海洋大學河海工程學系2002 工程數學 (一) 二 B 班期中考 OPEN BOOK

1. If $g(x) = \frac{1}{2}(x + \frac{3}{x})$, determine $g(g(g(\cdots g(2)))) =?$ and $g(g(g(\cdots g(-2)))) =?$ (5 %)
If $y_{n+1}(x) = \int_0^x 2s\, y_n(s)ds + 1$, find $y_1(x), y_2(x)$ and $y_3(x)$ (3 %) determine $\lim_{n\to\infty} y_n(s) =?$ (7 %) where $y_0(x) = 1$.

2. Solve the Euler-Cauchy equation (20 %) $x^2y''(x) - 3xy'(x) + 4y(x) = 0$, using (1). $y_1(x) = x^m$, (2). Solve another complementary solution by using L'Hospital's rule, (3). Solve another complementary solution by using $y_2(x) = y_1(x)u_1(x)$. (4). Solve another complementary solution by using Wronskian.

3. Explain the excitation, beating and resonance mathematically and physically. (10 %)

4. Given the following ordinary differential equations, (36 %)

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}, \tag{1}$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}, \tag{2}$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}, \tag{3}$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}, \tag{4}$$

plot the solutions and indicate the orthogonal relationships.

ODE	Equation.(1)	Equation.(2)	Equation.(3)	Equation.(4)
Separable (Y or N)				
Exact (Y or N)				
Integrating factor				
Homogeneous (Y or N)				
Solution				
Orthogonality (1,2,3,4)				

5. Given the following first order ODEs,

$$y_1'(x) + y_1(x) = 2\,cos(x)$$

$$y_2'(x) + y_2(x) = 2\,sin(x)$$

$$y_3'(x) + y_3(x) = e^{-x}$$

solve their particular solutions :
(a). Rewrite $y_1(x) = A\,cos(x - \alpha)$, where $A > 0$, $0 < \alpha < 2\pi$, determine A and α. (5%)
(b). Rewrite $y_2(x) = B\,sin(x - \beta)$, where $B > 0$, $0 < \beta < 2\pi$, determine B and β. (5%)
(e). Solve $y_3(x)$. (10 %)