that they are the same. (10%) 解 3.略

4. By using the Laplace transform, solve the integral equation for $y(t)$

$$y(t) = t^3 + \int_0^t \sin(t-\tau) y(\tau) d\tau. \quad (3)$$ (10%)　解 4. $y(t) = t^3 + \frac{1}{20} t^5$

5. Given a function $f(t)$,

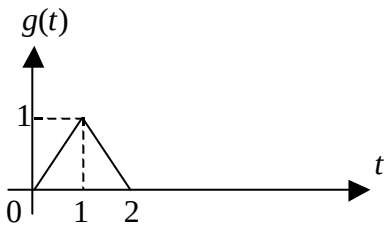
$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

determine $f(t) * f(t)$. (10%)

解 5. $f(t) * f(t) = g(t)$

6. Given a function $g(t)$,

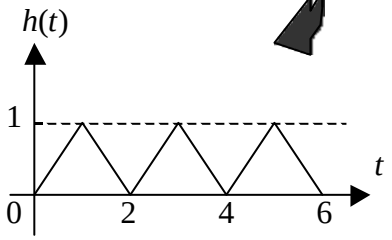
$$g(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$



determine $\mathcal{L}\{g(t)\}$. (10%)　解 6. $(1-e^{-s})^2 / s^2$

7. Given a function $h(t)$,

determine $\mathcal{L}\{h(t)\}$. (10%)



解 7. $\frac{1-e^{-s}}{s^2(1+e^{-s})}$

8. Explain the initial and final theorems in Laplace transform. (10%) 解 8. 略

9. Determine $\mathcal{L}\{\delta(t-3)\}$ and $\mathcal{L}\{H(t-3)\}$, where δ and H are the Dirac-Delta function and Heaviside function, respectively. (10%)　解 9. e^{-3s}, $(1/s)e^{-3s}$

10. Determine $\mathcal{L}^{-1}\left\{\ell n \frac{s+a}{s-a}\right\}$. (10%)　解 10. $\frac{1}{t}(e^{at} - e^{-at})$