89 下 工數(二) 第一次期中考 90/3/29

1. Find $L^{-1}\left\{\dfrac{s+3}{s(s^2+1)}e^{-3s}\right\}$ (12%)

   Hint : $L[\delta(t-t_0)]=e^{-st_0}$, $L[H(t-t_0)]=\dfrac{1}{s}e^{-st_0}$

2. For the following 1^{st} order O.D.E

$$\frac{dy}{dx}=\frac{2x+3y-4}{-3x+2y+3}\ , \qquad (2)$$

use the method specified below to solve the general solution.
(No credit for other methods.)

(a) Use a transformation, $(x,y)\rightarrow(X,Y)$, so that equation (2) becomes a homogeneous equation. Then solve this homogeneous equation with $Y=vX$ (9%)

(b) Solve equation (2) as an exact equation (if not exact, find the integrating factor). If the solution passes $(x, y) = (1, 1)$, write down the specific solution. (12%)

3. Find the general solutions for the following ODEs :

(a) $\dfrac{d^2y}{dx^2}+9y=x\cos x$

(16%)

(b) $\dfrac{d^3y}{dx^3}+\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+y=\sin 2x+\cos 3x$ (17%)

4. For the boundary value problem (20%)

$$\frac{d^2y}{dx^2}=2x,\quad y(0)=2,\quad y(1)=0,$$

(a) Formulate the Green's function $G(x, z)$:

   (a1) governing equation (a2) boundary condition

   (a3) jump condition (a4) continuity condition

(b) Find $G(x, z)$

(c) Write the solution $y(x)$ in terms of the Green's function $G(x, z)$.

5. Solve the initial value problem (14%)

$$y\frac{d^3y}{dt^3}+\left(3\frac{dy}{dt}+y\right)\frac{d^2y}{dt^2}+\left(\frac{dy}{dt}\right)^2=e^{-t},$$

$$y(0)=1,\quad \frac{dy}{dt}(0)=\frac{d^2y}{dt^2}(0)=0,$$

for $y(t)$.